Name ______________________________

Student No. ________________________

ARTS & SCIENCE 2D06 DR. A.A. CHEN

DAY CLASS
DURATION OF EXAMINATION: 3 hours
MCMASTER UNIVERSITY FINAL EXAMINATION April 2008

THIS EXAMINATION PAPER INCLUDES **19** PAGES AND **19** QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

Special Instructions:

— Use of any electronic calculator is allowed.
— Write all answers on the exam paper itself.
— At the back there is a scratch page and a memory sheet of formulae. Hand this page in with the rest of your exam paper.
— Mark values are given beside each question.
— Complete steps must be explicitly written down to earn full marks on any question.

Question	Mark	Max	Question	Mark	Max
A1		3	B1		5
A2		3	B2		5
A3		3	B3		5
A4		3	B4		5
A5		3	B5		5
A6		3	B6		5
A7		3	B7		5
A8		3	B8		5
A9		3	B9		5
A10		5	TOTAL		57

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PART A: Do all of the following questions in part A.

[3] A1. A 60 kg swimmer climbs onto a styrofoam block, which is floating in the water and whose density is 150 kg/m^3. If the water level rises right to the top of the styrofoam block when the swimmer is on it, calculate the block's volume.

[3] A2. A sinusoidal wave travelling in the positive x-direction has a displacement given by y = (2.5 cm)sin($2.5x - 134t$), where x is in meters and t is in seconds. What are the frequency, wavelength, and speed of this wave?

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[3] A3. Consider a large tank filled with water to a height of 10 meters. Suppose that the tank is pressurized from the top with a pressure of two times that of the atmosphere. What is the speed of the water emerging from a small hole at the base of the tank? (Assume that the area of the top of the tank is much bigger than the area of the hole.)

[3] A4. A tube of glowing gas shines light at two distinct wavelengths, namely, at 550 nm and 400 nm. If this tube is used in a double-slit experiment, what is the lowest-order bright fringe from the 550 nm light that will fall on a 400 nm dark fringe? What is the order of this 400 nm dark fringe?

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[3] A5. A 60-g mass is attached to a spring and undergoes simple harmonic motion. Its maximum acceleration is 15 m/s^2 and its maximum speed is 3.5 m/s. Determine the angular frequency, the spring constant, and the amplitude of the motion.

[3] A6. A hydrogen atom is in its ground state when its electron absorbs 48 eV in an interaction with a photon. What is the final energy of the electron after the absorption?

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[3] A7. In the Bohr hydrogen atom, what is the energy of the most energetic photon that can be emitted as the electron jumps between two neighboring energy levels? What are the n values for these two levels?

[3] A8. A particle is trapped in an infinite square well (a rigid box) of width 1 nm. If the energy difference between the ground state and the first excited state is 1.13 eV, find out whether the particle is an electron or a proton.

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[3] A9. Since nothing can escape from a black hole, its existence can only be inferred indirectly from its gravitational effects on the movement of objects in the hole's vicinity. Suppose that astronomers determine by this method that a particular object has a mass of 30 times the mass of the Sun, and occupies a region in space with a diameter of 1500 km. Can they conclude that this object is a black hole? (Derive your answer.)

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[5] A10. Discuss the different "metaphors" used by Newton and Einstein to describe the phenomenon of gravity. What are the differences/similarities? In what sense, if any, is one picture better or "more right" than the other?

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A10. continued (extra page if needed)

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PART B: Do ANY 5 of the following 9 questions. Each has the same mark value.

[5] B1. A balloon contains a gas of density ρ_g and is built to lift a mass M, which includes the balloon materials but *not* the gas. Find an algebraic expression that gives the minimum mass of gas required in terms of M, ρ_g and ρ_a (the atmospheric density).

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[5] B2. The simple harmonic motion of a particle is given by $x = (45 \text{ cm})\sin(\pi \text{t} + \pi/6)$, where x is in cm and t is in seconds.

(a) Find the time at which the potential energy is twice the kinetic energy.

(b) What is the particle's position at this time?

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[5] B3. A cylindrical block floats in water, with its circular sides facing up and down. The side length of the submerged portion is given by ℓ. The block has a total mass M and diameter d.

(a) Find an algebraic expression for the length ℓ of the submerged portion of the block in terms of M, d, and the water density ρ.

(b) If the block is pushed downwards (without completely submerging it) and then released, it will undergo simple harmonic motion. Find the period of the motion in terms of ℓ and the acceleration of gravity g.

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[5] B4. Consider an electron confined in a one-dimensional infinite rigid box of length L. Suppose that the electron is in a state whose wavefunction is given by $\Psi(x) = \mathrm{A}\ \sin(3\pi x/L)$, where A is the normalization constant.

(a) Draw the probability density (also known as the probability distribution) for this wavefunction as a function of position x. (Don't worry about determining the actual value of A for your plot.)

(b) Suppose you make a measurement of the electron's position in the box. From your plot in part (a), determine the probability that the electron will be found in the left half of the box.

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[5] B5. Consider an electron whose momentum has a magnitude of p. The direction of the momentum is unknown.

(a) What is a reasonable estimate of the *uncertainty* in the electron's momentum? Explain your answer.

(b) Using your result from part (a), determine the minimum kinetic energy possible for an electron confined to a one-dimensional region of atomic dimensions (about 0.1 nm). Does your result roughly make sense in light of what we know about the energies of atomic levels (of hydrogen, say).

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[5] B6. (a) Show that the density of matter needed to produce a black hole is inversely proportional to the square of the black hole's mass.

(b) How much mass would be required to make a black hole by squeezing matter to the density of water?

CONTINUED ON PAGE 15

[5] B7. Describe the key assumptions made by Bohr in his model of the hydrogen atom. Why is this model now called "semiclassical"? Discuss the Bohr atom's successes and shortcomings, and its importance in the historical development of our understanding of the atom.

Marks awarded will depend on content, organization, grammar, clarity, and the quality of your writing.

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[5] B8. The advent of quantum physics represented a major departure from the classical way of thinking about the natural world. Identify 2-3 of the main distinguishing features of quantum physics, and compare and contrast these with the framework of classical (Newtonian) physics.

Marks awarded will depend on content, organization, grammar, clarity, and the quality of your writing.

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[5] B9. In what way(s) is the theory of General Relativity a generalization/extension of Special Relativity? In other words, why did Einstein deem a General theory to be necessary?

Marks awarded will depend on content, organization, grammar, clarity, and the quality of your writing.

END OF EXAM QUESTIONS

Extra Page (turn it in with your exam copy)

Memory Sheet (turn it in with your exam copy)

Solution for quadratic equation: $x = (-b \pm \sqrt{b^2 - 4ac})/2a$
Area of sphere: $A = 4\pi r^2$
Volume of sphere: $V = \frac{4}{3}\pi r^3$
First two terms of Taylor series: $(1+x)^a \simeq 1 + ax$

Schwarzschild radius: $R_s = 2GM/c^2$
Hydrostatic law of pressure with depth: $\Delta P = \rho g \Delta y$
Archimedes' principle of buoyancy: $F_B = \rho_f V g$
Fluid flow speed: $A \cdot v = const$ where A is cross-section area of tube
Bernoulli's equation: $P + \frac{1}{2}\rho v^2 + \rho g y = const$
Oscillation period for SHM: $T = 2\pi\sqrt{m/k} = 2\pi/\omega$
Simple harmonic motion: $x(t) = A\ \cos(\omega t + \phi)$
Simple Pendulum: $T = 2\pi\sqrt{L/g}$
Wave speed: $v = f\lambda = \omega/k$
Travelling wave: $y(x,t) = A\ \sin(kx - \omega t + \phi)$ where $k = 2\pi/\lambda$, $\omega = 2\pi/T$
2-slit interference pattern: $I = I_0 \cdot \cos^2(\pi d sin\theta/\lambda)$
Photon energy: $E = hc/\lambda$
Energy levels of H atom: $E_n = -13.6eV/n^2$
Wavelengths emitted by H atom: $\frac{1}{\lambda_n} = R(\frac{1}{n^2} - \frac{1}{m^2})$
de Broglie relation: $\lambda = h/p$
Schrodinger equation: $\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-U)\psi = 0$
Particle in a box: energy levels $E_n = n^2h^2/8mL^2$
Uncertainty principle: $\Delta p \cdot \Delta x > \hbar$ $\quad \Delta E \cdot \Delta t > \hbar$

Physical constants:

Gravitation constant $G = 6.67 \times 10^{-11} Nm^2/kg^2$
Speed of light $c = 3.00 \times 10^8$ m/sec
Planck's constant $h = 6.626 \times 10^{-34}$ J-sec and $\hbar = h/(2\pi)$
Rydberg constant $R = 1.097 \times 10^7\ \mathrm{m}^{-1}$
Standard atmospheric pressure at sea level $P_0 = 1.013 \times 10^5$ Pa
Density of water $\rho = 1000\ \mathrm{kg/m^3}$
Density of air at sea level $\rho_0 = 1.3\ \mathrm{kg/m^3}$
mass of electron $= 9.1 \times 10^{-31}$ kg
mass of proton $= 1.67 \times 10^{-27}$ kg
1 eV (electron volt) $= 1.6 \times 10^{-19}$ Joules
charge on electron $e = 1.6 \times 10^{-19}$ coulomb
Mass of Sun $M_\odot = 2.0 \times 10^{30}$ kg

THE END