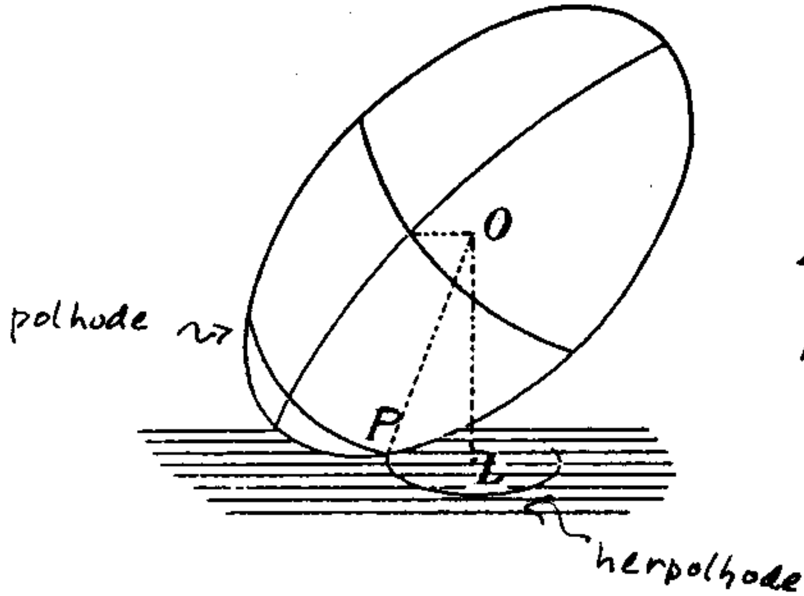


Fig. 1.

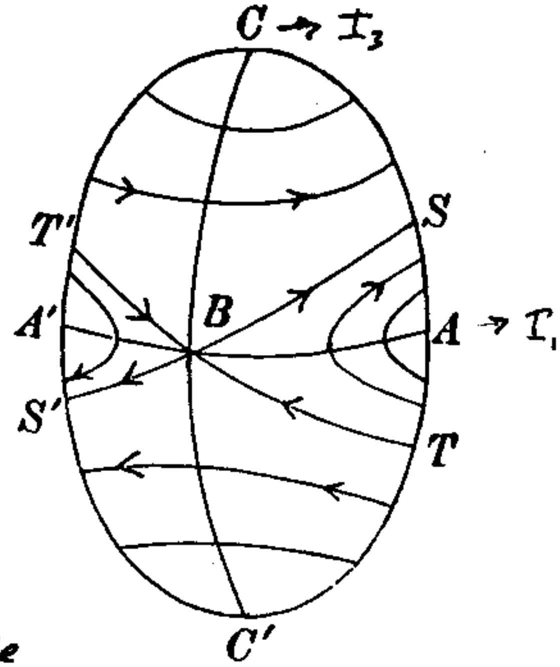


Fig. 2.

The terms polhode and herpolhode are due to Poinsot, *Théorie nouvelle de la rotation des corps*, 1834 and 1852.

$$(1)\quad \underset{\sim}{\omega}\cdot\underset{\sim}{L} = I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_3^2 = 2T$$

$$(2)\quad \underset{\sim}{L}\cdot\underset{\sim}{L} = I_1^2\omega_1^2 + I_2^2\omega_2^2 + I_3\omega_3^2 = L^2$$

$$\underset{\sim}{\omega}\cdot\underset{\sim}{\omega} = \omega_1^2 + \omega_2^2 + \omega_3^2 = \omega^2 \quad \text{not constant.}$$

Suppose $I_1 > I_2 > I_3$ Form $I_1 \cdot (1) - (2) \Rightarrow$

$$\underbrace{2T I_1 - L^2}_{>0} = I_2(I_1 - I_2)\omega_2^2 + I_3(I_1 - I_3)\omega_3^2 > 0$$

$$\frac{\omega_2^2}{\beta^2} + \frac{\omega_3^2}{\gamma^2} = +\text{ve} \quad \text{Ellipse}$$

The path followed by $\underset{\sim}{\omega}^2$, projected onto the y-z plane is an ellipse

$$\underbrace{2T I_3 - L^2}_{<0} = I_1(I_3 - I_1)\omega_1^2 + I_2(I_3 - I_2)\omega_2^2 < 0$$

$$\underbrace{2T I_2 - L^2}_{>\text{ or } <0} = I_1 \underbrace{(I_2 - I_1)}_{<0}\omega_1^2 + I_3\underbrace{(I_2 - I_3)}_{>0}\omega_3^2 \quad \text{indefinite}$$

$$\frac{\omega_1^2}{\alpha^2} - \frac{\omega_3^2}{\gamma^2} = \pm\,\text{Const.} \qquad \text{hyperbola}$$

So we have also shown that $2T I_3 - L^2 < 0$

$2T I_1 - L^2 > 0$

$$\therefore I_1 > \frac{L^2}{2T} > I_3$$

and I_2 lies in between (by assumption).

Let's revisit the ellipsoid of Inertia — rescaled

$$\langle \underset{\sim}{\omega}\, I\, \underset{\sim}{\omega} \rangle = 2T \qquad \underset{\sim}{L} = I \underset{\sim}{\omega}$$

$$\langle \underset{\sim}{x_p}\, I\, \underset{\sim}{x_p} \rangle = K \qquad \underset{\sim}{x_p} = \sqrt{\frac{K}{2T}}\, \underset{\sim}{\omega}$$ lies on ellipsoid

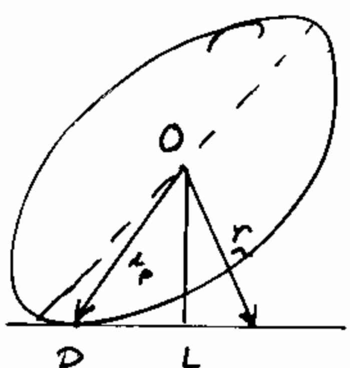
$$I_1 x^2 + I_2 y^2 + I_3 z^2 = K$$

Tangent plane: $(I_1 x)\, dx + (I_2 y)\, dy + (I_3 z)\, dz = 0$

$$\langle d\underset{\sim}{r} \cdot I \underset{\sim}{x_p} \rangle = 0 \qquad \langle d\underset{\sim}{r} \cdot \underset{\sim}{L} \rangle = 0$$

Vectors $\underset{\sim}{r}$ such that $\langle \underset{\sim}{r} - \underset{\sim}{x_p} \cdot \underset{\sim}{L} \rangle = 0$ lie in the plane

$$\underset{\sim}{r} \cdot \underset{\sim}{L} = \underset{\sim}{x_p} \cdot \underset{\sim}{L} = \underset{\sim}{x_p}\, I \underset{\sim}{\omega} = \sqrt{\frac{K}{2T}}\,(2T) = \sqrt{2TK}$$



Special case: $\underset{\sim}{r} = \underset{\sim}{q}$, parallel to $\underset{\sim}{L}$:

$$\underset{\sim}{q} \cdot \underset{\sim}{L} = |q| \cdot L \qquad |q|^2 = \frac{2TK}{L^2}$$

$$OL = \underset{\sim}{q} \qquad PL = \underset{\sim}{\rho}$$

$$|\rho|^2 = x_p^2 - q^2 = \frac{K}{2T}\omega^2 - \frac{2TK}{L^2}$$

$$|\rho|^2 = \frac{K}{2T}\left(\omega^2 - \left(\frac{2T}{L}\right)^2\right)$$

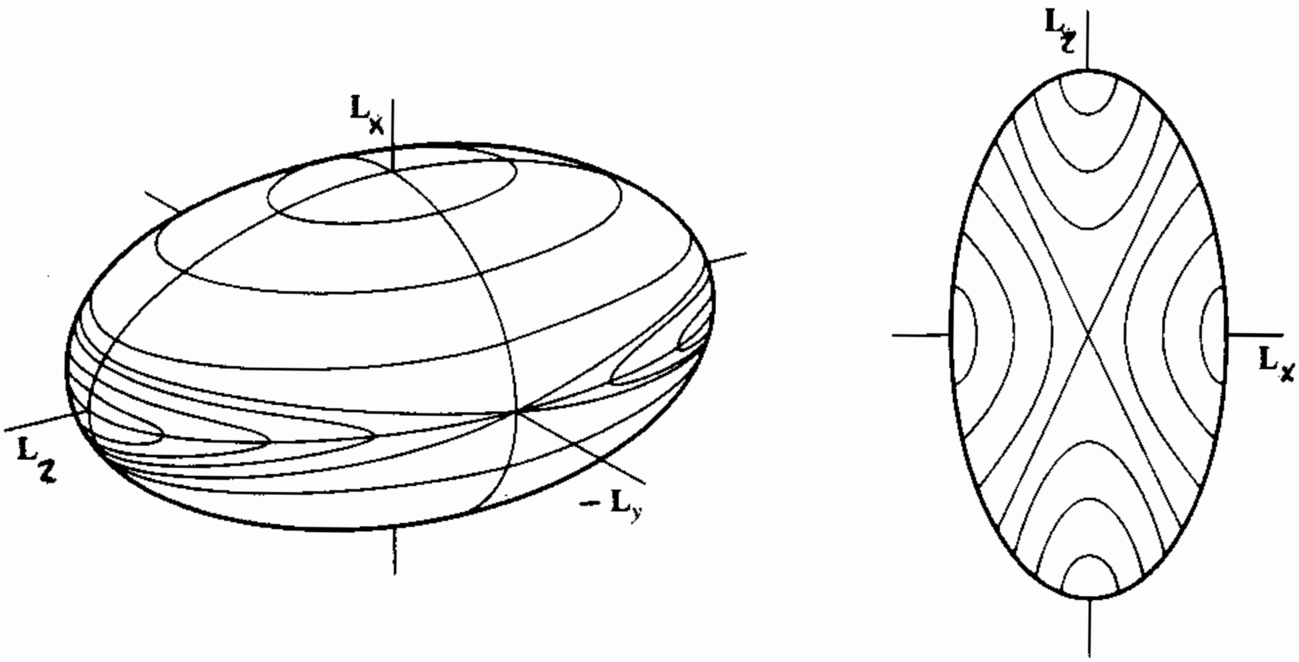
ω^2 is bounded

$\Rightarrow$ upper and lower limits on $|\rho|^2$.

$$\sqrt{2TI_3} < L < \sqrt{2TI_1}.$$

The sphere is outside the ellipsoid on the L_x axis and inside the ellipsoid along L_z. Figure 5.5 depicts curves where the sphere intersects the ellipsoid for various values of L. Fig. 5.5a shows a perspective view and Fig. 5.5b shows the view as seen from the L_y axis. The curves that appear as straight lines on Fig 5.5b correspond to the case where $L = \sqrt{2TI_2}$.

With the help of this geometrical construction, something can be said about the possible motions of a free asymmetric body. It is easy to see that a steady rotation



(a) (b)

FIGURE 5.5 (a) The kinetic energy, or Binet, ellipsoid fixed in the body axes, and some possible paths of the **L** vector in its surface. (b) Side view of Binet ellipsoid.

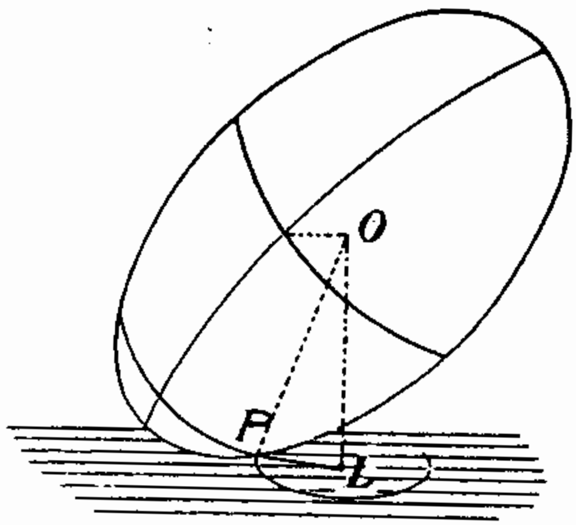


Fig. 1.

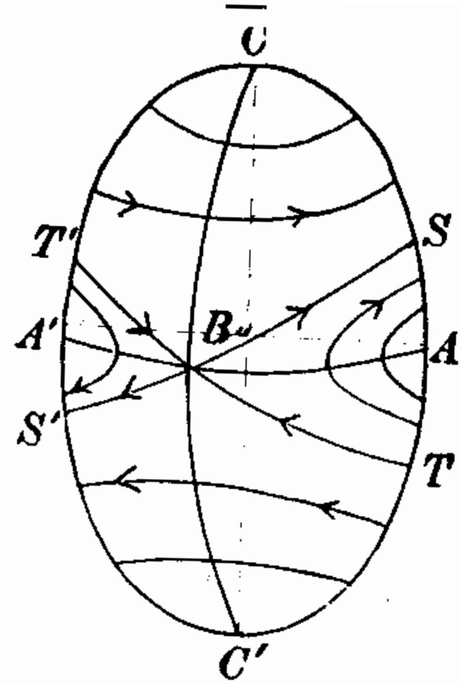


Fig. 2.

OA is x axis, moment I_1

OB is y axis, moment I_2

OC is z axis, moment I_3

Polhodes are the paths of $\underset{\sim}{\omega}^2$ on the ellipsoid

Projecting onto $\omega_1 \omega_3$ plane → curves become hyperbolae

Lines SS' and TT' become the asymptotes

(paths in special case $I_2 = L^2/2T$)

Projecting onto $\omega_1 \omega_2$ plane (OC direction)

paths → ellipses

Same for projecting onto $\omega_2 \omega_3$ plane

SS' and TT' divide the two families of ellipses,

those centred around A, and C.

For rotation around A: $\left.\begin{array}{l} 2T = I_1 \omega_1^2 \\ L^2 = (I_1 \omega_1)^2 \end{array}\right\} L^2/2T = I_1$

maximum value