David St-Amant Farmer
PHYS 4E03

March 23, 2020

Assignment #7: Solutions

$\boxed{1}$ (4.1)

① We have $\mu_{\Omega^-} = 3\mu_s = 3\left(-\frac{e}{3}\frac{1}{2M_s}\right) \approx -2.02\,\mu_N$

$\Rightarrow \mu_s \approx -0.673\,\mu_N$, $\mu_N \approx 0.105155\ e\cdot fm$
and $\hbar c = 1 \approx 197.327\ MeV\cdot fm$

Then, $\mu_p = \frac{4}{3}\mu_u - \frac{1}{3}\mu_d \approx 2.793\,\mu_N$

$\mu_n = \frac{4}{3}\mu_d - \frac{1}{3}\mu_u \approx -1.913\,\mu_N$

$\Rightarrow \mu_{up} \approx 1.852\,\mu_N$ and $\mu_{down} \approx -0.972\,\mu_N$

② Since we assume $\mu_i = \frac{Q_i}{2M_i}$, we get:

$\Rightarrow \boxed{M_u} = \frac{2e}{3}\cdot\frac{1}{2\mu_u} \approx 0.18\,\frac{e}{\mu_N} \approx \boxed{338\ MeV}$

$M_d = \frac{-1e}{3}\cdot\frac{1}{2\mu_d} \approx 0.171\,\frac{e}{\mu_N} \approx 321\ MeV$

$M_s = \frac{-1e}{3}\,\frac{1}{2\mu_s} \approx 0.248\,\frac{e}{\mu_N} \approx 465\ MeV$

③ Using these values for μ_u, μ_d and μ_s:

- $\mu_{\Lambda^0} \approx -0.673\,\mu_N$
- $\mu_{\Xi^0} \approx -1.515\,\mu_N$
- $\mu_{\Sigma^-} \approx -1.072\,\mu_N$
- $\mu_{\Xi^-} \approx -0.573\,\mu_N$
- $\mu_{\Sigma^0} \approx 0.811\,\mu_N$
- $\mu_{\Sigma^+} \approx 2.694\,\mu_N$

$\boxed{2}$ (4.3) We start with $\frac{d\hat{\sigma}_a}{dq^2} = -\frac{d\hat{\sigma}}{dt} = \frac{2\pi\alpha^2}{q^4}\left(\frac{\hat{e}_a^2}{e^2}\right)\left[\frac{(\hat{s}-q^2)^2+\hat{s}^2}{\hat{s}^2}\right]$

We assume incoherent scattering from the proton's quarks, neglecting their mass as well.

We have $\underline{\hat{s} \simeq -2\hat{p}\cdot K \simeq -2xp\cdot K \simeq xs}$ since $\hat{p}^\mu \equiv xp^\mu$, $x \in [0,1]$. Also, $x = \frac{q^2}{2M\nu}$

$$\Rightarrow \frac{d\sigma}{dq^2} = \sum_a \frac{d\sigma_a}{dq^2} = \sum_a \left(\int_0^1 dx\, P_a(x) \frac{d\hat{\sigma}_a}{dq^2}\right)$$

To impose $x = \frac{q^2}{2M\nu}$, insert $1 \equiv \int d\nu\, \delta\left(\nu - \frac{q^2}{2Mx}\right)$

$$\Rightarrow \frac{d\sigma}{dq^2} = \sum_a \int_0^1 dx \int d\nu\, P_a(x) \frac{d\hat{\sigma}_a}{dq^2}\, \delta\left(\nu - \frac{q^2}{2Mx}\right)$$

$$\Rightarrow \frac{d\sigma}{dq^2\, d\nu} = \sum_a \int_0^1 dx\, P_a(x) \frac{d\hat{\sigma}_a}{dq^2}\, \delta\left(\nu - \frac{q^2}{2Mx}\right)$$

We can simplify $\frac{d\hat{\sigma}_a}{dq^2} = \frac{2\pi\alpha^2}{q^4}\left(\frac{\hat{e}_a^2}{e^2}\right)\underbrace{\left(\frac{(\hat{s}-q^2)^2+\hat{s}^2}{\hat{s}^2}\right)}_{(+)}$

$$\Rightarrow (+) = 1 + \left(\frac{\hat{s}-q^2}{\hat{s}}\right)^2$$

$$= 1 + \left(1 - \frac{q^2}{xs}\right)^2$$

$$\Rightarrow \frac{d\sigma}{dq^2\,d\nu} \boxed{=} \frac{2\pi\alpha^2}{q^4}\int_0^1 dx \left[\sum_a P_a(x)\left(\frac{\hat{e}_a^2}{e^2}\right)\right]\left[1 + \left(1 - \frac{q^2}{xs}\right)^2\right]\delta\left(\nu - \frac{q^2}{2mx}\right)$$

We define $P(x) \equiv \sum_a P_a(x)\frac{\hat{e}_a^2}{e^2}$

$$\boxed{=} \frac{2\pi\alpha^2}{q^4}\int_0^1 dx\, P(x)\left(1 + \left(1 - \frac{q^2}{xs}\right)^2\right)\delta\left(\nu - \frac{q^2}{2mx}\right)$$

Use $\delta(b(x)) = \dfrac{\delta(x - x_0)}{\left|\frac{\partial b}{\partial x}\right|_{x_0}}$

$$\Rightarrow \delta\Big(\underbrace{\nu - \frac{q^2}{2mx}}_{=0}\Big) = \frac{\delta\left(x - \frac{q^2}{2m\nu}\right)}{\left|\frac{q^2}{2mx_0^2}\right|} = \frac{2mx_0^2}{q^2}\,\delta\Big(x - \underbrace{\frac{q^2}{2m\nu}}_{x_0}\Big)$$

$$\Rightarrow x_0 = \frac{q^2}{2m\nu}$$

Thus, $\frac{d\sigma}{dq^2 d\nu} = \frac{2\pi\alpha^2}{q^4}\int_0^1 dx\, P(x)\left(1+\left(1-\frac{q^2}{xs}\right)^2\right)\frac{2Mx_0^2}{q^2}\,\delta\!\left(x-\frac{q^2}{2M\nu}\right)$

$$\boxed{=}\ \frac{2\pi\alpha^2}{q^4}P(x)\left(1+\left(1-\frac{q^2}{xs}\right)^2\right)\frac{2Mx^2}{q^2}\Bigg|_{x=\frac{q^2}{2M\nu}}$$

We can now use $\nu = w - w'$ and from (4.2) we have $s = -2M\cdot p = 2wM$, $t = -4ww'\sin^2\left(\frac{\theta}{2}\right)$
$= -q^2$

$$\boxed{=}\ \frac{2\pi\alpha^2}{q^4}P(x)\left(1+\left(1-\frac{2M(w-w')}{2Mw}\right)^2\right)\frac{4ww'\sin^2\left(\frac{\theta}{2}\right)}{2M(w-w')^2}$$

$$= \frac{2\pi\alpha^2}{q^4}P(x)\left(1+\left(\frac{w'}{w}\right)^2\right)\left(\frac{w'}{w}\right)\frac{2}{M}\frac{\sin^2(\theta/2)}{\left(1-\frac{w'}{w}\right)^2}$$

$$= \frac{4\pi\alpha^2}{Mq^4}\left(\frac{w'}{w}\right)P(x)\sin^2(\theta/2)\underbrace{\left[\frac{(w^2+w'^2)}{w^2}\cdot\frac{w^2}{(w-w')^2}\right]}_{(+)}$$

$$(+) = \frac{w^2+w'^2}{\nu^2} = \frac{(w-w')^2+2ww'}{\nu^2} = 1+\frac{2ww'}{\nu^2}$$

$$\Rightarrow \frac{d\sigma}{dq^2\,d\nu} = \frac{4\pi\alpha^2}{M q^4}\left(\frac{w'}{w}\right) P(x)\left(\sin^2\!\left(\frac{\theta}{2}\right) + \underbrace{\frac{2ww'\sin^2(\theta/2)}{\nu^2}}_{=\,\frac{q^2}{2\nu^2}\,=\,\frac{xM}{\nu}}\right)$$

$$\boxed{= \frac{4\pi\alpha^2}{M q^4}\left(\frac{w'}{w}\right) P(x)\left(\sin^2\!\left(\frac{\theta}{2}\right) + \frac{xM}{\nu}\right)\Bigg|_{x = \frac{q^2}{2M\nu}}}$$

■

$\boxed{3}$ (4.4) We want to show $\int_0^1 \frac{dx}{x}\left(F_2^{ep}(x) - F_2^{en}(x)\right) = \frac{1}{3}$ } (*)

From (4.33), $F_2(x) = x\,P(x)$ where from (4.31) $P(x) = \frac{4}{9}\left(P_u + P_{\bar{u}}\right) + \frac{1}{9}\left(P_d + P_{\bar{d}} + P_s + P_{\bar{s}}\right)$

$$\Rightarrow (*) = \int_0^1 \frac{dx}{x}\left(x P^p - x P^n\right) = \int_0^1 dx\left(P^p - P^n\right)$$

$$= \int_0^1 dx \left[\frac{4}{9}\left(\left(P_u^p - P_{\bar{u}}^n\right) - \left(P_u^n - P_{\bar{u}}^p\right) \right) + \frac{1}{9}\left(\left(P_d^p - P_{\bar{d}}^n\right) - \left(P_d^n - P_{\bar{d}}^p\right) + \left(P_s^p - P_{\bar{s}}^n\right) - \left(P_s^n - P_{\bar{s}}^p\right) \right) \right]$$

Here we assume $P_{\bar{u}} \approx P_{\bar{d}} \approx P_{\bar{s}}$ for p and n

$$\Rightarrow (*) = \int_0^1 dx \left[\frac{4}{9}\left(\left(P_u^p - P_{\bar{u}}^p\right) - \left(P_u^n - P_{\bar{u}}^n\right) \right) + \frac{1}{9}\left(\left(P_d^p - P_{\bar{d}}^p\right) - \left(P_d^n - P_{\bar{d}}^n\right) + \left(P_s^p - P_{\bar{s}}^p\right) - \left(P_s^n - P_{\bar{s}}^n\right) \right) \right]$$

Now use the integrals on p. 114

$$= \frac{4}{9}\left(2-1\right) + \frac{1}{9}\left(1-2+0+0\right)$$

$$= \frac{4}{9} - \frac{1}{9} = \boxed{\frac{1}{3}} \blacksquare$$