Arts & Science 2D06

Mid-Year Exam 2017 December Name: Solutions

Time allowed: 2.5 hours. No books or notes allowed. An electronic calculator may be used. *Complete solutions must be shown to obtain full marks for any of the problems.*

Formulae:

Solution for quadratic equation: $x = (-b \pm \sqrt{b^2 - 4ac})/2a$

Taylor series: $(1+x)^a \simeq 1 + ax$ for small x

Constant acceleration: $x = x_0 + v_0 t + \frac{1}{2}at^2$, $v = v_0 + at$, $v^2 = v_0^2 + 2ax$

$\sum F = ma$ $F_{AB} = -F_{BA}$ unit vectors $(\mathbf{i}, \mathbf{j}, \mathbf{k})$

Kinetic friction: force $f = \mu_k N$ Static friction: force $f \leq \mu_s N$

Momentum: $\mathbf{p} = m\mathbf{v}$ Kinetic energy: $K = (1/2)mv^2$

Centripetal $a_c = v^2/r$ Rotational kinetic energy: $K = (1/2)I\omega^2$

Moment of inertia of disk: $I = (1/2)MR^2$ Moment of inertia of sphere: $I = (2/5)MR^2$

Gravitational potential energy: $U = mgy$

Spring potential energy: $U = (1/2)kx^2$

Elastic collisions: $v_1 = \frac{(m_1 - m_2)}{(m_1 + m_2)}u_1$, $v_2 = \frac{2m_1}{(m_1 + m_2)}u_1$

Newton's universal law of gravity: $F_g = GMm/r^2$

Gamma factor: $\gamma = (1 - v^2/c^2)^{-1/2}$

Lorentz transformation: $x' = \gamma(x - vt)$, $t' = \gamma(t - \frac{v}{c^2}x)$.

Reverse Lorentz tranformation: $x = \gamma(x' + vt')$, $t = \gamma(t' + \frac{v}{c^2}x')$

Velocity addition: $u' = \frac{(u - v)}{(1 - uv/c^2)}$ Momentum: $p = \gamma mv$

Rest-mass energy: $E = mc^2$ Kinetic energy: $K = (\gamma - 1)mc^2$

Total energy: $\sqrt{p^2c^2 + m^2c^4}$

Numerical Constants:

$c = 300,000$ km/sec $= 3.00 \times 10^8$ m/sec (speed of light)

$g = 9.8$ m/s^2 (acceleration of gravity near surface of Earth)

$m_p = 1.67 \times 10^{-27}$ kg (mass of the proton)

PART A: Do all of the following short questions.

A1. [3] A warehouse for parking spaceships is 80 meters long. How fast should a 160-m-long spaceship move in order to fit (for a short moment) in the warehouse?

$L_o = 160\,m$, $L = 80\,m$, $V = ?$

$L = \frac{L_o}{\gamma} \rightarrow \sqrt{1 - V^2/c^2} = \frac{L}{L_o}$

$1 - V^2/c^2 = \left(\frac{L}{L_o}\right)^2$

$\frac{V}{c} = \sqrt{1 - \left(\frac{L}{L_o}\right)^2} = \sqrt{1 - \left(\frac{80}{160}\right)^2} = 0.87$

$\therefore \underline{V = 0.87c \quad (\text{or } V = 2.6 \times 10^8\ m/s)}$

A2. [3] A train ~~with~~ accelerates uniformly at 0.6 m/s^2 for 20 seconds, starting from a velocity v_i. During this time, it travels a distance of 600 m. Find the train's final velocity.

$a = 0.6\ m/s^2$, $t = 20\ s$, $\Delta x = 600\ m$

$x = x_i + v_i t + \frac{a}{2} t^2 \rightarrow \Delta x = v_i t + \frac{a}{2} t^2$

$V_i = \frac{\Delta x - a/2\, t^2}{t} = \frac{600 - \frac{0.6}{2}(20)^2}{20} = 24\ m/s$

$\therefore \quad V_f = V_i + at = 24 + (0.6)(20) = \underline{36\ m/s}$

A3. [3] The circular base of a merry-go-round has a mass of 500 kg and a radius of 5 m. Approximating this base as a thin solid disk, find its rotational kinetic energy, if a point on the disk's outermost edge has a centripetal acceleration of 0.5 m/s^2.

$$K_{rot} = \frac{1}{2} I \omega^2 \quad , \quad I = \frac{1}{2} M R^2 = \frac{1}{2}(500)(5)^2 = 6250 \text{ kg}\cdot\text{m}^2$$

$$\omega = \frac{V}{R} \, , \quad V = \sqrt{R a_c}$$

$$\therefore \quad \omega = \frac{\sqrt{R a_c}}{R} = \frac{\sqrt{a_c}}{\sqrt{R}} = \frac{\sqrt{0.5}}{\sqrt{5}} = 0.32 \text{ s}^{-1}$$

$$\therefore \quad K_{rot} = \frac{1}{2}(6250)(0.32)^2 = \underline{320 \text{ J}}$$

A4. [3] A particle moving along the x axis experiences a force given by $F(x) = -Ax^2$, where A is a constant. Find the amount of work done by this force as the particle travels between two positions x_1 to x_2. (Express your answer in terms of x_1 ~~to~~ and x_2.) If this is the only force acting on the particle, what is the change in the particle's kinetic energy between these same positions?

$$F(x) = -Ax^2$$

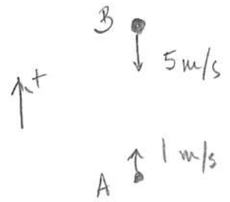
$$W = \int_{x_1}^{x_2} F(x)\,dx = -A \int_{x_1}^{x_2} x^2\,dx = -A \left. \frac{x^3}{3} \right|_{x_1}^{x_2}$$

$$\underline{W = A\left(\frac{x_1^3}{3} - \frac{x_2^3}{3}\right)}$$

• Work-energy theorem : $\Delta K = W = \underline{A\left(\frac{x_1^3}{3} - \frac{x_2^3}{3}\right)}$

A5. [3] Consider two cars: car A is traveling north at 1 m/s, while car B is traveling south at 5 m/s. After they collide, car A has a velocity of 2 m/s, going south. What kind of collision occurred (*i.e.*, elastic, inelastic, or totally inelastic), if the cars have the same mass?

Conservation of momentum : $m_A V_{A_0} + m_B V_{B_0} = m_A V_{Af} + m_B V_{Bf}$



$m_A = m_B \Rightarrow$ $V_{A_0} + V_{B_0} = V_{Af} + V_{Bf}$

$1 - 5 = -2 + V_{Bf}$

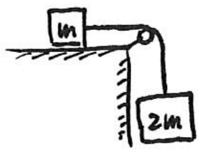
$V_{Bf} = -2$ m/s

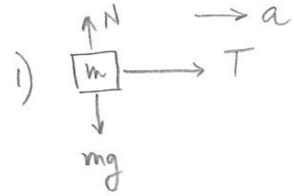
Since $V_{Af} = V_{Bf}$, the cars are stuck together

$\Rightarrow$ the collision was totally inelastic.

A6. [3] Choose a topic or subtopic from Term 1 that you found especially interesting; it can be from the physics content or the historical/philosophical material. Using just the space below, explain your choice.

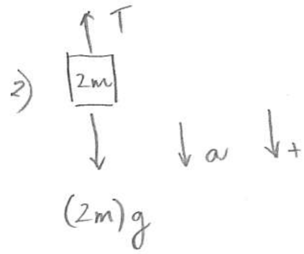
A7. [3] A mass m sits on a horizontal, frictionless table, and is connected by a string to a second, hanging mass $2m$, as shown in the figure. Find the acceleration of the system and the tension in the string. (Ignore the mass of the string and the friction related to the pulley.)





$\Downarrow$

$T = ma$



$\Downarrow$

$2mg - T = 2ma$

$\therefore \quad 2T = 2mg - T$

$\Rightarrow \quad \underline{T = \frac{2}{3} mg}$

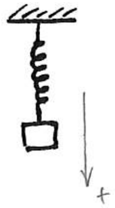
and $\quad \frac{2mg}{3} = ma$

$\Rightarrow \quad \underline{a = \frac{2g}{3}}$

PART B: Do ANY 3 of the following 5 questions (your choice; 5 marks each; please indicate clearly your choices – otherwise, the first three attempted ones will be marked).

B1. [5] Consider a mass m that is hanging from a spring of spring constant k, as shown in the figure below.

(a) How far is the spring stretched by the mass (from the spring's unstretched/relaxed position)?



in equilibrium,

$$\vec{F}_{net} = 0$$

$$\vec{F}_g + \vec{F}_{spring} = 0$$

$$mg - kx = 0 \quad \Rightarrow \quad x = \frac{mg}{k}$$

(b) Suppose that you push up on the mass and lift it at constant speed until the spring returns to its unstretched length. Using your answer for part (a), determine how much work you have done against gravity.

Against gravity alone, applied force : $F_a = -F_g = -mg$ (up)

$$\therefore \; W_g = F \Delta x \cos\theta = F_a (x_f - x_i)(1)$$

$$= (-mg)(0 - x) \qquad \text{(let unstretched position be } x=0\text{)}$$

$$= mgx = \frac{m^2 g^2}{k}$$

(c) How much work have you done against the spring? Also, find the *total* work that you have exerted in pushing the mass.

Against the spring alone, applied force : $F_a = -F_{spring} = -(-kx) = kx$

$$\therefore \; W_s = \int_x^0 F_a(x)\,dx = \int_x^0 kx\,dx = k \left.\frac{x^2}{2}\right|_x^0 = -\frac{kx^2}{2} = -\frac{k}{2}\left(\frac{mg}{k}\right)^2$$

$$\Rightarrow W_s = -\frac{m^2 g^2}{2k}$$

and $$W_{total} = W_g + W_s = \frac{m^2g^2}{k} - \frac{m^2g^2}{2k} = \frac{m^2g^2}{2k}$$

B2. [5] A book and a hoop ($I = MR^2$) are both released from rest at the top of a ramp, which is at a vertical height h above the ground. The ramp is inclined at an angle of $30°$ with respect to the ground. The book slides down the ramp, while the ring rolls down, without slipping. Through careful measurements, you discover that both objects have the same speed by the time they reach the bottom of the ramp. (Do not ignore friction)

(a) Find this final speed. Express your answer in terms of h (*i.e.*, leave h in as a parameter).

• hoop :

$$E_f = E_i$$

$$\frac{1}{2}I\omega^2 + \frac{1}{2}Mv^2 = Mgh$$

$$\frac{1}{2}\left(\cancel{M}\cancel{R^2}\right)\left(\frac{v^2}{\cancel{R^2}}\right) + \frac{1}{2}\cancel{M}v^2 = \cancel{M}gh$$

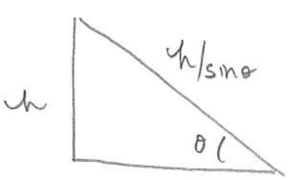
$$\frac{1}{2}v^2 + \frac{1}{2}v^2 = gh$$

$$v^2 = gh$$

$$v = \sqrt{gh}$$

(b) Find the coefficient of kinetic friction between the ramp's surface and the book.

• book :



$$E_i = E_f + |W_{fr}|$$

$$mgh = \frac{1}{2}mv^2 + \left|F_{fr}\cdot\left(\frac{h}{\sin\theta}\right)\cos\pi\right|$$

$$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2 + \mu_k(\cancel{m}g\cos\theta)\left(\frac{h}{\sin\theta}\right)(+1)$$

$$\mu_k\left(\frac{gh}{\tan\theta}\right) = gh - \frac{1}{2}v^2$$

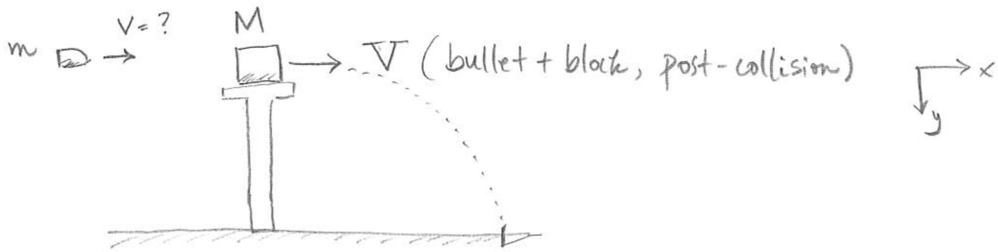
$$\mu_k = \left(\frac{\tan\theta}{\cancel{gh}}\right)\left[\cancel{gh} - \frac{1}{2}(\cancel{gh})\right]$$

$$= (\tan 30°)\left(1 - \frac{1}{2}\right)$$

$$= 0.29$$

B3. [5] One way to determine a bullet's speed is to shoot it horizontally into a block of wood sitting on a post, which is standing vertically. The block stops the bullet, falls from the post, and lands on the ground. We then measure the horizontal distance between the landing point and the post, and calculate the bullet's speed.

Now suppose that a bullet of mass 10 g is fired into a 3.5-kg block, which is on a 1.8-m-tall post. The block is observed to land 2.1 m from the bottom of the post. What is the bullet's speed? (Ignore air resistance and the friction between the block and the post.)



- bullet into block : totally inelastic collision

$$\vec{p}_i = \vec{p}_f$$

$$p_{ix} = p_{fx}$$

$$mv = (m+M)V \Rightarrow V = \frac{mv}{m+M} = \frac{(0.01)(v)}{0.01+3.5} = (2.8\times10^{-3})v$$

- After collision, System has constant speed in x-direction $= V$

$$\Rightarrow x = x_0 + Vt \Rightarrow V = \frac{\Delta x}{t} = \frac{2.1}{t} \quad \text{———} \quad (*)$$

- to find t, use y-direction :

$$y = y_0 + \cancelto{0}{v_{0y}t} + \frac{g}{2}t^2$$

$$t = \sqrt{\frac{2\Delta y}{g}} = \sqrt{\frac{2\cdot1.8}{9.81}} = 0.61 \text{ sec}$$

- Sub this in (*) :

$$V = \frac{2.1}{0.61} = 3.4$$

$$2.8\times10^{-3}v = 3.4$$

$$\Rightarrow \underline{v = 1230 \text{ m/s}}$$

B4. [5] Consider two events, labeled A and B, say. Suppose that in some reference frame, event B happens 2 microseconds after event A; and that furthermore event B takes place 1.5 km from event A (*i.e.*, let $\Delta x = x_B - x_A = 1.5$ km). (1 microsecond = 10^{-6} second)

(a) How fast must an observer be moving along the $+x$ axis so that these two events occur *simultaneously* from her/his point of view?

(b) Is it possible for event B to happen *before* event A for some observer? (Explain/derive your answer.)

(a) Is it possible for an observer to see events A and B happen *at the same place*? Why or why not?

(a) "Simultaneously" $\Rightarrow \Delta t' = t_B' - t_A' = 0$; let $\Delta t = t_B - t_A = 2\ \mu\text{sec}$

$$\Delta t' = \gamma\left(\Delta t - \frac{v}{c^2}\Delta x\right) = 0$$

$$\frac{v}{c^2}\Delta x = \Delta t$$

$$v = c^2\frac{\Delta t}{\Delta x} = (3\times10^8)^2\left[\frac{2\times10^{-6}}{1.5\times10^3}\right] = 1.2\times10^8\ \text{m/s}$$

(or $0.4c$)

(b) "B" before "A" $\Rightarrow t_B < t_A \Rightarrow \Delta t' = t_B' - t_A' < 0$

$$\therefore\ \Delta t - \frac{v}{c^2}\Delta x < 0 \text{, for some } v$$

$$\frac{v}{c^2}\Delta x > \Delta t$$

$$v > \frac{\Delta t}{\Delta x}c^2$$

$$v > 0.4c$$

$\therefore$ yes, it's possible.

(c) "at the same place" $\Rightarrow \Delta x' = x_B' - x_A' = 0$

$$\Delta x' = \gamma(\Delta x - v\Delta t) = 0$$

$$\Delta x - v\Delta t = 0 \Rightarrow v = \frac{\Delta x}{\Delta t} = \frac{1.5\times10^3}{2\times10^{-6}} = 7.5\times10^8\ \text{m/s}$$

So $v = 2.5c > c$, which is NOT possible.

B5. [5] So far, we have studied physics mainly by emphasizing the ideas/theories that have been developed toward understanding natural phenomena. Yet historically experiments have been important in directing the evolution of these ideas – as should be the case in any "healthy" science. Discuss two such experiments, and their impact on the development of theories discussed this term. (Note: "Experiments" may include observations, *e.g.*, in astronomy.)

Marks awarded will depend on content, organization, grammar, clarity, and the quality of your writing.

Some examples :

- Galileo – incline plane : falling things have equal acceleration ; kinematics
- Michelson - Morley : absence of aether ; special relativity
- Galileo - telescope observations : hints of universality of terrestrial physical laws ; heliocentrism.

... etc.

36

36 marks total

(Happy Holidays!)

Extra Page for Scratch Work