Arts & Science 2D06

Mid-Year Exam **2013 December** **Name:**

Time allowed: 3 hours. No books or notes allowed. An electronic calculator may be used. *Complete solutions must be shown to obtain full marks for any of the problems.*

Formulae:

Solution for quadratic equation: $x = (-b \pm \sqrt{b^2 - 4ac})/2a$

Taylor series: $(1+x)^a \simeq 1 + ax$ for small x

Constant acceleration: $x = x_0 + v_0 t + \frac{1}{2}at^2$, $v = v_0 + at$, $v^2 = v_0^2 + 2ax$

$\sum F = ma$ $F_{AB} = -F_{BA}$ unit vectors (**i, j, k**)

Kinetic friction: force $f = \mu_k N$ Static friction: force $f \leq \mu_s N$

Momentum: $\mathbf{p} = m\mathbf{v}$ Kinetic Energy: $K = (1/2)mv^2$

Rotational Kinetic Energy: $K = (1/2)I\omega^2$

Moment of inertia of hoop: $I = MR^2$

Gravitational Potential: $U = mgy$ Spring potential: $U = (1/2)kx^2$

Elastic collisions: $v_1 = \frac{(m_1 - m_2)}{(m_1 + m_2)}u_1$, $v_2 = \frac{2m_1}{(m_1+m_2)}u_1$

Centripetal acceleration: $a_c = v^2/r$

Newton's universal law of gravity: $F_g = GMm/r^2$

Gamma factor: $\gamma = (1 - v^2/c^2)^{-1/2}$

Lorentz transformation: $x' = \gamma(x - vt)$, $t' = \gamma(t - \frac{v}{c^2}x)$.

Reverse Lorentz tranformation: $x = \gamma(x' + vt')$, $t = \gamma(t' + \frac{v}{c^2}x')$

Velocity addition: $u' = \frac{(u-v)}{(1-uv/c^2)}$

Rest-mass Energy: $E = mc^2$ Kinetic energy: $K = (\gamma - 1)mc^2$

Total Energy: $\sqrt{p^2c^2 + m^2c^4}$

Numerical Constants:

$c = 300,000$ km/sec $= 3.00 \times 10^8$ m/sec (speed of light)

$G = 6.67 \times 10^{-11}$ N m^2/kg^2 (Newton's law of gravity constant)

$g = 9.8$ m/s^2 (acceleration of gravity near surface of Earth)

PART A: Do all of the following short questions.

A1. [3] The total energy of a particle is twice its rest energy. Find the speed of the particle in terms of c.

A2. [3] A ball attached to a string is whirled at a constant speed v in a circular path of radius R. If v is doubled, how does this change affect the ball's acceleration a?

A3. [3] A ball is released from rest and bounces back up to 70% of its starting height. Find the fraction of the ball's total energy that was lost in the bounce.

A4. [3] A hoop of radius 0.40 m and mass 0.9 kg is rolling without slipping on a horizontal surface. The hoop has a constant linear/translational speed of 18 m/s and is heading toward an incline of slope 25°. How far up the incline will the hoop roll?

A5. [3] A 60-kg rollerblader pushes off a wall of a campus building, and acquires a speed of 3 m/s. How much work is done on the rollerblader? What is the change in the her total energy? Describe how energy is conserved for her. (Ignore friction.)

A6. [3] At $t = 0$, a particle leaves the origin with a velocity of 5 m/s in the positive y direction, with an acceleration given by $\mathbf{a} = (3\mathbf{i} - 5\mathbf{j})$ m/s^2. When the particle reaches its maximum y coordinate, what is the particle's velocity?

(*Hint*: the y-component of the particle's velocity is zero when its maximum y is reached.)

A7. [3] Suppose that in a frame S', two events happen at the same time but are separated by 500 km. Find the space and time intervals (*i.e.*, Δx and Δt) in frame S if S' is moving at $0.5c$ with respect to S, in the positive x direction.

A8. [3] On a horizontal icy (frictionless) street, Santa is riding on a sleigh pulled by reindeer, with a speed of 8.2 m/s. Their combined weight is 485 kg. As the sleigh passes in front of a house in Santa's village, an elf on the second floor drops a 64-kg gift bag vertically onto Santa's lap. What is the new speed of the sleigh?

PART B: Do ANY 4 of the following 7 questions (your choice; 5 marks each).

B1. [5] A particle moves with a constant acceleration of 3 m/s^2. At $t = 4$ seconds, it is at $x = 100$ m, while at $t = 6$ seconds, it has a velocity $v = 15$ m/s. What is the particle's position at $t = 6$ seconds?

B2. [5] Suppose that the force acting on a brick of mass 6.0 kg is given by the function $F(t) = 4t^2 + 3$ N.

(a) Sketch this force as a function of time, starting from $t = 0$.

(b) If the particle starts from rest, calculate its velocity after 5 seconds.

B3. [5] (a) Show that in Newtonian physics, the momentum p and kinetic energy K of a particle are related through

$$p = \sqrt{2mK}$$

(b) Now show that in Special Relativity, the momentum p and kinetic energy K of a particle are related through

$$p = \sqrt{(K/c)^2 + 2mK}$$

(c) Consider a 2-kg book thrown at 10 m/s. Show that the relativistic expression for the book's momentum can be approximated by the Newtonian expression.

B4. [5] Consider a pendulum comprised of a ball of mass M on a (very light) string of length L. The pendulum is released horizontally from rest and swings downward. Derive an equation for the tension T in the string, as a function of M and the angle θ. (θ is the angle of the pendulum with respect to its equilibrium vertical position, such that $\theta = 90^\circ$ at the start.)

B5. [5] A block of mass m lies on a horizontal surface and is attached to a spring of constant k, as shown in the figure. The spring is compressed a distance d from its equilibrium position and released from rest.

(a) If the block comes to rest when it reaches the equilibrium position for the first time, find the coefficient of friction between the surface and the block.

(b) If instead the block comes to rest after traveling only a distance equal to $d/2$ from its release position, now what is the coefficient of friction?

B6. [5] Describe three characteristics of a “good” law (or set of laws) in physics. Support your discussion with examples from our lecture content of this past term.

Marks awarded will depend on content, organization, grammar, clarity, and the quality of your writing.

B7. [5] Isaac Newton and Albert Einstein were two of the notable thinkers whose ideas we discussed this term. Choose one of them: then discuss the importance of his work to our knowledge of the physical world, and the broader impact of his thought on a cultural/societal level.

Marks awarded will depend on content, organization, grammar, clarity, and the quality of your writing.

44 marks total (Happy Holidays!)

Extra Page for Scratch Work