- Questions for tutorial (SHM)

Question 14.8: 2 equal masses, attached to separate identical springs next to one another.

- One mass: $x_0 = 20\,cm$
- the other: $x_0 = 10\,cm$

which mass reaches equilibrium first ($x = 0$)?

first notice that since $\omega = \sqrt{\frac{k}{m}}$ and $T = 2\pi\sqrt{\frac{m}{k}}$ → they have the same Period and angular frequency.

upon release: SHM → general solution $x = A\cos(\omega t + \phi)$

The acceleration is given by

$$a = \frac{d^2x}{dt^2} = -\omega^2 A\cos(\omega t + \phi)$$

the acceleration upon release is maximum at the time of release $t = 0$

$$a_{max} = \omega^2 A \quad \text{(magnitude)}$$

So mass (1) has a larger initial acceleration than mass (2), since ω is independent of x_0

→ however, (1) also has a longer distance to cover

→ they should reach $x = 0$ at the same time.

Question 14.13 Does a car bounce on its springs faster when it is empty or when it is fully loaded

"faster" → higher ω

$\omega \propto \sqrt{\frac{1}{m}}$ → ω is smaller when the car is loaded

Problem 14.27

2 kg mass → oscillates according to $x = 0.65 \cos 8.40t$

x in meter
t in seconds

(a) amplitude : maximum value of x : 0.650 m

(b) frequency : $\omega = 8.40\ s^{-1}$

$$f = \frac{\omega}{2\pi} = \frac{8.40}{2\pi} = 1.34\ s^{-1}$$

(c) ① maximum speed :

$$V = -(0.65)(8.4) \sin 8.40t$$

$$V_{max} = 0.65 \cdot 8.4 = 5.5\ m/s$$

② or $k = \omega^2 \cdot m = 141.12\ N/m$

$E = U_{max} = \frac{1}{2} k x_{max}^2 =$

$\frac{1}{2}(141.12)(0.65)^2 = 30\ J.$

~~(d)~~ the total energy is $E = K_{max} = \frac{1}{2} m V_{max}^2 = 30\ J$

(d) ~~$V^2 = V_0^2 \left(1 - \frac{x^2}{A^2}\right)$~~ Potential energy

~~$A^2V^2 = A^2V_0^2 - x^2V_0^2$~~ $U =$

@ x : $E = \frac{1}{2}mV^2 + \frac{1}{2}kx^2 = \frac{1}{2} m V_{max}^2$

$$V^2 = \frac{m V_{max}^2 - kx^2}{m}$$

$$= V_{max}^2 - \frac{k}{m}x^2 = V_{max}^2 - \omega^2 x^2$$

$= (5.5)^2 - (8.4)^2 (0.26)^2$

$= 30.25 - (70.56)(0.068)$

$= 30.25 - 4.766$

$= 25 \Rightarrow V = 5\ m/s$

e) $K = \frac{1}{2}mV^2 = 25\ J$

$U = E - K = 5\ J$