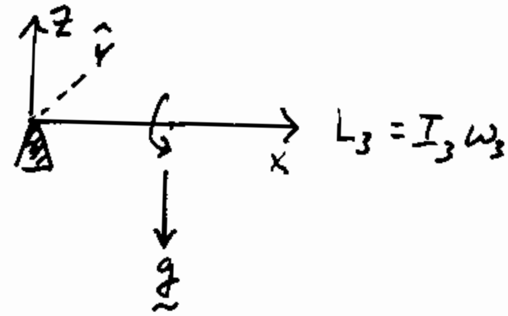
Horizontal Precession of Gyroscope:

Elementary view



$\underset{\sim}{r} = (\ell, 0, 0)$

$\underset{\sim}{F} = (0, 0, -mg)$

$\underset{\sim}{\Gamma} = \underset{\sim}{r} \times \underset{\sim}{F} = mg\ell(0\hat{y}) = mg\ell\,\hat{\phi} = \dfrac{mg\ell}{|L|}(0\hat{z} \times \underline{L}) = \dfrac{dL}{dt}$

$\dot{\alpha} = mg\ell / |L|$ precesses around $0z$.

More detailed study: $\dot{\alpha}_0 = \dfrac{P_\alpha - P_\gamma \cos\beta_0}{I_1 \sin^2\beta_0} \to \dfrac{P_\alpha}{I_1}$ IF $\beta_0 = \pi/2$

But $P_\gamma \cos\beta \simeq P_\alpha$ near β_0 so must be careful.

β_0 determined by $\left(\dfrac{\partial V}{\partial \beta}\right)_{\beta_0} = 0$

$\cos^2\beta_0$: $P_\alpha^2 - 2P_\alpha P_\gamma \cos\beta_0 + P_\gamma^2 - P_\alpha P_\gamma \dfrac{\sin^2\beta_0}{\cos\beta_0} + I_1 mg\ell \dfrac{\sin^4\beta_0}{\cos\beta_0} = 0$

$(P_\gamma \cos\beta_0)^2 - (P_\gamma \cos\beta_0) P_\alpha \underset{(1+\cos^2\beta_0)}{(2\cos^2\beta_0 + \sin^2\beta_0)} + \underline{P_\alpha^2 \cos^2\beta_0}$ small

$+ I_1 mg\ell \underbrace{\sin^4\beta_0}_{\simeq 1 - 2\cos^2\beta_0} \cos\beta_0 = 0$

Rule: keep $(P_\gamma \cos\beta_0)$ or P_α

but drop relative order $\cos^2\beta_0$ terms.

$(P_\gamma \cos\beta_0)^2 - P_\alpha (P_\gamma \cos\beta_0) + I_1 mg\ell \cos\beta_0 \simeq 0$ (order of $\cos^2\beta_0$)

$\cos\beta_0 \{ P_\gamma^2 \cos\beta_0 - P_\alpha P_\gamma + I_1 mg\ell \} = 0$

$\cos\beta_0 \simeq \dfrac{P_\alpha}{P_\gamma} - \dfrac{I_1 mg\ell}{P_\gamma^2}$ is solution for minimum of $V_{eff}(\beta)$

Hence, if we look for solutions with $\beta_0 \simeq \pi/2$

We have (1) $\cos\beta_0 \simeq \dfrac{p_\alpha}{p_\gamma} - \dfrac{I_1 mgl}{p_\gamma^2}$ $\quad \beta_0 \simeq \pi/2 - \dfrac{p_\alpha}{p_\gamma} + \cdots$

(2) $p_\gamma = \underset{\sim}{L}\cdot\hat{\underset{\sim}{e}}_\gamma = \underset{\sim}{L}\cdot\hat{e}_3 \Rightarrow |L| \approx p_\gamma$ lies close to the symmetry axis

(3) $\dot{\gamma}_0 = p_\gamma\left(\dfrac{1}{I_3} + \dfrac{\cot^2\beta}{I_1}\right) - \dfrac{p_\alpha\cos\beta}{I_1\sin^2\beta} \simeq \dfrac{p_\gamma}{I_3} = \dfrac{|L|}{I_3}$

is the mean rate of spin around symmetry axis

(4) precession rate $\dot{\alpha}_0 = \dfrac{p_\alpha - p_\gamma\cos\beta}{I_1\sin^2\beta} \simeq \dfrac{mgl}{p_\gamma} = \dfrac{mgl}{|L|}$

varies inversely with $|L|$.

(5) $\Omega^2 = \dfrac{p_\alpha p_\gamma - I_1 mgl}{I_1^2\cos\beta_0} \simeq \dfrac{p_\alpha p_\gamma}{I_1^2 \cancel{\cos\beta_0}\, p_\alpha/p_\gamma} \simeq \dfrac{p_\gamma^2}{I_1^2}$

nutation rate $\Omega \simeq \dfrac{p_\gamma}{I_1} = \dfrac{|L|}{I_1}$ varies linearly with L.

The combination of rapid nutation with slow precession is called pseudo-regular precession.

(The nutation is hardly visible.)