14.3 – 14.5 including p.4 (1)

Jan 25, 2019

Simple harmonic motion (cont'd) (14.2)

- Velocity and acceleration

$\frac{d^2x}{dt^2} = -\omega^2 x$, $\omega = \sqrt{\frac{k}{m}}$

Review:

$x = A\cos(\omega t + \phi)$

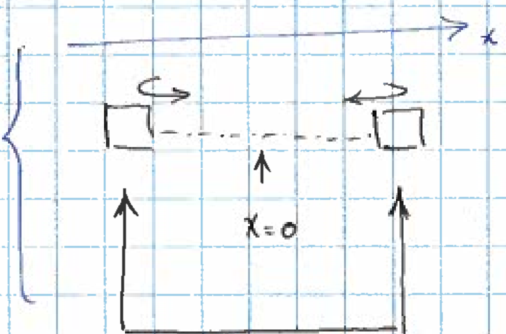
$V = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi)$

$a = \frac{d^2x}{dt^2} = -\omega^2 A \cos(\omega t + \phi) = -\omega^2 x$

are cyclic

$\rightarrow$ $V_{max} = \omega A$, where ? at $x = 0$

and $a_{max} = \omega^2 A$, at $x = \pm A$



a_{max}

Note that $V = 0$ at the endpoints (for a moment)

example : 14-~~14~~ 18 (to get a feel for size of SHM acceleration)

(demo) tuning fork : ~~264~~ 441 Hz cycles/second (vibrating) $A = 1.5\,mm$ (at the tip) } max acceleration of the tip?

$a_{max} = \omega^2 A$

ω : $f = \cancel{264}\ 441\ Hz = \cancel{264}\ 441\ s^{-1} = \frac{1}{T}$ } $\omega = \frac{2\pi}{T} = 2\pi f = \cancel{1659}\ 2771\ s^{-1}$

$a_{max} = \omega^2 A = (\cancel{1659}\ 2771)^2 (1.5 \times 10^{-3} m)$

$f \left[\frac{cycles}{sec}\right]$

$T \left[\frac{sec}{cycle}\right]$ } $f = \frac{1}{T}$

$= \cancel{4127}\ 11\,517\ m/s^2$

$\sim \cancel{420}\ 1200\ g's$ (!)

Energy of SHM (Use spring as example, but results can be generalized)

SPRING: $K = \frac{1}{2}mv^2$ and $U = \frac{1}{2}kx^2$

total energy $E = K + U$, let $\phi = 0$ for convenience

$$= \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$= \frac{1}{2}m(\omega^2A^2\sin^2\omega t) + \frac{1}{2}k(A^2 \cancel{\omega} \cos^2\omega t)$$

$$= \frac{1}{2}m\omega^2A^2\sin^2\omega t + \frac{1}{2}kA^2\cos^2\omega t$$

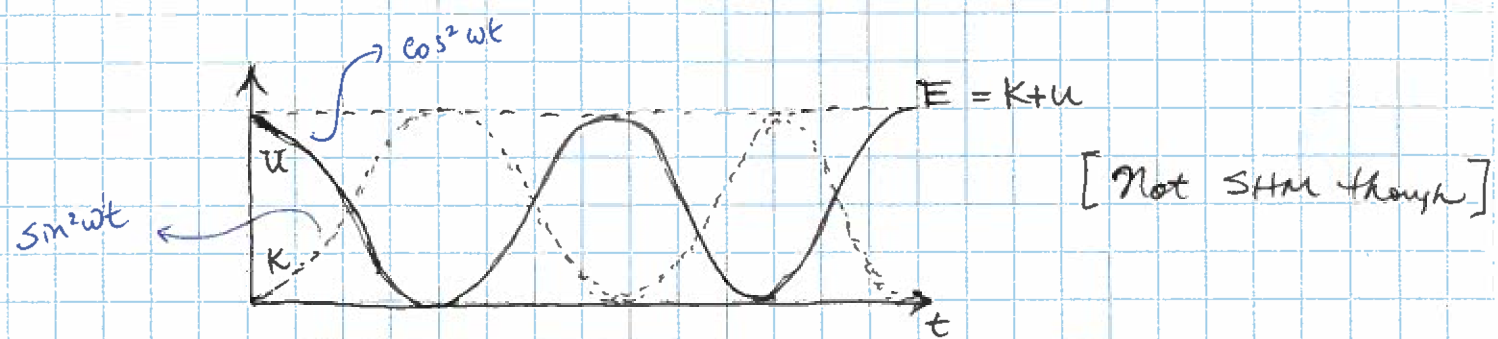
$$= \frac{1}{2}\cancel{m}\frac{k}{\cancel{m}}A^2\sin^2\omega t + \frac{1}{2}kA^2\cos^2\omega t$$

$$= \frac{1}{2}kA^2(\sin^2\omega t + \cos^2\omega t)$$

$$= \frac{1}{2}kA^2$$

1 Constant total E 2 $\propto$ square of the amplitude.

K and U oscillate out of phase relative to each other:



Where is $K = U$? $\frac{1}{2}mv^2 = \frac{1}{2}kx^2 \Rightarrow x = \pm\sqrt{\frac{m}{k}}\,v$

WHEN?

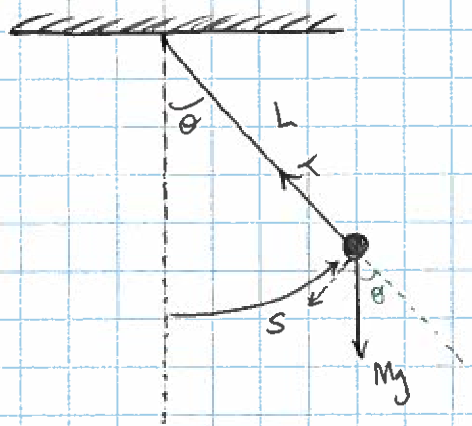
$\Rightarrow A\cos\omega t = \pm\sqrt{\frac{m}{k}}\,A\omega\sin\omega t$

$\cos\omega t = \pm\,\cancel{\sqrt{\;}}\,\sin\omega t \Longrightarrow \tan\omega t = \pm 1$

$\therefore\ \omega t = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4} \ldots \Rightarrow t = \frac{\pi}{4\omega}, \frac{3\pi}{4\omega}, \frac{5\pi}{4\omega} \ldots$

Another example of a system that has SHM :

The Pendulum



$s = L \cdot \theta$ displacement

restoring force $F = -mg\sin\theta$ (like $-kx$)

equation of motion } → Newton's 2nd law

$\therefore \quad ma = \cancel{m}\frac{d^2s}{dt^2} = -\cancel{m}g\sin\theta$

$$L\frac{d^2\theta}{dt^2} = -g\sin\theta$$

recall

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta \qquad \left(\text{SHM: } \frac{d^2x}{dt^2} = (-\omega^2)x\right) = -\frac{k}{m}x)$$

?

approximation : $\sin\theta \approx \theta$ (radians) for small θ

$- \frac{\theta^3}{3!} + \dots$

$$\text{so} \quad \frac{d^2\theta}{dt^2} \cong -\frac{g}{L}\theta$$

$\Rightarrow$ solution $\theta = \cancel{\theta_0}\, A\theta_0 \cos(\omega t + \phi)$ with $\omega = \sqrt{\frac{g}{L}}$

period of oscillation $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$

- ind. of mass , ind. of A θ_0

→ Galileo, Huygens

applications {
→ pendulum clock (accurate) (grandfather clock)
↳ until ~ 1927 → Quartz clock
→ Use pendulum for accurate measurement of g.
(at different locations)