Arts and Science 2D06 - 2016 Mid-Year Exam - Solutions

PART A

A1. $K = (\gamma - 1)mc^2 \Rightarrow \gamma = \dfrac{K + mc^2}{mc^2} \Rightarrow \dfrac{1}{\sqrt{1 - v^2/c^2}} = \dfrac{K + mc^2}{mc^2}$

$$1 - \frac{v^2}{c^2} = \left(\frac{mc^2}{K + mc^2}\right)^2 \Rightarrow v = \sqrt{1 - \left(\frac{mc^2}{K + mc^2}\right)^2}\, c$$

$$\therefore v = \sqrt{1 - \left[\frac{(1.67\times10^{-27})(9\times10^{16})}{(1.6\times10^{-7}) + (1.67\times10^{-27})(9\times10^{16})}\right]^2}\, c$$

$$\Rightarrow \underline{v = 0.9999996\ c}$$

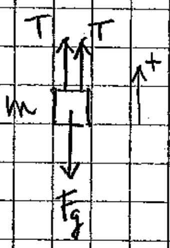
A2. $169\ \text{km/h} = 169\ \dfrac{\text{km}}{\text{h}}\left(\dfrac{1000\ \text{m}}{\text{km}}\right)\left(\dfrac{1\ \text{hr}}{3600\ \text{s}}\right) = 46.9\ \text{m/s}$

$$x = v_{0x}t \Rightarrow t = \frac{x}{v_{0x}} = \frac{20\ \text{m}}{46.9\ \text{m/s}} = 0.4\ \text{s}$$

$$y = y_0 + v_{0y}t + \frac{a}{2}t^2 \quad ; \quad \text{let } y_0 = 0,\ a = +g \quad (v_{0y} = 0)$$

$$\Rightarrow y = \frac{9.8}{2}(0.4)^2 = \underline{0.78\ \text{m}}$$

A3.



$$\Sigma F = 2T - F_g = ma_c$$

$$2T - mg = m\frac{v^2}{R}$$

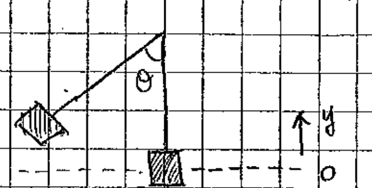
$$T = \frac{1}{2}m\left(\frac{v^2}{R} + g\right)$$

$$\therefore T = \frac{1}{2}(70)\left(\frac{5^2}{6} + 9.8\right) = \underline{489\ \text{N}}$$

A4. $E_0 = U_0 = mgh = (2.2\times10^{-3})(9.81)(25) = 0.54\ \text{J}$

$E_f = K_f = \frac{1}{2}mv^2 = \frac{1}{2}(2.2\times10^{-3})(10)^2 = 0.11\ \text{J}$

$\therefore$ 0.43 J were lost $\Rightarrow$ fraction $= \dfrac{0.43}{0.54} = \underline{0.80}$



Before release : $E_0 = mgh_0 = mgL(1-\cos\theta)$

1) Just before collision : Find velocity

$$E_1 = \frac{1}{2}mv_1^2$$

$$E_0 = E_1 \Rightarrow \frac{1}{2}\cancel{m}v_1^2 = \cancel{m}gL(1-\cos\theta)$$

$$v_1 = \sqrt{2gL(1-\cos\theta)}$$

2) right after collision : find velocity

$$P_1 = P_2 \Rightarrow mv_1 = 2mv_2 \Rightarrow v_2 = \frac{1}{2}\sqrt{2gL(1-\cos\theta)}$$

3) blocks rise to max. height : $(2\cancel{m})gh_3 = \cancel{\tfrac{1}{2}}(\cancel{2m})\left(\frac{1}{2}\sqrt{2gL(1-\cos\theta)}\right)^2$

$$\Rightarrow h_3 = \frac{1}{\cancel{2g}} \cdot \frac{\cancel{2g}L(1-\cos\theta)}{4} = \underline{\frac{L(1-\cos\theta)}{4}}$$

A6. ______

A7. $\omega \propto \sqrt{t}$; $\omega = \frac{d\theta}{dt} \Rightarrow \theta = \int \omega\, dt \quad \therefore \theta \propto t^{3/2}$

$$\therefore \left.\begin{aligned} \theta_1 &\propto 1^{3/2} \\ \theta_3 &\propto 3^{3/2} \end{aligned}\right\} \frac{\theta_3}{\theta_1} = \frac{3^{3/2}}{1} = 5.2$$

$$\therefore \underline{\theta_3 = (5.2)\,\theta_1}$$

PART B

31. (a) $a = \frac{dv}{dt} \Rightarrow v = \int a\,dt = \int_0^1 a_0\left(1 - \frac{t^2}{2}\right) dt$ (@ $t=0$, $v=0$)

$$\Rightarrow \quad V = a_0 \left[t - \frac{t^3}{6} \right]_0^1 = (15)\left(1 - \frac{1}{6}\right) = \underline{12.5 \text{ m/s}}$$

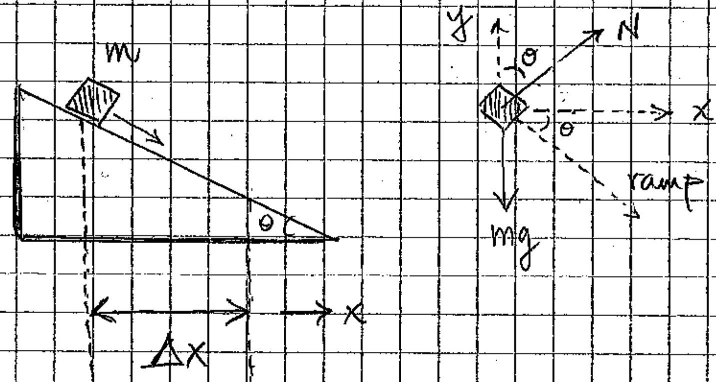
(b) Velocity after one hour = Velocity at $t = 3$ seconds

$$\therefore \quad V = a_0 \left[t - \frac{t^3}{6} \right]_0^3 = (15)\left(3 - \frac{27}{6}\right) = \underline{-22.5 \text{ m/s}}$$

(c) $x = \int v\,dt = a_0 \int_0^3 \left(t - \frac{t^3}{6}\right) dt = a_0 \left(\frac{t^2}{2} - \frac{t^4}{24}\right)_0^3$ (@ $t=0$, $x=0$)

$$\therefore \quad X = (15)\left(\frac{9}{2} - \frac{81}{24}\right) = \underline{16.9 \text{ m}}$$

32.



(a) X-direction : $N \sin\theta = m a_x$

$$(\cancel{m}g\cos\theta)\sin\theta = \cancel{m}a_x$$

$$a_x = g\sin\theta\cos\theta$$

$$a_x = \frac{g}{2}\sin 2\theta \quad \text{(constant)}$$

$$\therefore \quad X = X_0 + \cancel{V_{0x}}t + \frac{a_x}{2}t^2 \quad \text{(assume } V_{0x} = 0\text{)}$$

$$\Rightarrow \quad t = \left(\frac{4 \cdot \Delta X}{g\sin 2\theta}\right)^{1/2}$$

$\therefore$ for a given ΔX : t is minimized if $\sin 2\theta = 1 \Rightarrow \underline{\theta = \pi/4}$

(b) $t = 2\sqrt{\frac{\Delta X}{g}}$

B3. @ top : $\sum F = m a_c$

$$T + mg = \frac{m v_{top}^2}{L} \quad \downarrow +$$

String is no longer taut when $T = 0$

$\therefore$ Speed at top must be at least:

$$\frac{\cancel{m} v_{top}^2}{L} = \cancel{m} g$$

$$v_{top} = \sqrt{gL}$$

Conservation of energy : $E_{bottom} = E_{top}$

let $y = 0$ @ bottom $\Rightarrow$ $K_{bottom} = K_{top} + U_{top}$

$$\frac{1}{2}\cancel{m} v_b^2 = \frac{1}{2}\cancel{m} v_{top}^2 + \cancel{m} g(2L)$$

$$v_b = \sqrt{v_{top}^2 + 4gL}$$

So $v_b = \sqrt{gL + 4gL} = \underline{\sqrt{5gL}}$, as required

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B4. Let L_0 = proper length of spaceship

1) measurement of spaceship #1 : $L_1 = \frac{L_0}{\gamma_1} = \sqrt{1 - \frac{(0.8c)^2}{c^2}}\, L_0$

$$\Rightarrow L_1 = 0.6 L_0$$

2) measurement of spaceship #2 : $L_2 = \frac{L_0}{\gamma_2} = 1.5 L_1$

$\therefore \sqrt{1 - \frac{v_2^2}{c^2}}\, \cancel{L_0} = 1.5(0.6 \cancel{L_0}) \Rightarrow \underline{v_2 = 0.44c}$

B5. 1) heavenly things made of aether $\neq$ "Earth stuff"
aether obeys different laws than "Earth laws" $\}\rightarrow$ Both changed w/ Galileo / Newton

2) Aether's rest frame $\Leftrightarrow$ absolute space ; only frame in which speed of light $= 3 \times 10^8$ m/s $\}\rightarrow$ aether made redundant, then abolished by Einstein.