1996 CAP University Prize Exam

Wednesday, February 7
2.00 - 5.00 pm

Calculators are allowed.

Each question should be written in a different booklet, with the question number and your name and University clearly written on the first page.

You should attempt as many questions as possible, in whole or in part.

Questions are of equal value.

Constants:

$\varepsilon_o = 8.85 \times 10^{-12}$ F/m

$c = 3.0 \times 10^8$ m/s

$g = 9.8$ m/s^2

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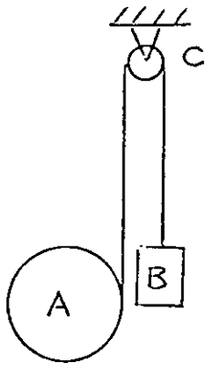
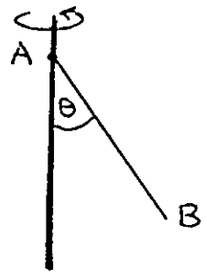
1.



A is a disk with radius 5 cm, mass 2 kg, and B is a block with mass 5 kg. They are connected by a long light cord wrapped around A which passes over the light, frictionless pulley C. How long does it take for B to fall 2 m if the system is released from rest?

2.



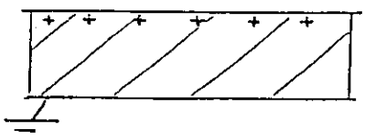
Rod AB has length 0.5 m, and is attached by a frictionless pin at A to a vertical shaft which rotates at 75 rpm.

(a) At what angle ϑ to the vertical will the rod hang in the steady state?

(b) If the rod is hanging vertically ($\vartheta = 0$) when the rotation suddenly begins, what will be the speed of the rod's centre when ϑ has increased to 30^0?

3. A famous practical joke makes use of a suitcase, inside which is mounted a flywheel which is rapidly spinning about a horizontal axis. Describe what a porter experiences when he tries to turn a corner carrying the suitcase. If the flywheel has a mass of 5 kg, a radius of gyration of 20 cm, and is spinning at 600 rpm, suggest a way the porter should carry the suitcase to be able to go round a right-angled bend in just 3 seconds.

4.

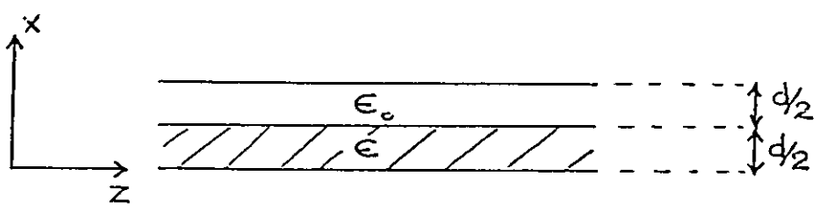


The space between the parallel plates is filled with a lossy dielectric having permittivity $\varepsilon = 4\varepsilon_0$ and conductivity $\sigma = 10^{-10}\Omega^{-1}m^{-1}$. The plates are separated by 5 mm, and each plate has a length of 10 cm and a width of 10 cm. A charge of 10^{-6}C is distributed uniformly over the upper plate at time t=0, and the lower plate is grounded. Assume that the electric field between the plates is uniform, with negligible fringing.

(a) Determine the current density between the plates at t>0.

(b) Prove that the magnetic field in the dielectric is zero.

5.



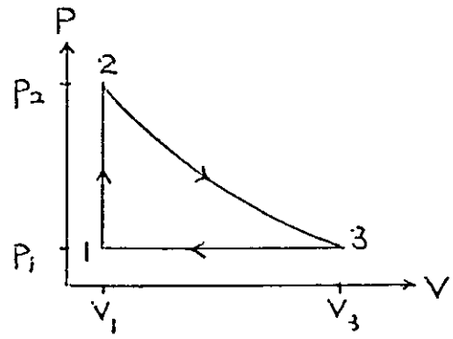
Two metal plates occupy the yz plane at x=0 and x=d. The space between them is half filled by a dielectric of permittivity $\varepsilon = 4\varepsilon_0$ and half by free space. A TE wave propagates in the z direction between the plates.

(a) What are the electric and magnetic fields between the plates?

(b) If the cut-off frequency of the lowest mode is 3 GHz, what is the plate separation d?

6. A proton (rest mass 938 MeV/c^2) moving with speed 0.9c strikes a stationary neutron (rest mass 940 MeV/c^2) forming a deuteron (rest mass 1876 MeV/c^2) and a gamma ray of energy 250 MeV. What is the speed of the deuteron, and at what angle to the proton's direction does the gamma ray move?

7.



An ideal monatomic gas is carried round the cycle shown in the figure. 1-2 is at constant volume, 2-3 is adiabatic, 3-1 is at constant pressure, V_3 is $8V_1$, and n moles of the gas are used.

(a) What is the heat input, the heat output, and the efficiency of the cycle, in terms of P_1, V_1, n, and R?

(b) Compare the efficiency with the efficiency of a Carnot cycle operating between the same extremes of temperature.

8. A system consists of 5 distinguishable non-interacting particles, each of which can occupy either of two states, of energy zero and ε. In an ensemble of such systems in thermal equilibrium there are twice as many systems having one particle in the upper state as systems having two.

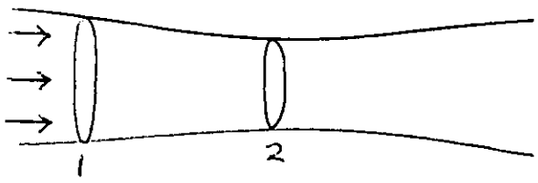
(a) What is the average energy of a system at this temperature?

(b) If, instead of being distinguishable, the particles are identical bosons, what would be the average system energy at this temperature?

9. A venturi meter is a flowmeter for gases (and liquids). It consists of a pipe of smoothly varying radius. The pressure difference is measured between two points.

   (a) What assumptions must you make about the flow in order to derive a useable algebraic model? Assume that the pipe is horizontal.

   (b) Calculate the mass flow rate of an ideal gas (air) given the following values:



Pressures	$p_1 = 200$ kPa, $p_2 = 190$ kPa
Diameters	$d_1 = 5$ cm, $d_2 = 4$ cm
Temperature	$T_1 = 25^0$C
Gas constant	$R = 8.314$ J K^{-1} mol^{-1}
Air	$MW = 28.97 \times 10^{-3}$ kg/mol
C_p/C_v	$\gamma = 1.400$

10(a) If α is small, the lowest eigenvalue of the Hamiltonian

$$H = \frac{p_x^2}{2m} + \frac{1}{2}m\omega^2x^2 + \alpha\frac{m^2\omega^3x^4}{\hbar}$$

is approximately given by

$$E = E_o + \alpha E_1 + \alpha^2 E_2 \ .$$

Calculate E_1 and E_2 using perturbation theory.

(b) If a particle is in the lowest eigenstate of H prior to a change in α from zero to 0.1, calculate the probability of it subsequently being found in an excited state if (i) $d\alpha/dt >> \omega$, and (ii) $d\alpha/dt << \omega$.

($\langle n+1|x|n\rangle = \left(\frac{(n+1)\hbar}{2m\omega}\right)^{1/2}$ for a harmonic oscillator)