**Physics 3C3 – Analytical Mechanics**

**Lecture Titles Winter 2010**

1. Introductory remarks; Review of Newton's laws; $N$-particle system: Center of momentum
. coordinate system; $\mathbf{L} = \mathbf{L}_{CM} + \mathbf{L}'$.
2. **Central forces**; Kepler's laws for motion of a planet. Elliptical orbits.
3. Hyperbolic orbits in $1/r$ potential for positive $E$. Rutherford's experiment.
4. **Scattering theory**, Two-particle problem equivalent to one-body with effective mass.
. Scattering angle and differential cross-section.
5. Transformation from inertial to a **rotating coordinate system**. Coriolis and centrifugal
. forces.
6. Coriolois force on a falling mass; Foucault pendulum.
7. d'Alembert's principle concerning forces of constraint. **Lagrange equations** of motion.
8. Examples of Lagrange equations, and their relation to Calculus of Variations.
9. **Hamilton's principle** of Stationary Action as basis of mechanics. Lagrange eqs. with
. constraints. Least squares fits with constraints, by analogy.
10. Cyclic (ignorable) coordinates. Canonical momentum $p = \partial L/\partial \dot{q}$. The **Hamiltonian**
. as a Legendre transform of $L$. Hamilton's equations of motion.
11. **Small oscillations** around equilibrium: coupled oscillators. Orthogonality of normal
. modes.
12. Modal matrix. Transform to normal coodinates.
13. Waves on a chain of $N$ point masses: tight binding model Lagrangian. **Normal modes**
. of the $N$-site chain; solution by Chebyscheff polynomials.
14. **Rigid Boby Dynamics**: Inertia tensor, principal axes and moments of inertia.
. Ellipsoid of inertia. $\mathbf{L}$ not parallel $\vec{\omega}$.
15. Euler's equations. Precession of symmetric top. Stability of rotation of an asymmetric
. rigid body about certain axes.
16. Poinsot's geometrical picture for torque free motion of an asymmetric rigid body.
17. **Euler angles** as dynamical variables for the symmetric top.
18. Torque free motion of the symmetric top: body fixed frame vs. inertial frame.
19. Symmetric top in gravitational field: precession and nutation.
20. Horizontal (pseudo-regular) precession of a gyroscope.
21. **Hamiltonian Dynamics**: Hamilton's equations in phase space; charged particle in an
. EM field: canonical momentum $\vec{p} = m\vec{v} - q\vec{A}/c$
22. **Canonical transformations** in phase space: e.g. $F_1(q, Q)$ for harmonic oscillator.
23. Four types of generators $F_1, \cdots F_4$ which preserve structure of Hamilton's equations.
24. **Hamilton-Jacobi** eqn. for Principal function $S(q, P, t)$. Relation to wave mechanics.
25. Hamilton's Characteristic function $W(q, P) - Et$ when $H(q, p) = E$. Separable systems.
26. **Action-angle variables** for separable and periodic systems; e.g. Kepler problem;
. Bohr-Sommerfeld quantisation rule $J_i = n_i h$.
27. **Poisson Brackets**, reln. to commutators in QM. $H(q, p)$ generates time development.
. Symplectic structure of phase space. Fundamental Poisson brackets $[q_i, p_j] = \delta_{ij}$.
28. Active and passive views of canonical transforms. Relation to Heisenberg picture in QM.
. Canonically invariant quantities: Poisson brackets, phase space volume, action.
29. Liouville's theorem. Similarity of classical mechanics to geometrical optics and Hamilton
. Jacobi theory to wave optics. The virial theorem.

30. **Chaos in classical mechanics.** Logistic equation as example. Feigenbaum plot;
. bifurcations, fractal structure; e.g. Cantor set.
31. Toroidal structure of phase space for integrable systems; regular and chaotic orbits
. in the Henon-Heiles potential.
32. Poincaré maps (sections of phase space); nested tori and onset of chaos.
33. 1D and 2D autonomous systems; elliptical and hyperbolic orbits near stable/unstable
. fixed points.
34. Review of Transformation theory
35. Review of Action-Angle variables. Classical perturbation theory.
36. Adiabatic conditions and fast perturbations.