Arts & Science 2D06

Mid-Year Exam **2018 December** **Name:** Solutions

Time allowed: 2.5 hours. No books or notes allowed. An electronic calculator may be used. *Complete solutions must be shown to obtain full marks for any of the problems.*

Formulae:

Solution for quadratic equation: $x = (-b \pm \sqrt{b^2 - 4ac})/2a$

Taylor series: $(1+x)^a \simeq 1 + ax$ for small x

Constant acceleration: $x = x_0 + v_0 t + \frac{1}{2}at^2$, $v = v_0 + at$, $v^2 = v_0^2 + 2ax$

$\sum F = ma$ $F_{AB} = -F_{BA}$ unit vectors (**i**, **j**, **k**)

Kinetic friction: force $f = \mu_k N$ Static friction: force $f \le \mu_s N$

Momentum: $\mathbf{p} = m\mathbf{v}$ Kinetic energy: $K = (1/2)mv^2$

Centripetal $a_c = v^2/r$ Rotational kinetic energy: $K = (1/2)I\omega^2$

Moment of inertia of disk: $I = (1/2)MR^2$ Moment of inertia of sphere: $I = (2/5)MR^2$

Gravitational potential energy: $U = mgy$

Spring potential energy: $U = (1/2)kx^2$

Elastic collisions: $v_1 = \frac{(m_1 - m_2)}{(m_1 + m_2)}u_1$, $v_2 = \frac{2m_1}{(m_1+m_2)}u_1$

Newton's universal law of gravity: $F_g = GMm/r^2$

Gamma factor: $\gamma = (1 - v^2/c^2)^{-1/2}$

Lorentz transformation: $x' = \gamma(x - vt)$, $t' = \gamma(t - \frac{v}{c^2}x)$.

Reverse Lorentz tranformation: $x = \gamma(x' + vt')$, $t = \gamma(t' + \frac{v}{c^2}x')$

Velocity addition: $u' = \frac{(u-v)}{(1-uv/c^2)}$ Momentum: $p = \gamma mv$

Rest-mass energy: $E = mc^2$ Kinetic energy: $K = (\gamma - 1)mc^2$

Total energy: $\sqrt{p^2c^2 + m^2c^4}$

Numerical Constants:

$c = 300,000$ km/sec $= 3.00 \times 10^8$ m/sec (speed of light)

$G = 6.67 \times 10^{-11}$ N m^2/kg^2 (Newton's law of gravity constant)

$g = 9.8$ m/s^2 (acceleration of gravity near surface of Earth)

$M_E = 5.98 \times 10^{24}$ kg (mass of Earth)

PART A: Do all of the following short questions.

A1. [3] A 5.0-kg object moving at 7.5 m/s bumps into a 3.5-kg object that is at rest. The collision is totally inelastic. What percentage of the system's initial kinetic energy is lost during the collision?

$$\vec{P}_i = \vec{P}_f$$

$$(5)(7.5) + (3.5)(0) = (5+3.5)V_f$$

$$8.5\,V_f = 37.5$$

$$V_f = 4.4 \text{ m/s}$$

$$\left.\begin{aligned} K_i &= \tfrac{1}{2}(5)(7.5)^2 = 140.6 \text{ J} \\ K_f &= \tfrac{1}{2}(8.5)(4.4)^2 = 82.3 \text{ J} \end{aligned}\right\} \quad \frac{K_i - K_f}{K_i} = \frac{58.3}{140.6} = 0.41$$

$\rightarrow$ $\underline{41\% \text{ of } K_i \text{ is lost.}}$

A2. [3] Suppose you have discovered a new planet, and have determined that the distance between its centre and its moon is 4.2×10^8 km. If the moon's (circular) orbit around this planet is found to be 1.8 Earth days, what is the mass of your planet?

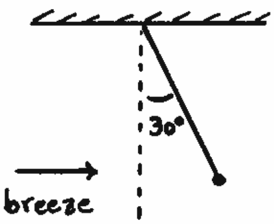
$$F = \frac{GM\cancel{m}}{r^{\cancel{2}}} = \frac{\cancel{m}v^2}{\cancel{r}}$$

$$M = \frac{v^2 r}{G}$$

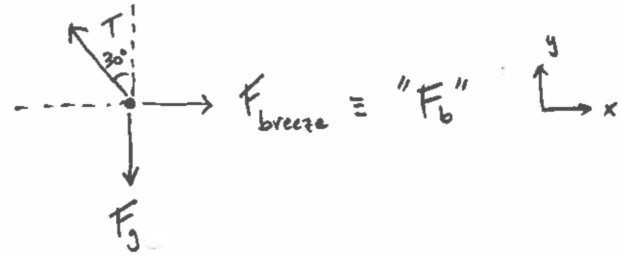
Now, $$V = \frac{2\pi r}{T} = \frac{2\pi(4.2\times10^8 \times 10^3 \text{ m})}{1.8 \text{ day}\left(\frac{24 \text{ hr}}{\text{day}}\right)\left(\frac{3600 \text{ s}}{\text{hr}}\right)} = 1.7\times10^7 \text{ m/s}$$

$$\therefore \quad M = \frac{(1.7\times10^7)^2(4.2\times10^{11})}{6.67\times10^{-11}} = \underline{1.8\times10^{36} \text{ kg}}$$

A3. [3] A 0.5-g ladybug hangs by a thread. Due to a horizontal breeze, the ladybug hangs without moving, as shown in the figure. Find the magnitudes of the force exerted by the breeze on the ladybug, and of the tension in the thread.



Free body diagram :



• y: $T\cos 30° = F_g \rightarrow T = \dfrac{mg}{\cos 30°} = \dfrac{(0.5\times10^{-3})(9.81)}{\cos 30°} = \underline{5.7\times10^{-3}\,N}$

• x: $T\sin 30° = F_b \rightarrow F_b = (5.7\times10^{-3})\sin 30° = \underline{2.9\times10^{-3}\,N}$

A4. [3] Consider an object whose angular velocity is given by the formula $\omega = Ct^2$, where C is a constant. What are its angular position and angular acceleration at $t =$ 3.5 seconds, if the angular position is zero at $t = 0$ and $C = 4$ rad/s^3?

• Angular position : $\omega = \dfrac{d\theta}{dt} = Ct^2 = 4t^2$

$$\int d\theta = \int 4t^2\,dt$$

$$\theta = \frac{4t^3}{3} + \text{constant} \quad (\text{constant} = 0 \text{ since } \theta = 0 \text{ @ } t=0)$$

$$\Rightarrow \theta(t) = \frac{4t^3}{3} \rightarrow \theta(3.5) = \frac{4(3.5)^3}{3} = \underline{57.2 \text{ radians}}$$

• Angular acceleration : $\alpha = \dfrac{d\omega}{dt} = 4\dfrac{d(t^2)}{dt} = 8t$

$$\rightarrow \alpha(3.5) = 8(3.5) = \underline{28 \text{ rad/s}^2}$$

A5. [3] A sphere of mass 1.0 kg, radius 0.50 m, and linear/translational speed of 16 m/s rolls without slipping on a horizontal surface, and approaches an incline of slope 30°. How far up the incline will the sphere roll?

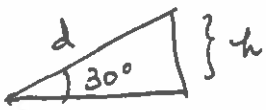
- Initial energy : $E_0 = K + K_{rot} = \frac{1}{2}(1)(16)^2 + \frac{1}{2}\left(\frac{2}{5}\cdot 1 \cdot \cancel{0.5^2}\right)\frac{16^2}{\cancel{0.5^2}}$

  $(\omega = \frac{v}{R})$

  $= 128 + 51.2 = 179.2 \text{ J}$

- Final energy : $E_f = U = mgh = mgd\sin 30°$

  $= (1)(9.81)(\sin 30°)d = 4.9\, d$



$\therefore \quad E_0 = E_f \Rightarrow 179.2 = 4.9\, d$

$\underline{d = 36.6 \text{ m}}$

A6. [3] Choose a topic or subtopic from Term 1 that you found especially interesting; it can be from the physics content or the historical/philosophical material. Using just the space below, explain your choice.

A7. [3] Suppose that in your reference frame, two firecrackers go off 200 meters apart and 2 seconds apart. Is this time interval a proper time? Explain your answer. Is there any other reference frame in which the time interval between the events is a proper time? If so, what is the speed of this frame relative to yours?

- The proper time is measured by one clock present at both events ⇒ both events must happen at the same place.

  $\therefore$ $\Delta t = 2$ seconds is not a proper time, since $\Delta x = 200\text{ m} \neq 0$.

- The frame in which both events occur at the same place ($\Delta x' = 0$) gives the proper time between them.

- Use $x' = \gamma(x - vt) \rightarrow \Delta x' = \gamma(\Delta x - v\Delta t) = 0$

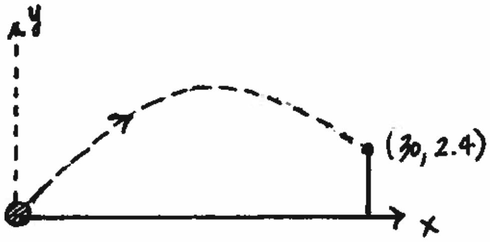
  Since $\gamma \neq 0$ always,

  $$\Delta x - v\Delta t = 0$$

  $$V = \frac{\Delta x}{\Delta t} = \frac{200}{2} = \underline{100 \text{ m/s}}$$

PART B: Do ANY 3 of the following 5 questions (your choice; 5 marks each; please indicate clearly your choices – otherwise, the first three attempted ones will be marked).

B1. [5] A soccer ball is kicked at an angle of 25° and hits the crossbar at $(x, y) = (30, 2.4)$ meters relative to its starting location (see figure). Find the initial speed of the soccer ball.



$\vec{V_o}$: $V_{ox} = V_o \cos\theta = 0.91\, V_o$

$V_{oy} = V_o \sin\theta = 0.42\, V_o$

$$X = X_o + V_{ox}t = 0.91 V_o t = 30 \Rightarrow t = \frac{33}{V_o} \qquad (1)$$

$$y = y_o + V_{oy}t - \frac{g}{2}t^2 = 2.4$$

$$0.42\, V_o t - 4.9 t^2 = 2.4 \qquad (2)$$

Sub (1) in (2) : $0.42\, \cancel{V_o}\left(\frac{33}{\cancel{V_o}}\right) - 4.9\left(\frac{33}{V_o}\right)^2 = 2.4$

$$13.9 - \frac{5336}{V_o^2} = 2.4$$

$$11.5\, V_o^2 = 5336 \longrightarrow \underline{V_o = 21.5 \text{ m/s}}$$

B2. [5] A 5.5-g bullet, flying horizontally at 340 m/s, strikes a 0.9-kg wooden block at rest on a horizontal table. The bullet passes through the block and exits with a speed of 120 m/s. The block then slides a distance of 50 cm along the table from its initial position. (Assume we can neglect the block's motion while the bullet was in the block.)

(a) What is the block's kinetic energy immediately after the bullet passes through?

$$\vec{P}_o = \vec{P}_f \rightarrow (0.0055)(340) = (0.0055)(120) + 0.9\,V_B$$

$$1.87 = 0.66 + 0.9\,V_B$$

$$V_B = 1.34 \text{ m/s}$$

$$\therefore K_{f,B} = \frac{1}{2}m_B V_B^2 = \frac{1}{2}(0.9)(1.34)^2$$

$$\rightarrow \underline{K_{f,B} = 0.81\,\text{J}}$$

(b) What is the coefficient of kinetic friction between the block and the table?

Work energy theorem : $W_f = \Delta K = K_f - K_i$

$$-fd = -\frac{1}{2}mV_i^2 = -\frac{1}{2}mV_B^2$$

$$\mu_k \cancel{m_B} g d = \frac{1}{2}\cancel{m_B} V_B^2$$

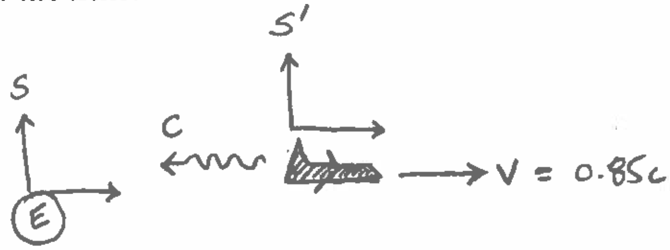
$$\mu_k = \frac{V_i^2}{2gd} = \frac{(1.34)^2}{2(9.81)(0.5)}$$

$$\rightarrow \underline{\mu_k = 0.18}$$

B3. [5] A rocket ship flying away from Earth at 0.85c emits light signals back towards Earth, once per second in its own reference frame.

(a) In Earth's reference frame, how far does the ship travel between the emission of one signal and the next?

(b) Again in Earth's reference frame, how much time goes by between the emission of one signal and the next?



$$\Rightarrow \gamma = \frac{1}{\sqrt{1-0.85^2}} = 1.9$$

(a) In Earth's frame: $\Delta t = \gamma \Delta t' = (1.9)(1) = 1.9$ sec.

$\therefore$ distance $= (0.85c)(1.9) = \underline{4.8 \times 10^8 \text{ m}}$

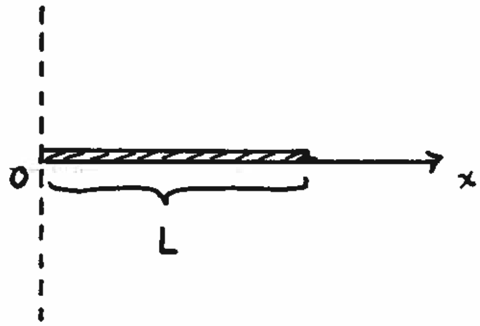
(b) From part (a): $\underline{\Delta t = \gamma \Delta t' = 1.9 \text{ sec}}$

or, use Lorentz transformation:

$$\Delta t = \gamma\left(\Delta t' + \cancelto{0}{\frac{v}{c^2}\Delta x'}\right)$$

$$= \gamma \Delta t' = 1.9 \text{ sec.}$$

B4. [5] A thin stick of length L is placed with one end at $x = 0$, as shown in the figure. The rod's linear density λ (*i.e.*, mass per unit length) is given by $\lambda = Ax^2$, where A is a constant. What is the stick's moment of inertia for rotations about an axis through $x = 0$ and perpendicular to the stick?



$$I = \int r^2 dm = \int x^2 dm$$

$$dm = \lambda(x)\,dx = Ax^2 dx$$

$$\therefore \quad I = \int_0^L Ax^4\,dx$$

$$= A \int_0^L x^4\,dx$$

$$= A \cdot \frac{x^5}{5}\Bigg|_0^L$$

$$= A\left(\frac{L^5}{5} - 0\right)$$

$$\Rightarrow \quad \underline{I = \frac{1}{5} A L^5}$$

B5. [5] Consider the statement, "Einstein's Special Relativity replaces Newtonian Mechanics," and discuss in what sense(s) you think this statement is reasonable and not reasonable (discuss one of each, at least).

Marks awarded will depend on content, organization, grammar, clarity, and the quality of your writing.

S.R. removes absolute time and distance, which are required in N.M. (totally different picture of space + time)

Predictions of S.R. agree w/ experiments and observations; for large velocities, N.M. predicts wrongly.

At low speeds, N.M. is plenty good enough.

In the low-speed limit, S.R. equations reduce to N.M. equations → "contains" better than "replaces"

... etc.

36 marks total (Happy Holidays!)

Extra Page for Scratch Work