AS2D06

Sample Project Report

FLUID VISCOSITY & TERMINAL VELOCITY:
AN INTRODUCTION AND EXTENSION.

ARTS AND SCIENCE 2D6
MONDAY, MARCH 15, 1999

INTRODUCTION

In most introductory physics courses, the effects of friction on falling and other moving objects are often ignored with regard to basic kinematics and dynamics. Most likely, this is done in order to first build a strong foundation of the basics such as velocity, acceleration, projectile motion, Newton's laws, etc. Such a strategy is probably quite effective. However, the assumption to ignore frictional forces is quite unrealistic in many 'real-life' situations. This project serves to investigate falling objects beyond the traditional introductory treatment, by analyzing the effects of various interacting forces. This will be done, first by discussing fluid viscosity and terminal velocity of steel balls falling through oil. What has been learned from that experiment, will then be extended to the analysis of fluid viscosity and terminal velocity of a beach ball falling through air. Both have similar concepts involved, and both will attempt to derive terminal velocities, coefficients of viscosity, and the related Reynold's number. However, these situations are by no means identical, as will be demonstrated in this independent project.

FLUID VISCOSITY & TERMINAL VELOCITY

PURPOSE

By measuring the terminal velocities of steel balls falling through a viscous oil, fluid "friction" or viscosity, and its effects on fluid motion can be investigated and experimentally determined. By calculating the Reynold's number for the gathered data, the validity of Stoke's law and subsequently the equation for terminal velocity, can be determined.

PREPARATORY QUESTIONS

You might start here with a simple statement of Stoke's Law and the Reynolds number before plunging in

$\rho(\text{steel}) = 7.80\text{x}10^{-6}$ kg/mm^3
$\rho(\text{oil}) = 0.80\text{x}10^{-6}$ kg/mm^3

Question 1

<u>log V_T vs. log R plot</u>

$V_T = 2R^2g(\rho - \rho_f)/9\eta$ $\quad V_T = 2/9R^2g(\rho - \rho_f)/\eta$ $\quad V_T = R^2[(2/9)\bullet(g/\eta)(\rho - \rho_f)]$

$\log V_T = \log\{R^2[(2/9)\bullet(g/\eta)(\rho - \rho_f)]\}$ $\quad \log V_T = \log R^2 + \log[(2g/9n)(\rho - \rho_f)]$

$\log V_T = 2\log R + \log[(2g/9n)(\rho - \rho_f)]$ => $\mathbf{y = mx + b}$ (where $y = \log V_T$, $x = \log R$)

$\therefore$ theoretical value for slope is 2 and y-intercept = $\log[(2g/9n)(\rho - \rho_f)]$

<u>V_T vs. R^2 plot</u>

$V_T = 2R^2g(\rho - \rho_f)/9\eta$ $\quad V_T = [2g/9\eta](\rho - \rho_f)R^2$ => $\mathbf{y = mx + b}$

$\therefore$ theoretical value for slope is $(2g/9n)(\rho - \rho_f)$ and y-intercept = 0

Question 2

To find units of η, the following equation can be used,

$V_T = 2/9R^2g(\rho - \rho_f)/\eta$ $\quad \eta = 2/9R^2g(\rho - \rho_f)/V_T$

So, when merely concerned with units => $\eta = (\text{mm})^2(\text{mm/s}^2)(\text{kg/mm}^3 - \text{kg/mm}^3) / (\text{mm/s})$

$= (\text{mm}^2/\text{s})(\text{kg/mm}^3) = \text{kg/mm}\bullet\text{s}$ = units of η

To convert to standard units, multiply by a factor of 10^3 (to convert to metres)

Question 3

Water is not appropriate because it is not viscous enough, and the steel balls would travel much too fast to record accurate measurements. There needs to be a greater drag force.

$V_T = 2/9R^2g(\rho - \rho_f)/\eta = (1.00\text{mm})^2(9800\text{mm/s}^2)(6.8\text{x}10^{-6}\text{kg/mm}^3) / (1.00\text{x}10^{-6}\text{kg}\bullet\text{mm/s})$

$= 14808.89$ mm/s (which is much too fast)

It could be said that the length of cylinder required would have to be extremely large to record measurements.

METHODS AND OBSERVATIONS

Following the procedure in the lab manual "Physics 1B03 Coursepack," the apparatus was set up as discussed there. This was already done prior to our lab session. The distance between the two timing marks on the glass cylinder was measured, and found to be (301 ± 1)mm. Then, using the vernier caliper, the diameter of each steel ball was measured independently by each partner. The steel ball was held just below the surface of the oil (being careful to avoid any attached air bubbles), near the central axis of the cylinder, and subsequently released. As the ball fell through the specified timing marks, a stopwatch was used to measure the time taken for the ball to travel between the two marks. The time measurement was repeated by retrieving the using a small magnet until two times that agreed to within 0.1 seconds were observed.

Table 1: Diameters and times for steel balls falling through oil

BALL DIAMETER		TIME TO FALL (S)	TIME TO FALL (S)
VANCE'S RESULT (MM)	JONATHAN'S RESULT (MM)		
1.58	1.58	22.43	22.43
2.36	2.34	10.10	10.10
0.32	0.318	5.75	5.75
4.00	4.00	3.90	3.90

Question 4

The eye level of the person doing the timing should initially be in the plane of the circumferential top timing mark and should be in the plane of the lower timing mark by the time the ball has fallen, in order to avoid parallax error. This is to ensure that the time the ball takes to fall that distance is measured accurately. If the eye level of the timer is not directly on the same plane as the ball, the time measured would be incorrect because he/she would be viewing a different distance that the ball falls since they are watching the ball at an angle. For example, if the eye level of the timer is above the timing mark, it would look as though the ball crossed the line after it actually did, and therefore the timer would obtain an incorrect time.

Also, since the timer is viewing the ball through a liquid filled cylinder, distortion due to refraction could be present and therefore it would make it harder to determine the exact moment when the ball crosses the line if the timer's eye level is not in the plane of the circumferential timing mark.

Question 5

The oil level should be several centimetres above the top timing mark so that the ball has time to reach its terminal velocity before the timing begins. Therefore, it is ensured that the ball has reached its terminal velocity before it reaches the first timing mark.

RESULTS AND DISCUSSION

The average radius of the steel ball is determined by dividing the average diameter by 2.

e.g. Radius of smallest ball $= \frac{1}{2}$ [(measurement 1 + measurement 2) / 2]

$= \frac{1}{2}$ [(1.58 + 1.58)mm / 2] = 0.790mm

The terminal velocity for each steel ball is determined as follows:

e.g. Terminal velocity for smallest ball => $V_T = d/t$ where d = 301mm, t = 22.43s

V_t = 301mm/22.43s = 13.41mm/s

logR = log (0.790) = -0.102 log V_T = log (13.4) = 1.13 $R^2 = (0.790\text{mm})^2 = 0.624\ \text{mm}^2$

Table 2: Terminal velocities for steel balls falling through oil

AVERAGE BALL RADIUS R (MM)	TERMINAL VELOCITY V_T (MM/S)	LOG R	LOG V_T	R^2 (MM^2)
2.00	77.2	0.301	1.89	4.00
1.60	52.4	0.204	1.72	2.56
1.18	29.8	0.0719	1.47	1.39
0.790	13.4	-0.102	1.13	0.624

units ?

Question 6

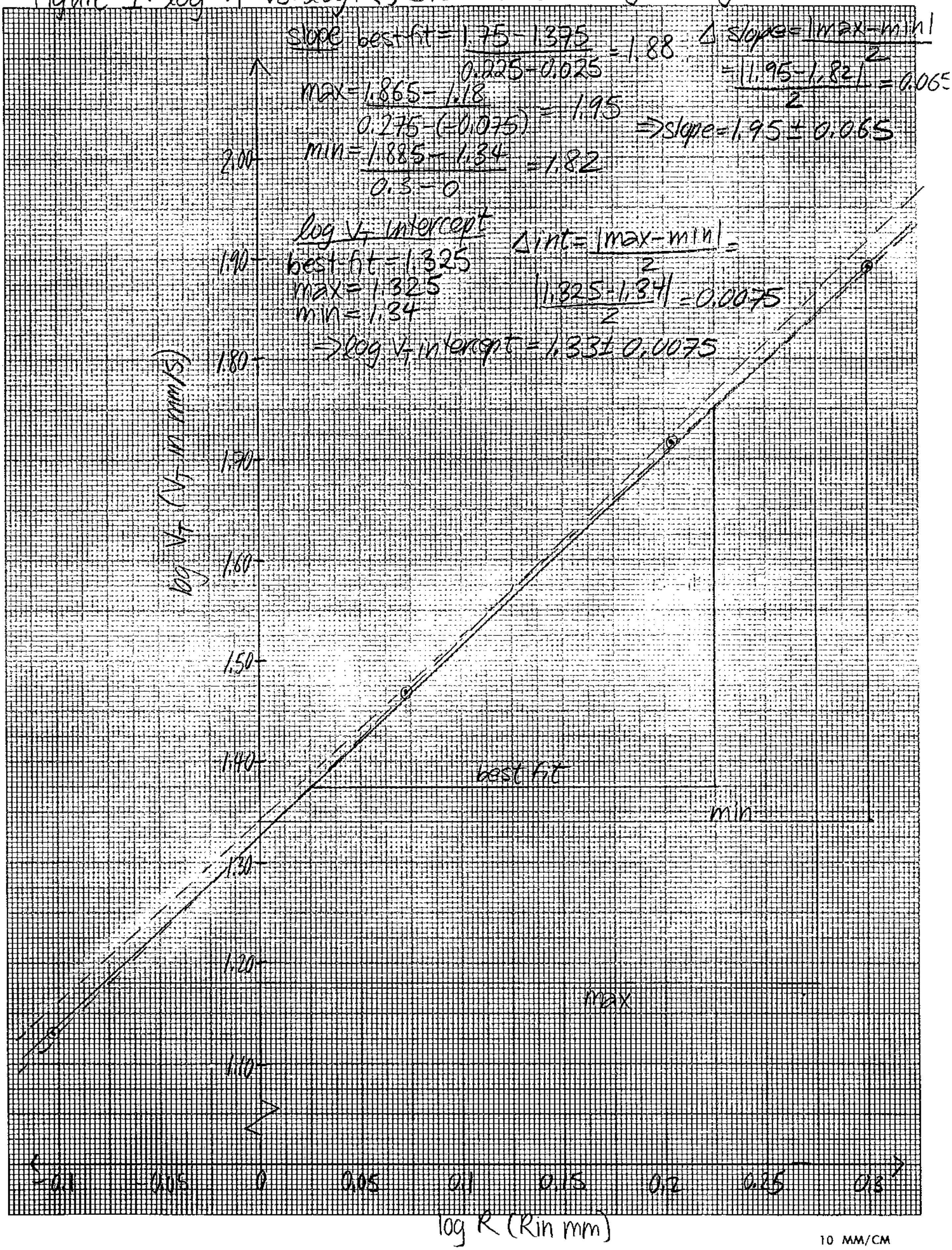
The slope of the log V_T vs. log R graph (Figure 1) was found to be 1.95 ± 0.065. To determine the uncertainty in the slope value, the scatter points were used to draw the extreme slope lines (The slope and uncertainty calculations are done directly on the graph - Figure 1). This was in agreement with the theoretical value when the uncertainty is taken into account, since the theoretical value was 2.

The best fit line was also drawn on the graph of V_T vs. R^2 (The slope and uncertainty calculations are done directly on the graph - Figure 2). Once again, the uncertainty was estimated by drawing the extreme lines using the scatter of the points. The slope of this graph was determined to be ~~19.4~~ 1.94 ± 1.35 and the y-intercept was found to be 1.0 ± 2.25.

Question 7

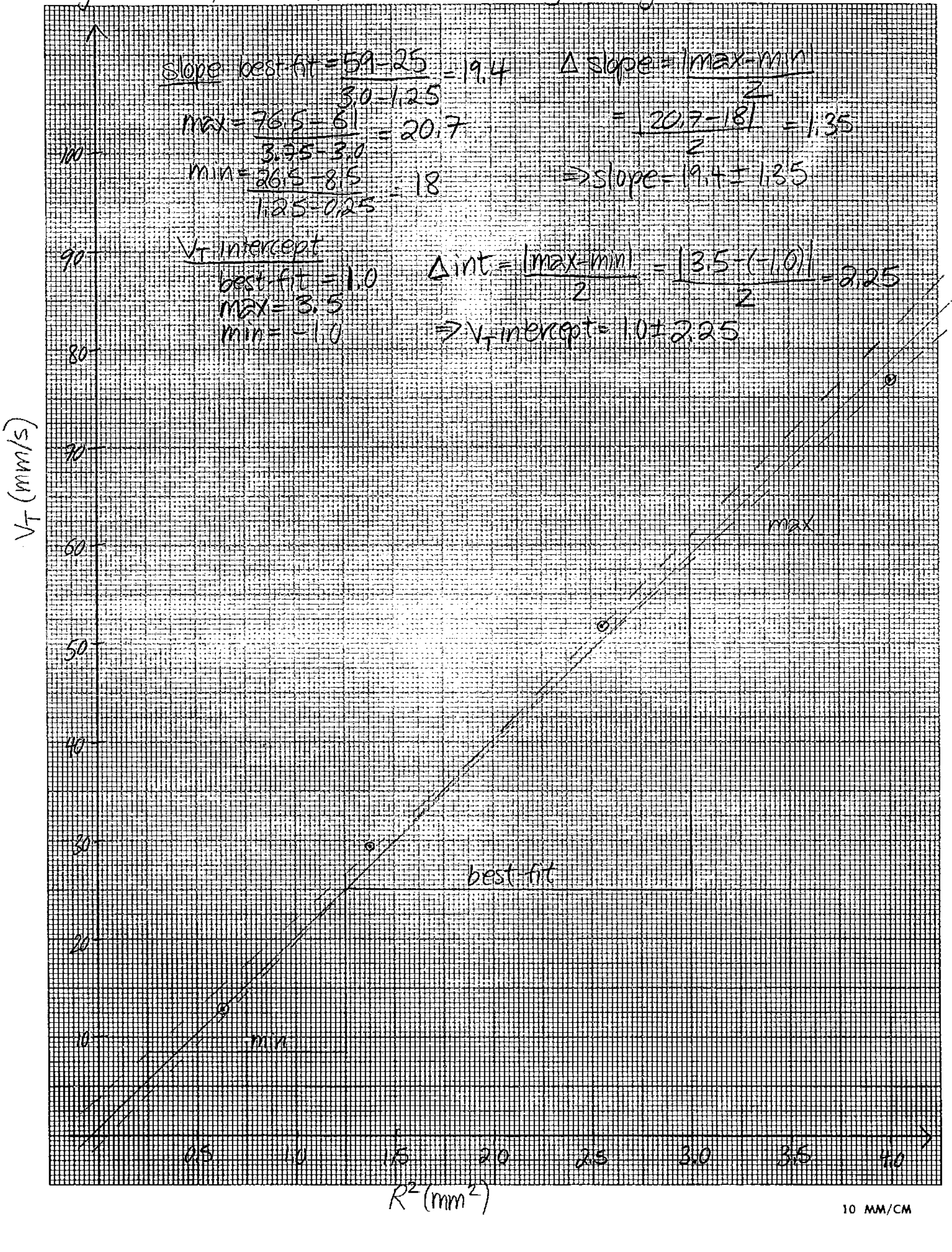
The intercept of the V_T vs. R^2 graph was found to be in agreement with the expected value of 0 when the uncertainty is taken into account.

Figure 1: log V_T vs. log R ; Steel balls falling through oil



10 MM/CM

Figure 2: V_T vs. R^2; Steel balls falling through oil.



Coefficient of Viscosity

The coefficient of viscosity for the fluid, η, was determined using the slope of the V_T vs. R^2 (m=19.4).

$V_T = 2R^2g(\rho - \rho_f)/9\eta$ $\quad \eta = 2g(\rho - \rho_f)R^2/9\ V_T$ [slope (m) of V_T vs. R^2 is V_T/R^2] => $\boldsymbol{\eta = 2g(\rho - \rho_f)/9m}$

$$\eta = 2g(\rho - \rho_f)/9m = 2(9800\ mm/s^2)(7.0x10^{-6} kg/mm^3)/9[19.8(mm \cdot s)^{-1}]$$
$$= 7.86x10^{-4}\ kg/mm \cdot s\ = 0.786\ kg/m \cdot s\ = 0.786\ Pa \cdot s$$

In order to determine the uncertainty in η, the scatter in the data points was used to draw extreme slope lines.

$$\sigma(\eta) = [\sigma(m)/m]\eta \qquad = [1.35(mm \cdot s)^{-1}/19.4]\eta = 0.0547\ kg/m \cdot s$$

$\therefore \eta = 0.786 \pm 0.0547\ Pa \cdot s$

Validity of Stoke's Law

Reynold's number was calculated using the largest steel ball and its velocity.

$RN = VR\rho_f / \eta = (77.2\ mm/s)(2.00mm)(0.8x10^{-6}\ kg/mm^3)/(7.86x10^{-4}\ kg/mm \cdot s) = 0.157$

Because this value of RN is appreciably less than 1.0, Stoke's Law is valid for determining the equation for terminal velocity in this case. Since the value for RN << 1 for the largest ball, it did not need to be calculated for all the balls.

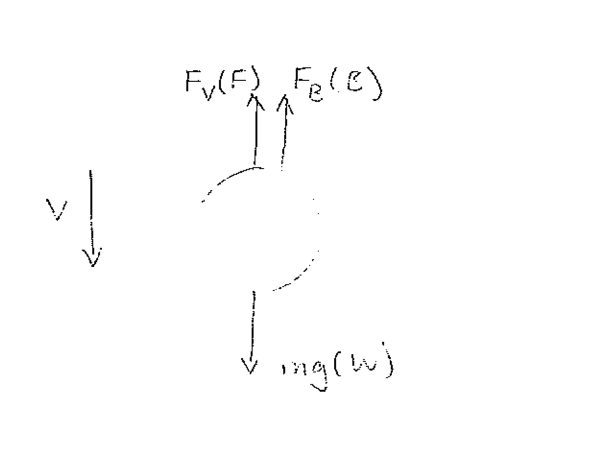


Figure 3 - forces on object falling through fluid

When the Reynold's number is appreciably less than 1, the flow around an object (steel sphere in this case) can be considered laminar, and the corresponding viscous force F_v (drag force) has been experimentally determined to be directly proportional to the speed of the object ($F_v \alpha v$), so that,

$F_v = kV$ $\quad$ and since $k = 6\pi r\eta$ $\quad$ $F_v = 6\pi R\eta V$ (known as Stoke's equation, which is valid in this case)

An object falling through a fluid under the action of gravity has three different forces acting on it, including mg (the force of gravity), F_b (buoyant force of the fluid), and F_v (viscous drag force). [see Figure 3] Given the object falling is a sphere, the following are true: $mg = 4/3\pi R^3\rho g$ (ρ = density of ball); $F_b = 4/3\pi R^3\rho_f g$, $F_v = 6\pi R\eta V$. The net force of this system is equal to (mass)(acceleration). Then, at terminal velocity, as the net forces approach and reach 0, the acceleration approaches and reaches 0 as well. It is at this point when terminal velocity can be solved:

$4/3\pi R^3\rho g - 4/3\pi R^3\rho_f g - 6\pi R\eta V_T = 0$ ==> $\boldsymbol{V_T = 2R^2g(\rho - \rho_f)/9\eta}$ (which can be used when RN << 1)

This belongs at the beginning.

The graph produces quite a nice result, suggesting terminal velocity was nicely reached & the measurement is valid

SOURCES OF ERROR

One of the factors that may have affected results is the purity of the oil. Since the oil for these trials is reused and the trials involve submerging the steel balls (by hand) into the oil prior to release, the purity may be affected. Since the oil may not have been pure, its viscosity could therefore be affected and thus produce experimental results that differ from predicted, theoretical results.

Another possible source of error could be human error. Since this particular lab depended on human reaction times which may vary from trial to trial, observations could have been affected. Kind of vague. Timing? parallax

The steel balls used in the experiment may also have been a source of error. The motion of the sphere is affected by any irregularities in the surface of the sphere. If the steel balls were not perfectly round, then they may travel at slightly different speeds depending on their orientation in space when they were dropped into the oil. Also, although it was made clear in the laboratory procedure to avoid air bubbles if possible, the possibility of their existence remains. Thus, they can be considered as a plausible source of error due to their affect on the falling sphere. This is really reaching.

That's the key word. One can go on thinking of tinier and tinier possible errors, but just list the ones that are biggest or most realistic

CONCLUSION

The viscosity of the oil was calculated by measuring the time it took for steel balls to fall a certain distance through the oil. Using this time and distance, terminal velocity could be calculated. The slope of the log V_T vs. log R graph (Figure 1) was found to be 1.95 ± 0.065, and consequently in agreement with the theoretical value when uncertainty is considered. The y-intercept of the graph of V_T vs. R^2 (1.0 ± 2.25) was also found to be in agreement with the theoretical value, when taking uncertainty into account. Using the slope from the V_T vs. R^2 graph, the value of η was determined to be 0.786 ± 0.0547 Pa•s. Reynolds number was calculated to be 0.157, and since it was much less than 1, Stoke's Law is valid for determining the equation for terminal velocity.

The study of the fall of an object near the Earth's surface is often begun by considering the gravitational force as the only force acting on the object. If the object is something like a steel sphere in a laboratory experiment, then this assumption is valid. Eventually however, it becomes necessary to consider the medium through which the object falls. For example, the fall of a steel sphere through air, and the fall of a steel sphere through oil involve similar dynamics, however, the effects of fluid friction are more easily observed in oil. The following investigation extends these dynamics to low-density objects falling in *air*.

FLUID VISCOSITY & TERMINAL VELOCITY: AN EXTENSION

PURPOSE

By measuring the terminal velocity of a beach ball falling through air, fluid "friction" or viscosity, and its effects on fluid motion can be investigated and experimentally determined. By calculating the Reynold's number for the gathered data, the validity of Stoke's law and subsequently the equation for terminal velocity, can be determined.

CHOICE OF EXPERIMENT SITE

Some of the factors that were considered when choosing a site for the experiment include:

height of drop
ability to measure distance intervals
amenability to set up apparatus
avoiding external factors such as breeze

There were two main possibilities for the site that were considered: stairwell at Health Sciences Foyer and the stairwell at Common's Marketplace.

Both sites provided for the absence of annoying breezes, however, it is with this factor that the similarities end and the differences begin. The Health Sciences Foyer does not easily allow for the measurement of distance intervals. Also, when measuring the time for the object to fall through a particular distance interval, a marker for the end of that interval (i.e. string, bar) is required. However, given that the stairwell is situated in the middle of open space, this requirement presents a difficulty in terms of setting up the apparatus. The surrounding structural arrangement (only certain height levels allowed for the attachment of string markers) only permitted 3 distance intervals. Also, the height of the drop is merely one floor.

At Common's Marketplace however, the stairwell spanned 3 floors (see Figure 4). The staircase was oriented such that it formed and surrounded a long rectangular prism through which an object could be dropped. This prism was reinforced by an iron cage latticework, which was conveniently divided into 12 equally spaced vertical distance intervals acting almost like a giant ruler. Also, because the staircase and the plateaus of each floor surrounded this cage, the apparatus required would be easy to set up.

Needless to say, our choice of site was Common's Marketplace. The exactness of the equally spaced distance intervals, the set-up of the apparatus, and other methodological factors will be discussed and scrutinized in the methods and observations section.

EXPERIMENTAL DESIGN, METHODS AND OBSERVATIONS

Each wall of the iron cage latticework was composed of 5 and 6 columns that spanned the entire height. Each column was divided into 6 intervals, however because adjacent columns had bars that bisected each of these intervals (see Figure 5a), the iron cage had 12 distance intervals altogether. The distance between 4 of the 12 separate intervals were measured, using the midpoints of the bars as starting and stopping points (see Figure 5b), and found to be 0.612 ± 0.001m, 0.611 ± 0.001m, 0.610 ± 0.001m and 0.613 ± 0.001m respectively.

At the top of the structure, a hockey stick was placed across the cage at the centre of one wall. An oversized length of fishing line that more than spanned the height of the drop, was attached to the beach ball and was hung over and perpendicular to the hockey stick. The line was then fed through the centre bar of the adjacent wall and firmly anchored. The set up has been diagrammed in Figure 6. This contraption acted as a pulley of sorts, that helped to keep the releasing point in the centre of the cage consistent.

Question: You need to know the mass and volume of your falling object. How will you measure these?

inflated mass? or uninflated?

How about buoyancy while on the scale

The mass and volume of the beach ball being used were measured. The mass of the beach ball was measured using a scale with digital readout, and was found to be 0.078 ± 0.001 kg. To understand how the volume of the beach ball was measured, a description of the beach ball is necessary. The beach ball is separated into 6 segments, and thus when one segment boundary is added to the opposite segment boundary, the circumference of the sphere is formed. Also, perpendicular to this diameter, another circumference marked by symmetrical logos can be found. Each partner measured each of these circumferences, and the following results were recorded

scale reading $= W - F_B$

Circumference 1: 108.0 ± 0.5cm (Jonathan) 107.5 ± 0.5cm (Vance)
Circumference 2: 108.0 ± 0.5cm (Jonathan) 108.0 ± 0.5cm (Vance)

The volume of the sphere can then be determined mathematically, and will be done in the Results and Discussion section.

It should also be noted that these trials were performed after midnight, to avoid human interruption, as well as the continuous closing and opening of doors. The apparatus having been set up, many practice trials were conducted in order to 'master' the technique. Considering that the ball would only take several seconds to traverse the height of the cage, preciseness of the measurements is key. For this reason, several precautions were taken. The same partner was responsible for releasing the ball, while the other consistently timed its descent. At the top, where the ball was released, the same bar was used as the marking point for release, and the ball was situated such that the horizontal circumference was eclipsed by the midpoint of the bar (see Figure 7). In other words half of the ball would appear above the bar, and half of the ball would appear below the bar. The logo on the beach ball was used as a marker for the central horizontal circumference. Thus, for each trial, the person releasing the ball stayed in the same position, used the same markers, and used the same techniques in order to level and situate the ball. The same ideology was used when measuring times during the ball's fall. That is, the time recorder, stopped the stopwatch as the middle of the ball (horizontal circumference) passed the midpoint of the given bar. It should also be noted that the position of the time recorder at each height was strategically chosen, so as to be situated in front of the central axis of the cage where the ball was most likely to pass during its descent. Furthermore, before the ball is released, it was ensured that the ball remained as still as possible without spinning, in order to increase the chances of the ball falling directly through the centre of the cage.

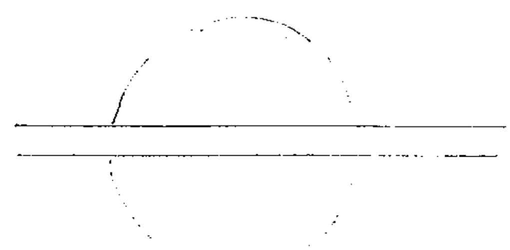


Figure 7 - view of ball releaser before release

Another method used to increase the accuracy of the results was the continuation of a timing rhythm. The time recorder would say "ONE - TWO - THREE - ONE - TWO" and would then start the timer when the next "THREE" would chronologically take place. So, as the stopwatch "beeped" signifying the next "THREE," the ball would be simultaneously dropped. This rhythm was practiced, and refined quite well.

After we felt confident about the set-up and measurement procedures, the trials began. For each of the 12 distance intervals, the trials were repeated until four times were obtained that agreed to within 0.1 seconds. The results (distance, time) were recorded in the following table. In order to avoid repetition, the following table includes a column for distance and average time as well, which will be calculated in the next section, Results and Discussion.

Table 3: Times, distances and velocities for beach ball falling through air

DISTANCE (M)	TIME 1 (S)	TIME 2 (S)	TIME 3 (S)	TIME 4 (S)	AVERAGE TIME (S)
0.612 ± 0.005	0.45	0.43	0.51	0.51	0.48 ± 0.021
1.224 ± 0.005	0.69	0.68	0.65	0.64	0.67 ± 0.012
1.836 ± 0.005	0.93	0.94	0.88	0.87	0.91 ± 0.018
2.448 ± 0.005	1.04	1.00	1.07	1.07	1.05 ± 0.017
3.060 ± 0.005	1.14	1.22	1.14	1.17	1.17 ± 0.019
3.672 ± 0.005	1.36	1.31	1.37	1.30	1.34 ± 0.018
4.284 ± 0.005	1.43	1.45	1.46	1.43	1.44 ± 0.0076
4.896 ± 0.005	1.57	1.58	1.59	1.56	1.58 ± 0.0071
5.508 ± 0.005	1.77	1.77	1.81	1.76	1.78 ± 0.011
6.120 ± 0.005	1.95	1.95	2.00	2.03	1.98 ± 0.020
6.732 ± 0.005	2.12	2.14	2.15	2.14	2.14 ± 0.0065
7.344 ± 0.005	2.35	2.33	2.35	2.27	2.33 ± 0.019

RESULTS AND DISCUSSION

The following procedure was used to find the average distance interval between the midpoints of bars.

$$D_{av} = (0.612\text{m} + 0.611\text{m} + 0.610\text{m} + 0.613\text{m})/4 = 0.612\text{m}$$

The uncertainty associated with the following can be determined using the following equation:

$$\sigma(D_{av}) = (\sigma^2_a + \sigma^2_b + \sigma^2_c + \sigma^2_d)^{½} / 4$$
$$= [4(0.001\text{m})^2]^{1/2} / 4 = 0.002\text{m} / 4 = 0.0005\text{m}$$
$$\therefore D_{av} = 0.612 \pm 0.0005\text{m}$$

The uncertainty in the average time can be determined using the following expression:

(using the times recorded for d = 1.836 ± 0.005)

$$\sigma(t_{av}) = [(1/n\text{-}1)(1/n)\sum(t - t_{av})^2]^{1/2}\text{ s}$$
$$= [(1/3\bullet4)\ ((0.93\text{-}0.91)^2 + (0.94\text{-}0.91)^2 + (0.88\text{-}0.91)^2 + (0.87\text{-}0.91)^2)]^{1/2}\text{ s} = 0.018\text{ s.}$$

The uncertainty associated with the circumferences of the beach ball, is found in much the same way as the average distance interval. The circumference of the ball is found to be 1.08 ± 0.0025m.

Given that the circumference = $\pi(2r)$ $\quad$ r = circumference / (2π) = 1.08m / (2)(3.14) = 0.172m

$\sigma(r) = r\,[\sigma(\text{circumference})/\text{circumference}] = (0.172\text{m})(0.0025\text{m}/1.08\text{m}) = 0.00046\text{m}$

$\therefore r = 0.172 \pm 0.00046\text{m}$

The volume of the beach ball is then equal to $4/3\ \pi r^3 = (4/3)(3.14)(0.172\text{m})^3 = 0.0213\text{m}^3$

The uncertainty of the volume can be determined using the following equation (where $y = ax^n$)

$\sigma(y) = (y)(n)[\sigma(x)/x] \implies \sigma(v) = (v)(3)[\sigma(r)/r] = (0.0213\text{m}^3)(3)(0.00046\text{m}/0.172\text{m}) = 0.00017\text{m}^3$

$\therefore v = 0.0213 \pm 0.00017\text{m}^3$

The density of the object (ρ) is equal to mass/volume:

$\rho = m/v = (0.078\text{kg})/(0.0213\text{m}^3) = 3.66\ \text{kg/m}^3$

The uncertainty of the density can be determined using the following equation:

$\sigma(\rho) = (\rho)[(\sigma(m)/m)^2 + (\sigma(v)/v)^2]^{1/2} = (3.66\ \text{kg/m}^3)[(0.001\text{kg}/0.078\text{kg})^2 + (0.00017\text{m}^3/0.0213\text{m}^3)^2]^{1/2}$

$= 0.055\ \text{kg/m}^3$

$\therefore \rho = 3.66 \pm 0.055\ \text{kg/m}^3$

To really nail this down, you might draw a parabola in with the appropriate value of a

Question: How do you know the thing is falling at terminal speed and not accelerating?

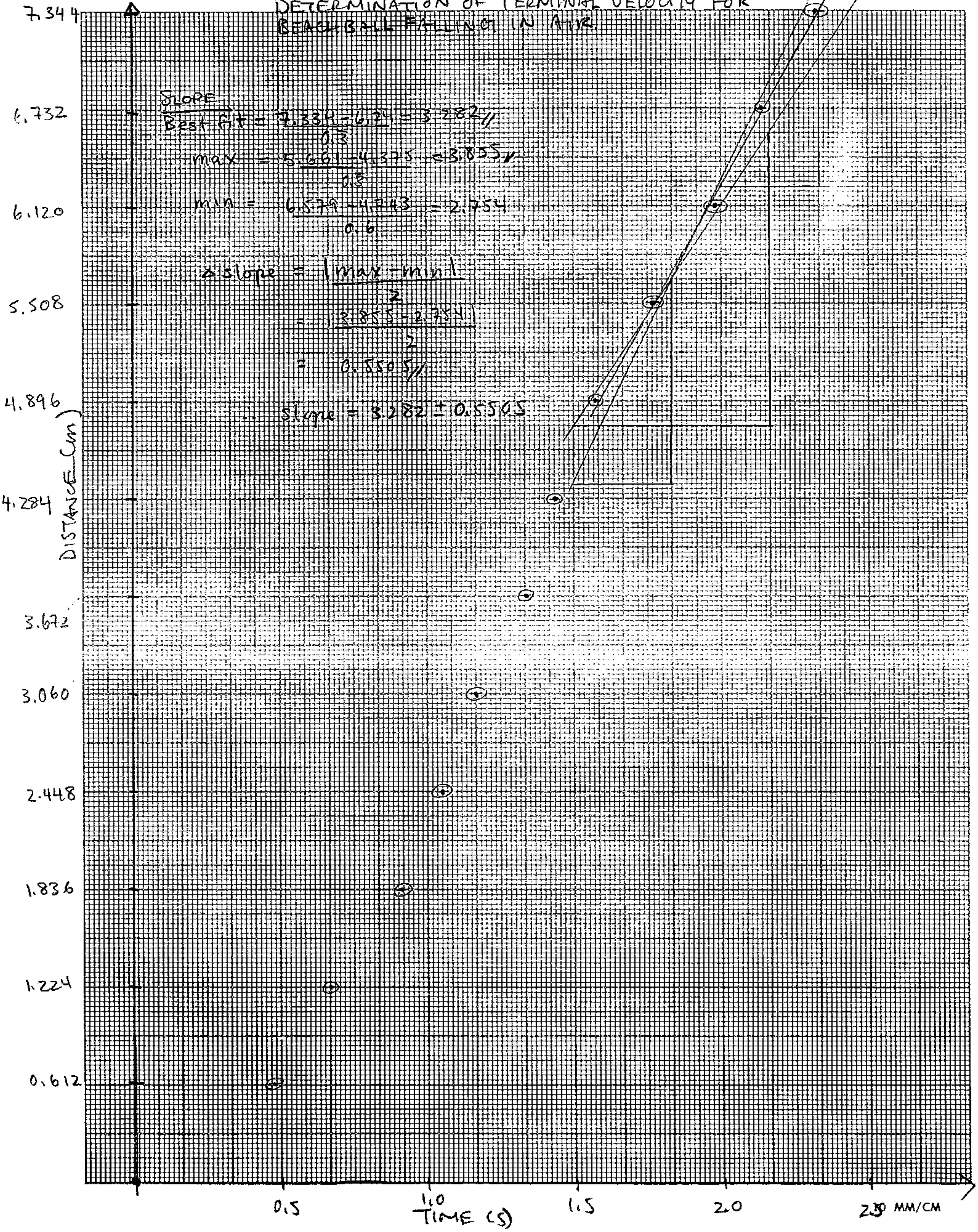
Figure 8 shows a plot of distance versus time. The slope and uncertainty calculations are done directly on the graph. The first eight points on the d-t plot follow a parabolic path, indicating that the position is a quadratic function of time, not linearly proportional to time, thus indicating the beach ball's acceleration. However, towards the end of the plot, the straight line relationship between distance and time clearly represents linear dependence, and therefore, the absence of acceleration. At this point, the object has reached terminal velocity, and continues traveling at the same velocity. The slope of this straight line consequently represents the ball's terminal velocity since $v = d/t$.

As shown in Figure 3, there are three forces acting upon the falling object which are weight (W), buoyancy (B) and friction/drag force (F). As the ball drops, it picks up speed, while the frictional force, which (when Stoke's Law is valid) is proportional to the velocity, increases. The velocity then becomes "terminal" when W = B + F. Thus, all interacting forces are cancelled, and the net force on the falling object becomes 0. From Newton's second law, it is known that a net force acting on a body will produce an acceleration. Conversely, if there are no net forces acting upon a body, there will be no acceleration. Thus, the acceleration of the ball decreases to zero and the ball reaches a constant velocity – the terminal velocity.

From the graph, it was determined that the terminal velocity of the beach ball in this investigation was 3.28 ± 0.551 m/s. $\therefore V_T = 3.28 \pm 0.551$ m/s

OK

FIGURE 8: DISTANCE vs. TIME
DETERMINATION OF TERMINAL VELOCITY FOR
BEACHBALL FALLING IN AIR.



air density

Coefficient of Viscosity

The coefficient of viscosity for air, η, was determined using the ball's terminal velocity and radius.

$V_T = 2R^2g(\rho - \rho_f)/9\eta \implies \boldsymbol{\eta = 2g(\rho - \rho_f)R^2/9\,V_T}$

1.29 0.046

$\eta = 2g(\rho - \rho_f)R^2/9\,V_T = 2(9.8\text{m/s}^2)(3.66\text{-}1.00\text{ kg/m}^3)(0.172\text{m})^2/9(3.28\text{m/s}) = 0.0522\text{ kg/m}\bullet\text{s} = \cancel{0.0522}\text{ Pa}\bullet\text{s}$

The uncertainty in the coefficient of viscosity is calculated as follows:

$\sigma(\eta) = (\eta)[(\sigma(\rho)/\rho)^2 + (\sigma(R)/R)^2 + (\sigma(V_T)/V_T)^2]^{1/2}$

$= (0.0522\text{ Pa}\bullet\text{s})[(0.055\text{ kg/m}^3/3.66\text{ kg/m}^3)^2 + (0.00046\text{m}/0.172\text{m})^2$

$+ (0.551\text{m/s} / 3.28\text{m/s})^2]^{1/2} = 0.00881\text{ Pa}\bullet\text{s} \qquad \therefore \eta = 0.0522 \pm 0.00881\text{ Pa}\bullet\text{s}$

Validity of Stoke's Law

The Reynold's Number can be calculated using the value for η isolated above and the following equation:

1.29

$RN = VR\rho_f/\eta = (3.28\text{ m/s})(0.172\text{m})(1.00\text{ kg/m}^3)/(0.0522\text{ kg/m}\bullet\text{s}) = \cancel{10.81}$ 15.6

The uncertainty associated with this value is calculated as follows:

$\sigma(RN) = (RN)[(\sigma(V)/V)^2 + (\sigma(R)/R)^2 + (\sigma(\eta)/\eta)^2]^{1/2}$

$= (10.81)[(0.551\text{ m/s} / 3.28\text{ m/s})^2 + (0.00046\text{m}/0.172\text{m})^2$

$+ (0.00881\text{Pa}\bullet\text{s}/0.0522\text{Pa}\bullet\text{s})^2]^{1/2} = 2.57 \qquad \therefore RN = 10.81 \pm 2.57$

Because this value of RN is appreciably greater than 1.0, Stoke's Law is therefore invalid for determining the equation for terminal velocity in this case.

For the previous experiment, whose Reynold's number was << 1.0, the flow around the sphere was laminar, and hence the viscous drag force is directly proportional to the speed of the sphere, that is, $F_v = kv$. When the Reynold's number is greater than 1, usually above a value of between 1-10, there is a turbulence behind the body known as the wake (track or path left by anything that has passed), rather than laminar flow where the flow is smooth and streamline. Turbulent flow is often characterized by erratic, small whirlpool-like circles called eddies, which absorb a great deal of energy. Although, the turbulence in this investigation may not be as erratic as this description suggests, neither can its movement be considered smooth. When turbulence is present, experiments show that drag force increases, not linearly, but as the square of speed, $F_v \propto v^2$. As can be imagined, the increase with speed is much more rapid than in the case of simple laminar flow. It is for this reason that the equation for terminal velocity, $V_T = 2R^2g(\rho - \rho_f)/9\eta$, from which the equation for η is derived, cannot be used for turbulent flow, that is, when the Reynold's Number is > 1. In the previous experiment, it was shown that one of the basics for deriving this equation for terminal velocity, involved the relationship, $F_v = kv$, since the viscous drag force is one of three forces acting on the falling object (see Figure 3). However, it has been established that this relationship for F_v

does not hold for turbulent flow where RN > 1, and thus Stoke's equation ($F_v = 6\pi R\eta V$) is invalid for determining the equation for terminal velocity in this case.

THEORETICAL RESULTS: COEFFICIENT OF VISCOSITY & REYNOLD'S NUMBER

Since it has been determined that Stoke's equation is invalid for determining the equation for terminal velocity, $V_T = 2R^2g(\rho - \rho_f)/9\eta$, it is reasonable to assume that the values calculated for the coefficient of viscosity and Reynold's number (both of which employed the formula for V_T) are invalid as well.

The **theoretical** coefficient of viscosity for air at 20°C is 1.8×10^{-5} Pa•s.
The **experimentally (invalid) determined** value for η was 0.0522 ± 0.00881 Pa•s.

Using the theoretical coefficient of viscosity for air at 20°C, a different Reynold's number can be isolated for:

Good

$RN = VR\rho_f/\eta = (3.28\ m/s)(0.172m)(1.00\ kg/m^3)/(1.8\times10^{-5}\ kg/m{\bullet}s) = 31342.22$

The uncertainty associated with this value is calculated as follows:

$\sigma(RN) = (RN)[(\sigma(V)/V)^2 + (\sigma(R)/R)^2]^{1/2}$

$= (31342.22)[(0.551\ m/s / 3.28\ m/s)^2 + (0.00046m/0.172m)^2]^{1/2}$

$= 5265.78$ $\therefore RN = 31342.22 \pm 5265.78$

Because this value of RN is incredibly greater than 1.0, Stoke's Law is therefore invalid for determining the equation for terminal velocity. When the Reynold's number approaches a value around 10^6, the drag force increases abruptly. Although the magnitude of this Reynold's Number is only in the order of 10^4, it is still quite high, and signifies that the viscous drag force cannot be merely defined by the laminar linear relationship that is the basis for the terminal velocity equation.

SOURCES OF ERROR

Because this experiment was performed on such a large scale, and involved such quick times, sources of error are very important when considering measurement uncertainties and the results. One of the main sources of error is human error or random error. Although it is well known to the experimenters that human or random error is not considered a genuine source of error and that it is usually accounted for in the uncertainty calculations, considering the nature of this experiment, it seems an imperative discussion topic. Human or random error was likely involved in levelling the ball prior to release, positioning the string over the hockey stick (so that the ball will be more likely to travel down the central axis), and the actual timing of the descent (which varies from trial to trial). Also, even though a rhythm was established between the ball releaser and the time recorder, it is very difficult to consistently remain synchronous.

However, the error sustained in these circumstances, through differing times of descent, was minimized through meticulousness, practice trials and also the number of trials performed. As the entire experiment including experimental setup, trials and measurements took place over a 4 hour period (midnight - 4 a.m.), sufficient time and care was provided for.

how about the fishing line? was it "slack" while falling -

Just as with the steel sphere in oil, the motion of the beach ball is likely affected by any irregularities in its surface shape. If the beach ball was not perfectly round, then it might travel at slightly different speeds depending on its orientation in space when dropped. One of the factors that might contribute to surface irregularities in the beach ball, is the fact that the multi-coloured segments of the ball are fused by a seam (that minimally protrudes above the surface of the ball). Furthermore, to make sure that the volume of the ball remained the same, it was ensured that the air level inside the ball (compressibility of the ball) was consistent. However, slight changes in this property may have affected the times of its descent.

Since the experiment was performed in an indoors environment, late at night, such things as human movement, breeze, and major temperature fluctuations were avoided. While it is known that the viscosity of gases increase with temperature, such effects are likely negligible, especially since the experiment site was quite controlled. Lastly, to the best of our ability, it was ensured that the point of release of the balls was in the centre of the cage, and that the ball was still prior to release. This helped to increase the chance of the ball traveling down the central axis, and decrease the chances of wild trajectories.

CONCLUSION

The viscosity of air was calculated by measuring the time it took a beach ball to fall certain distances through the air. Using these times and distances, terminal velocity could be calculated. The slope of the linear portion of the d vs. t plot represents the terminal velocity of the ball and was found to be 3.28 ± 0.551 m/s. Using the value of V_T / R^2, the value of η for air was determined to be 0.0522 ± 0.00881 Pa•s. The Reynolds number was calculated to be 10.81 ± 2.57, and because this value is appreciably greater than 1.0, Stoke's Law is therefore invalid for determining the equation for terminal velocity in this case. Thus the formula used to calculate the coefficient of viscosity ($V_T = 2R^2g(\rho - \rho_f)/9\eta$) is invalid, and thus the calculated value is also invalid. The theoretical coefficient of viscosity for air at 20°C is 1.8×10^{-5} Pa•s.

CONCLUSION

Having performed both experiments, it is clear that the differences between answering an intro physics question regarding a falling sphere, and analyzing the falling sphere in 'real-life' are great and often complex. One must always be careful, both in physics and any other field, to ensure that any related assumptions made are sound. Not only was the assumption regarding the absence of frictional forces on falling objects needed to be overcome, but also, the assumption regarding the type of motion a fluid is experiencing. The former assumption was quite obvious, but the latter assumption was something newly discovered. The Reynold's number was used to determine the validity of Stoke's Law, and subsequently the validity of the terminal velocity equation (which was used to determine the coefficients of viscosity). The validity of Stoke's Law is key, since it distinguishes between laminar and non-laminar (turbulent) motion, a difference that can drastically affect calculations. The key physical process in the flow of real fluids is the conversion of mechanical energy into heat that is the result of viscosity in either laminar or turbulent flow. As discussed before, the study of falling objects often involves considering merely the gravitational force. However, it eventually becomes necessary to say something about the medium through which the object falls, and the type of motion that medium experiences. In the analysis of two practical applications, this project has done just that.

Assessment

Good experiment, well executed + written up

Nice setup with beachball — well thought out

Some minor probs with the calculations etc that are noted in the relevant places.

19/20

Who Did What?

Brainstorming Ideas regarding possible experiments – Jonathan and Vance
Performing the 1st year Physics Lab (Steel Ball in Oil) – Jonathan and Vance
Research regarding "Fluid Viscosity and Terminal Velocity" – Jonathan and Vance
Setting up and performing the "Beach Ball" experiment – Jonathan and Vance
Taking pictures - Jonathan
Developing the pictures – Vance
Project write-up and graphs – Jonathan and Vance

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