Arts & Science 2D06

Mid-Year Exam 2007 Dec 12 Name:

Time allowed: 3 hours. No books or notes allowed. An electronic calculator may be used. *Complete solutions must be shown to obtain full marks for any of the problems.*

Formulae:

Solution for quadratic equation: $x = (-b \pm \sqrt{b^2 - 4ac})/2a$

Taylor series: $(1+x)^a \simeq 1 + ax$ for small x

Constant acceleration: $x = x_0 + v_0 t + \frac{1}{2}at^2$, $v = v_0 + at$, $v^2 = v_0^2 + 2ax$

$\sum F = ma$ $F_{AB} = -F_{BA}$ unit vectors (**i, j, k**)

Kinetic friction: force $f = \mu_k N$ Static friction: force $f \leq \mu_s N$

Momentum: $\mathbf{p} = m\mathbf{v}$ Kinetic Energy: $K = (1/2)mv^2$

Rotational Kinetic Energy: $K = (1/2)I\omega^2$

Gravitational Potential: $U = mgy$ Spring potential: $U = (1/2)kx^2$

Elastic collisions: $v_1 = \frac{(m_1 - m_2)}{(m_1 + m_2)}u_1$, $v_2 = \frac{2m_1}{(m_1 + m_2)}u_1$

Centripetal acceleration: $a_c = v^2/r$

Newton's universal law of gravity: $F_g = GMm/r^2$

Gamma factor: $\gamma = (1 - v^2/c^2)^{-1/2}$

Lorentz transformation: $x' = \gamma(x - vt)$, $t' = \gamma(t - \frac{v}{c^2}x)$.

Reverse Lorentz tranformation: $x = \gamma(x' + vt')$, $t = \gamma(t' + \frac{v}{c^2}x')$.

Velocity addition: $u' = \frac{(u-v)}{(1-uv/c^2)}$

Rest-mass Energy: $E = mc^2$ Kinetic energy: $K = (\gamma - 1)mc^2$

Numerical Constants:

$c = 300,000$ km/sec $= 3.00 \times 10^8$ m/sec (speed of light)

$G = 6.67 \times 10^{-11}$ N m^2/kg^2 (Newton's law of gravity constant)

$g = 9.8$ m/s^2 (acceleration of gravity near surface of Earth)

$M_E = 5.98 \times 10^{24}$ kg (mass of Earth)

$R_E = 6.37 \times 10^6$ m (radius of Earth)

PART A: Do all of the following short questions.

A1. [3] Suppose that a spaceship could be made to go constantly at the speed of 0.8c, and that it is sent to a star located 5 light years away from us. For the one-way trip to the star, calculate the time elapsed and the distance covered as measured from within the spaceship.

A2. [3] By how much would a person's mass increase if she were flying in a jet craft at the speed of sound (330 m/s)? Assume m = 60 kg.

A3. [3] A 6.0 kg object moving at 7.0 km/s collides inelastically with a 4.0 kg object that is initially at rest. What percentage of the initial kinetic energy of the system is lost during the collision?

A4. [3] A dog leaps to catch a ball. If the dog's jump was at 50° relative to the ground and its initial velocity was 6 m/s, what is the highest point of its trajectory?

A5. [3] A block starts from rest at the top of a 15° inclined plane and encounters a spring of constant 4500 N/m, rigidly attached to the plane (see figure). If the block's mass is 55 kg and it compresses the spring by 40.0 cm, find the distance the block travelled before it encountered the spring.

A6. [3] The position of an object as a function of time is given by $x = 4.1t^3$ - $2.2t^2$ + $1.7t$. What are the instantaneous velocity and acceleration of the object when $t = 2.0$ seconds? If the mass of the object is 2 kg, what is the force on the object at $t = 3.0$ seconds?

A7. [3] An artificial satellite revolves around the earth like a miniature moon. What must its speed of revolution and period be if it is to stay at a height of 3500 km from the earth's surface?

A8. [3] A 21 kg box needs be slid across the floor. If the coefficient of static friction between the box and floor is 0.37, what is the minimum force needed to start the box moving from rest?

A9. [3] The figure below shows a 100 kg block being released from rest from a height of 1.0m. It then takes 0.78 s to reach the floor. What is the mass of the block on the left side?

PART B: Do ANY 5 of the following 9 questions (your choice; 5 marks each).

B1. [5] For each of the following cases, sketch two graphs, one of vertical displacement vs. time elapsed, the other of vertical velocity vs. time elapsed (state your choice of +/- sign convention).

(a) a skydiver falls from a plane for a while, eventually opens the parachute, and finally reaches the ground.

(b) a marble drops from your hand and bounces three times before coming to rest on the floor.

(c) a baseball is thrown at $60°$ relative to the ground and falls into a deep valley.

B2. [5] A 5.0g bullet, travelling horizontally with a velocity of 350 m/s, is fired into a wooden block with mass 0.8 kg, initially at rest on a level surface. The bullet passes through the block and emerges with its speed reduced to 100 m/s. The block then slides a distance of 40 cm along the surface from its initial position.

a) Find the kinetic energy of the block at the instant after the bullet passes through it.

b) Find the coefficient of kinetic friction between the block and surface.

B3. [5] An enemy spaceship is travelling toward your ship with a speed of $0.5c$, as measured in your frame. It then fires a missile toward your ship at a speed of $0.8c$ relative to the enemy ship.

a) What is the missile's speed as measured in your frame?

b) If you know that the enemy ship is 9×10^6 km away from you when the missile is fired, how much time, as measured in your frame, will it take the missile to hit you?

B4. [5] At the same moment, one rock is dropped while another is thrown downward with an initial speed of 10 m/s from the top of a 200 m building. How much earlier does the thrown rock strike the ground?

B5. [5] The density λ (mass per unit length) of a thin rod of length L depends on position x, and is given by $\lambda{=}Ax$, where A is a constant. The rod is placed with one end at $x = 0$, as shown in the figure. Find the moment of inertia of the rod about an axis perpendicular to the rod through $x = 0$.

B6. [5] A hoop ($I = MR^2$) and a disk ($I = 0.5MR^2$) of equal radius R and equal mass M are rolling along at equal speeds along a horizontal surface. The objects then encounter an incline plane. Find the maximum vertical height that each object will reach as it climbs the plane. Which object goes up higher?

B7. [5] In studying the evolution of ideas in physics, we have seen a number of shifts in paradigm - or worldview - manifested in new ways of understanding certain aspects of the natural world. For example, Galileo's experiments led to the refutation of the Aristotelian idea that moving objects have an internal "will" that determines their motion. Describe two of such shifts, and discuss the key new developments (theoretical and/or experimental) that led to these changes.

Marks awarded will depend on content, organization, grammar, clarity, and the quality of your writing.

B8. [5] We first came across the notion of "natural law" or "the laws of physics" when discussing Newton's laws of motion. Identify and discuss two or three features that generally characterize such laws, and where relevant, compare and contrast these with other familiar notions of law from outside the context of physics/science.

Marks awarded will depend on content, organization, grammar, clarity, and the quality of your writing.

B9. [5] Great discoveries in physics often involve the realization that certain aspects of nature, which are seemingly very different and unrelated at first glance, are actually different manifestations of the same underlying physical principle or law. Identify and discuss one or two examples of this kind of "unification" that we've seen this term.

Marks awarded will depend on content, organization, grammar, clarity, and the quality of your writing.

52 marks total

Extra Page for Scratch Work