Separable ~~Integrable~~ Hamiltonians

For systems with 1 degree of freedom, we can always solve the equs of motion. e.g. by HJ Theory

For n degrees of freedom the HJ equ cannot in general be solved unless it is SEPARABLE

$$S(q_1 \ldots q_n, \alpha_1 \ldots \alpha_n) = \sum_{k=1}^{n} S_k(q_k, \alpha_1 \ldots \alpha_n)$$

n terms, each depending on just 1 coord.

One simple class of systems for which this is true is given by Hamiltonians which are the sum of n-independent parts

$$H(p_1 \ldots p_n, q_1, \ldots q_n) = \sum_{k=1}^{n} H_k(p_k, q_k)$$

e.g. n-uncoupled oscillators

then $H_k\left(\frac{\partial S}{\partial q_k}, q_k\right) = \alpha_k \qquad k=1, \ldots n$

$$\alpha = \alpha_1 + \alpha_2 + \ldots \alpha_n = H'$$

Sometimes H is not written in this form but in appropriate coords it becomes so.

If separation is possible then ~~each~~

$$p_k = \frac{\partial}{\partial q_k} S_k(q_k, \alpha_1 \ldots \alpha_n)$$

each p_k is function of just one q_k

If motion is periodic in each q_k then can introduce Action variables

$$I_k = \frac{1}{2\pi} \oint_C p_k (q_k, \alpha_1 \ldots \alpha_n)\, dq_k$$

sub I_k into S and then use

$$\Theta_k = \frac{\partial S}{\partial I_k} = \sum_{m=1}^{n} \frac{\partial}{\partial I_k} S_m (q_m, I_1 \ldots I_n)$$

angle variable conjugate to I_k.

then we have
$$\dot{I}_k = -\frac{\partial}{\partial \Theta_k} H'(I_1, \ldots I_n) = 0$$
$$\dot{\Theta}_k = \frac{\partial}{\partial I_k} H'(I_1, \ldots I_n) = \omega_k (I_1, \ldots I_n)$$

$$\Rightarrow \quad I_k = \text{const}$$
$$\Theta_k = \omega_k (I_1, \ldots I_n) + \delta_k$$