Art Sci 2D06 Tutorial

March 16th 2020

Question 1

For an object of macroscopic size (e.g a baseball, car airplane), an uncertainty in its momentum of only one percent yields a limiting uncertainty in its position that:

(a) is also one percent.

(b) is so small as to be immeasurable.

(c) is quite large.

(d) does not exist.

(e) is 50%

Question 2

An electron is confined to a region of space of length 0.1 nm ($\sim$ size of an atom). What is the minimum uncertainty of its momentum?

Assume $p \sim \Delta p_{min}$;

- What is the electron's speed?
- What is the kinetic energy (in eV?)

Question 3

An electron materalizes out of nowhere and out of nothing - How long will it stick around? (Assume it is at rest)

Useful Equations

Heisenberg's Position-Momentum uncertainty principle

$$\Delta x \Delta p \sim \hbar \tag{1}$$

- Δx is the uncertainty in position
- Δp is the uncertainty in momentum
- $\hbar$ is the reduced Planck's constant: $1.05 \times 10^{-34} J \cdot s = \frac{h}{2\pi}$

Heisenberg's Time-Energy uncertainty principle

$$\Delta E \Delta t \sim \hbar \tag{2}$$

- ΔE is the uncertainty in energy
- Δt is the uncertainty in time
- $\hbar$ is the reduced Planck's constant: $1.05 \times 10^{-34} J \cdot s = \frac{h}{2\pi}$

Mass-Energy Equivalence

$$E = mc^2 \tag{3}$$

- m is the mass of the particle
- E is the energy equivalent of the mass
- c is the speed of light: 3×10^8 m/s

Momentum Equation

$$p = mv \tag{4}$$

Kinetic energy equation:

$$K = \frac{p^2}{2m} \tag{5}$$

Mass of electron: 9.1×10^{-31} kg

Question 1 Solution

Using Heisenberg's position-momentum uncertainty principle:

$$\Delta x \cdot \Delta p \sim \hbar$$
$$\Delta x \sim \frac{\hbar}{\Delta p}$$

Since $\hbar << 1$, Δx is negligible - the answer is (b)

Question 2 Solution

Again, using Heisenberg's position-momentum uncertainty principle:

$$\Delta x \cdot \Delta p \sim \hbar$$
$$\Delta p \sim \frac{\hbar}{\Delta x}$$

To find the minimum possible uncertainty in momentum, we should assume the highest possible uncertainty for the position of the electron.

Given an electron in a 0.1 nm space, the maximum position uncertainty that it can have is $\Delta x = 0.1$ nm.

$$\Delta p_{min} \sim \frac{\hbar}{0.1 \times 10^{-9}}$$

$$\sim \frac{1.05 \times 10^{-34}}{0.1 \times 10^{-9}}$$

$$\sim 5.3 \times 10^{-25} \text{ kg} \cdot \text{m/s}$$

Next, we are asked to assume $p \sim p_{min}$; that is, the actual momentum of the electron is of similar magnitude as the minimum uncertainty of momentum we have just calculated.

If we want to find the velocity of this electron, let's use the usual momentum equation:

$$p = \Delta p_{min} = mv$$
$$v = \frac{m}{\Delta p_{min}} = \frac{5.3 \times 10^{-25}}{9.1 \times 10^{-31}}$$
$$v \sim 6 \times 10^5 \text{ m/s}$$

Which is approximately 1% of the speed of light!

If we want to find the kinetic energy:

$$K = \frac{p^2}{2m} = \frac{\Delta p_{min}^2}{2m}$$
$$= 1.5 \times 10^{-19} \text{ J} \sim 1 \text{ eV}$$

Which is of the right order of magnitude of energies for electrons in atoms.

Question 3 Solution

"Virtual particles" can appear from vacuum for an amount of time. This is because mass and energy have an equivalency based on Einstein's mass-equivalence relationship. Therefore, if there is an uncertainty in energy equal to the mass-energy of the electron, it can exist for a (short) time, governed by Heisenberg's time-energy uncertainty principle.

Let's first convert the mass of the electron to an energy, using the mass-energy equivalence equation:

$$E = mc^2$$
$$= (9.1 \times 10^{-31})(3 \times 10^8)^2$$
$$= 5.5 \times 10^{-14} \text{ J}$$

Now we can employ Heisenberg's time-energy uncertainty principle (and assume the uncertainty in energy is the mass-energy of the electron) to see how long it could exist for:

$$\Delta E \Delta t \sim \hbar$$
$$\Delta t \sim \frac{\hbar}{\Delta E} = \frac{1.05 \times 10^{-34}}{5.5 \times 10^{-14}}$$
$$\sim 10^{-21} \text{ s}$$

So if an electron does pop into existence out of nowhere, from nothing, it only sticks around for a very short amount of time, 10^{-21} s. Additionally, the heavier the particle, the less time it's able to stay in existence.