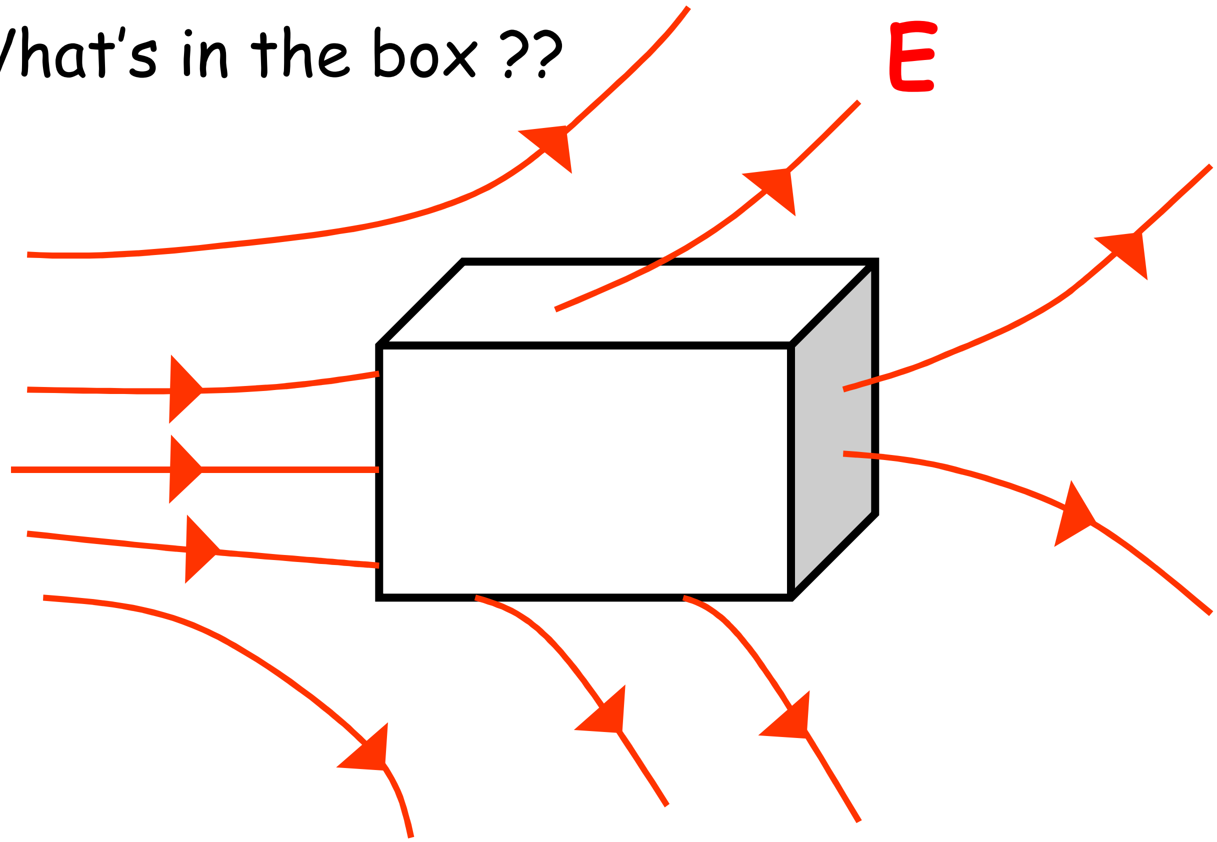
Gauss's Law

- Electric Flux
- Gauss's Law
- Examples

Text 24.1—24.2

Practice Problems:
Chapter 24,
Objective Questions 4, 11
Conceptual Questions 1, 2, 4, 6, 11
Problems 5, 6, 11, 15

What's in the box ??



Electric Flux Φ_E

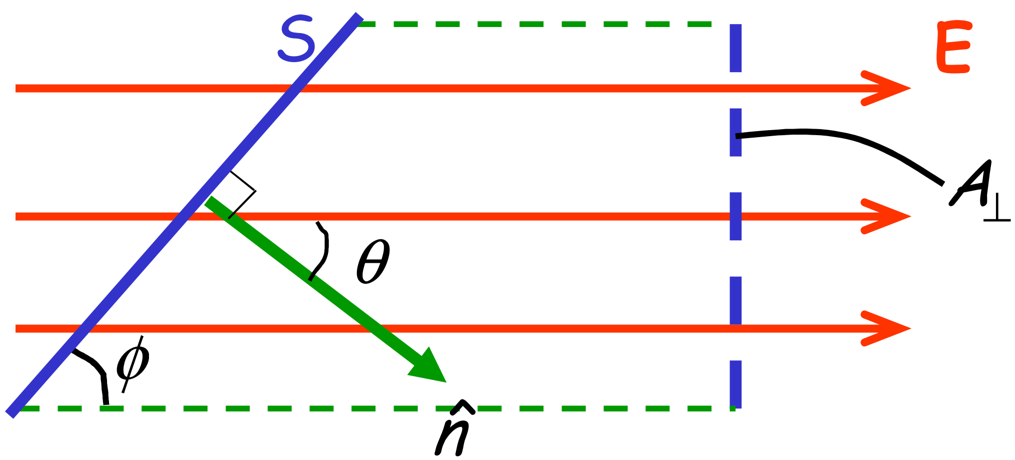
the "number of field lines through a surface S "

For <u>uniform</u> $\vec{E}$, define :

$$\Phi_E = \left|\vec{E}\right| \cdot A_{\perp}$$

(Units: N•m²/C)

where $A_{\perp}$ is the "area perpendicular to $\vec{E}$. "



Note : $A_{\perp} = A \sin\phi$

so... $A_{\perp} = A \cos\theta$

Notes:

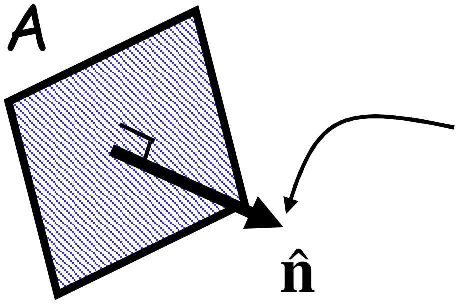
1) Φ_E is a scalar.

2) Units: N•m²/C

3) Φ_E is a *quantitative* measure of "the number of field lines through S."

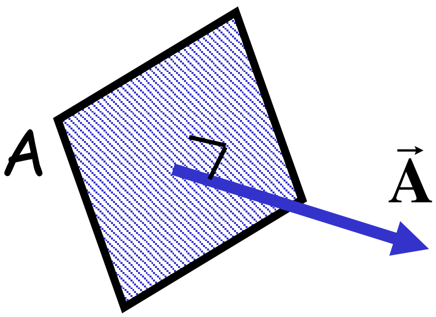
4) Φ_E refers to flux through some particular surface S."

Area A



Unit vector ⊥ surface
("unit normal")

Define "area vector": $\vec{\mathbf{A}} = (area) \cdot \hat{\mathbf{n}}$



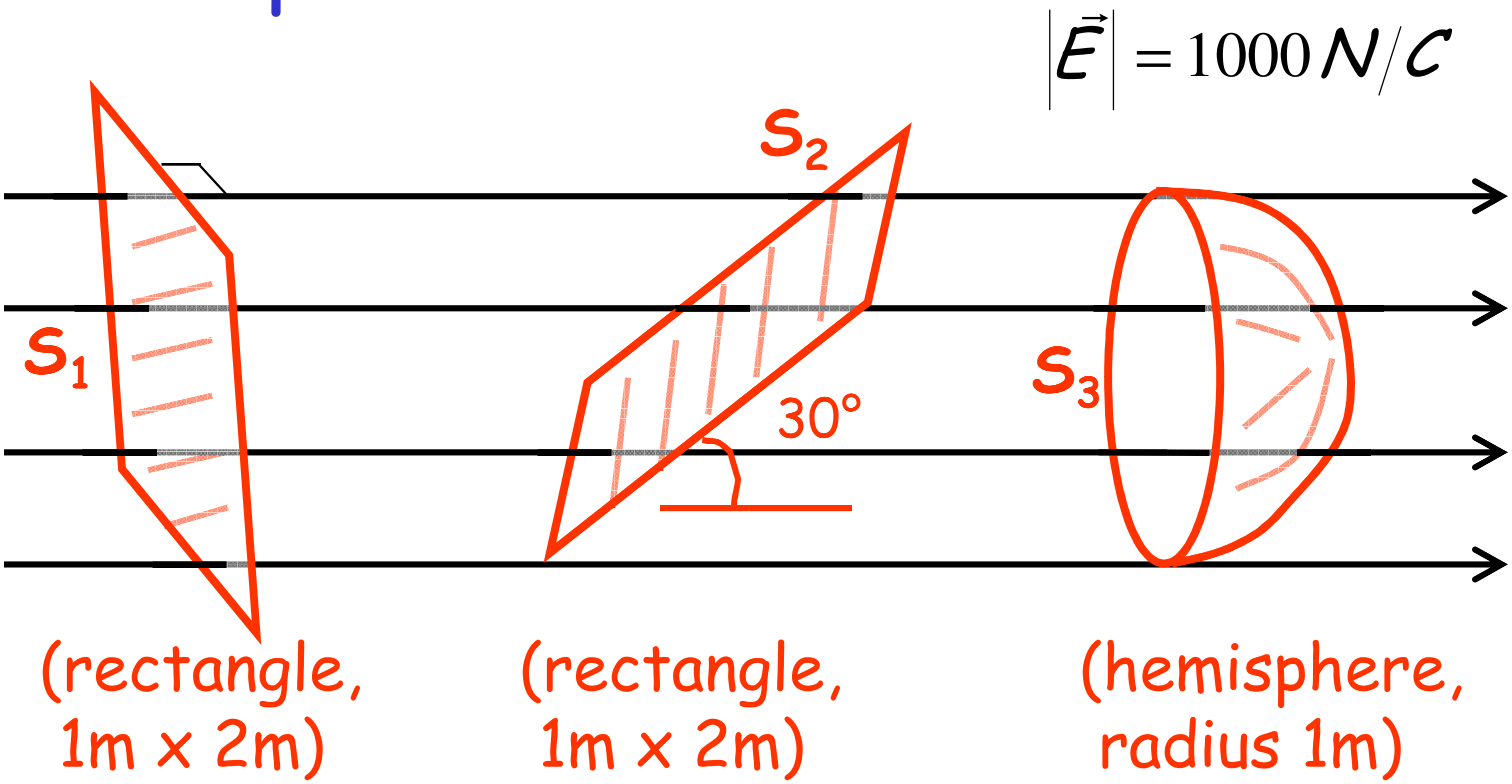
$|\vec{\mathbf{A}}| = \text{area}$

$\vec{\mathbf{A}} \parallel \hat{\mathbf{n}}$ *(perpendicular to surface)*

Then for uniform E,

$$\Phi_E = EA\cos\theta = \vec{E} \bullet \vec{A}$$

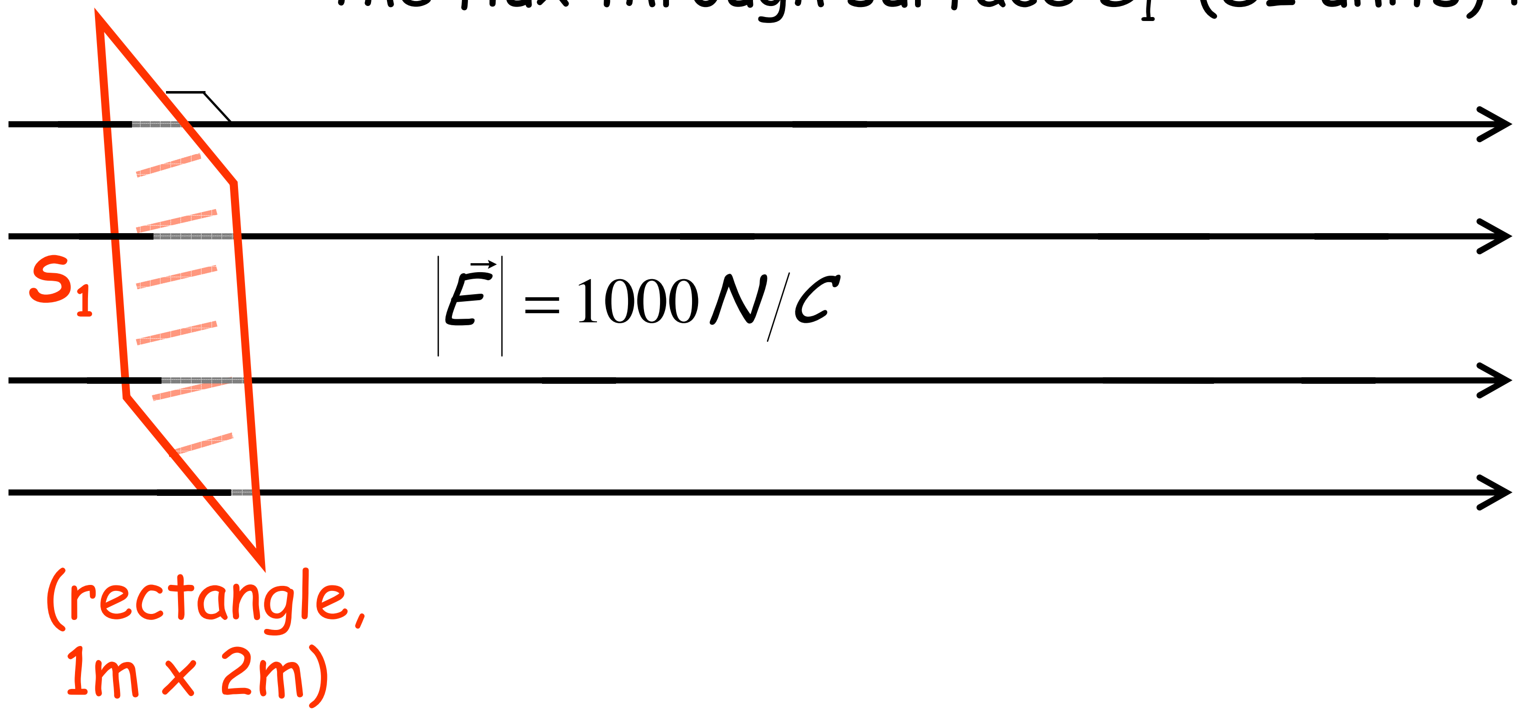
Examples:



Find: flux Φ_E through S_1, S_2, S_3.

Quiz

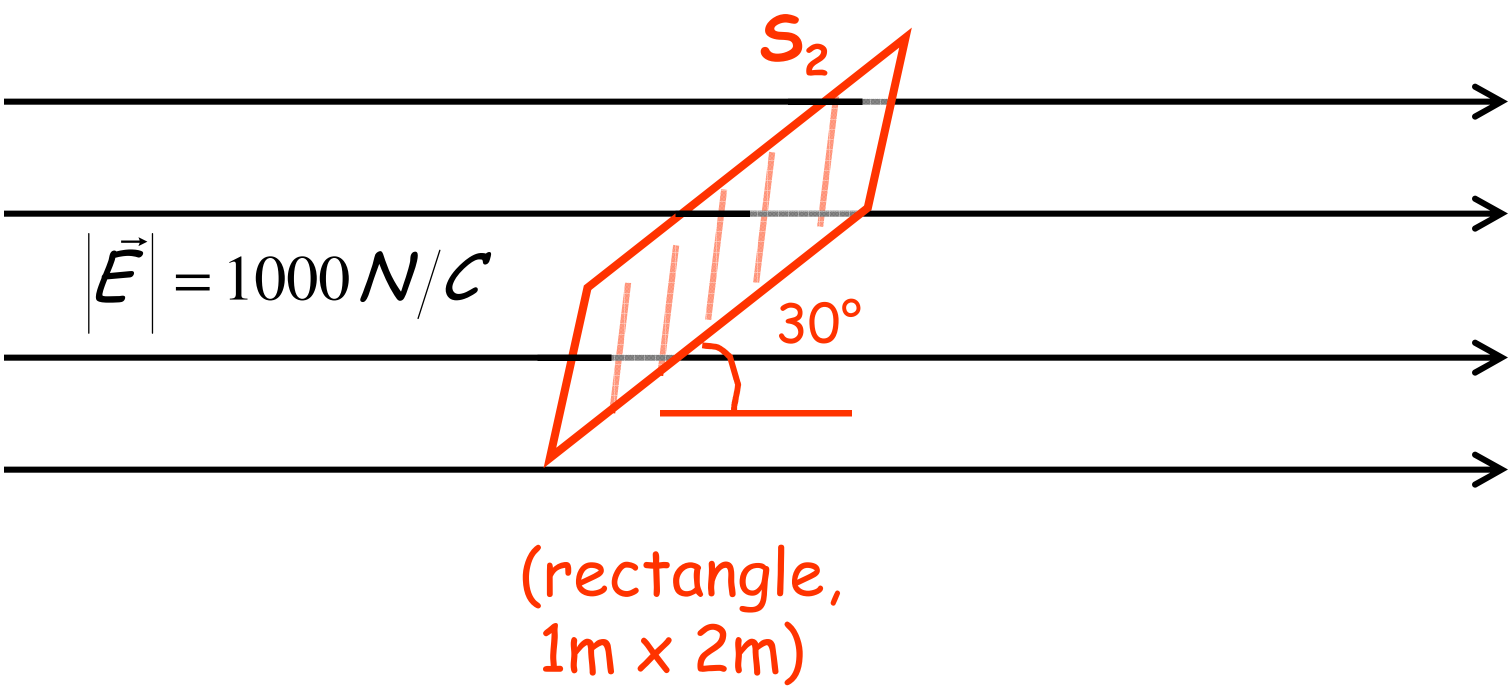
The flux through surface S_1 (SI units) is



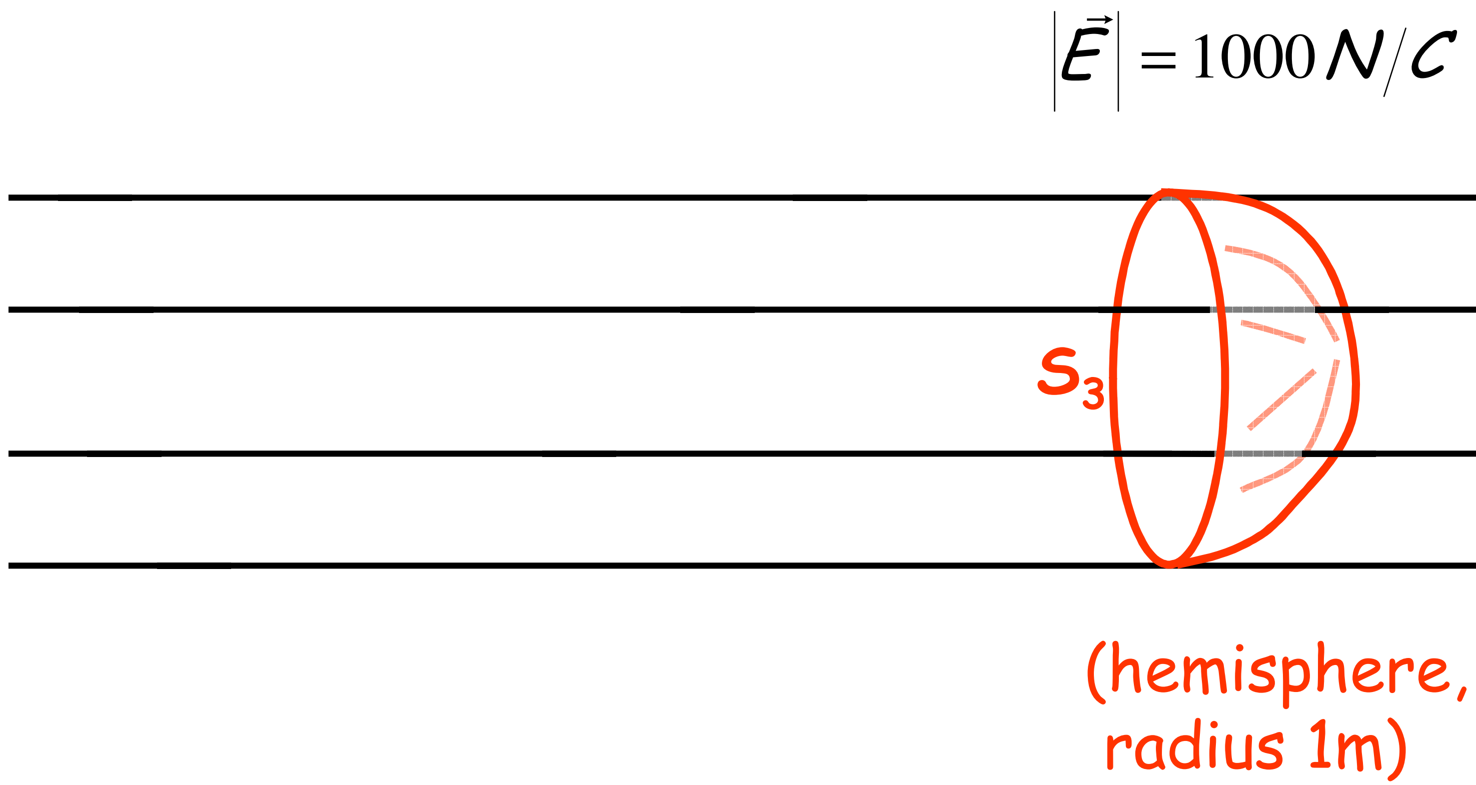
A) zero B) 1000 C) 2000 D) 4000 E) none of these

Quiz

The flux through surface S_2 (SI units) is



A) zero B) 1000 C) 2000 D) 4000 E) none of these


$\left|\vec{E}\right| = 1000\,N/C$
S3
(hemisphere,
radius 1m)

If E is not uniform, or S is not flat, then:

For a small surface $\mathbf{dA}$,

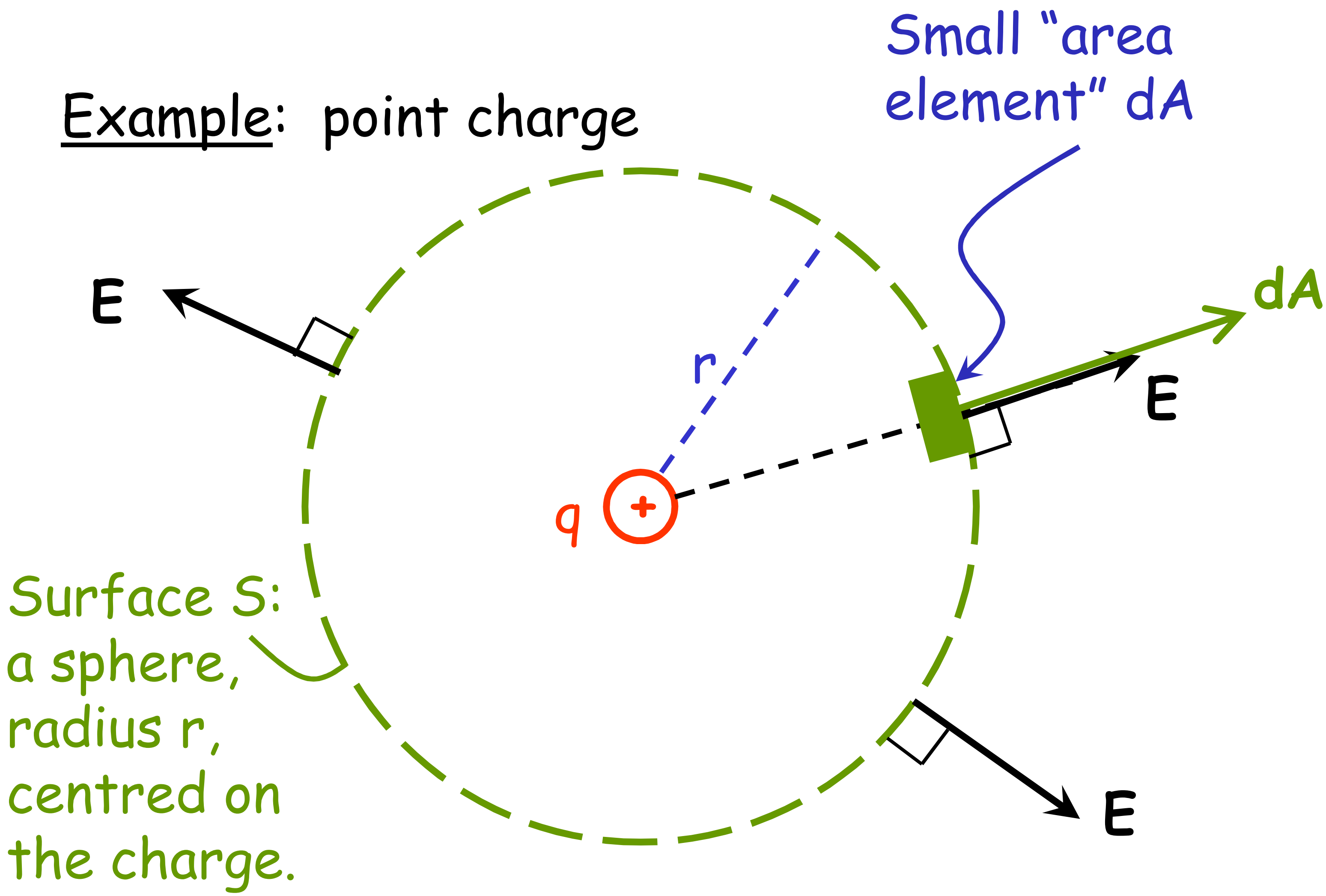
$$d\Phi_E = \mathbf{E} \bullet \mathbf{dA}$$

For the whole surface,

$$\Phi_E = \int_S \mathbf{E} \bullet \mathbf{dA}$$

$$= \int_S \left|\vec{E}\right| \cos\theta \, dA$$

<u>Example</u>: point charge



Flux:

$$\oint_{sphere} \vec{\mathbf{E}} \cdot \mathbf{d}\vec{\mathbf{A}} = \oint_{sphere} E\, dA \quad \textit{(why?)}$$

$$= E \oint_{sphere} dA \quad \textit{(why?)}$$

$$=$$

Gauss's Law

For a <u>*closed*</u> *surface S:* take the outward direction as the positive direction for $\mathbf{d}\vec{\mathbf{A}}$; then…

$$\text{Total flux through S} = \frac{\text{net charge enclosed}}{\varepsilon_o}$$

$$\varepsilon_o = \text{"permittivity of vacuum"} \approx 8.85\times10^{-12}\ \frac{\text{m}^2}{\text{N}\cdot\text{C}^2}$$

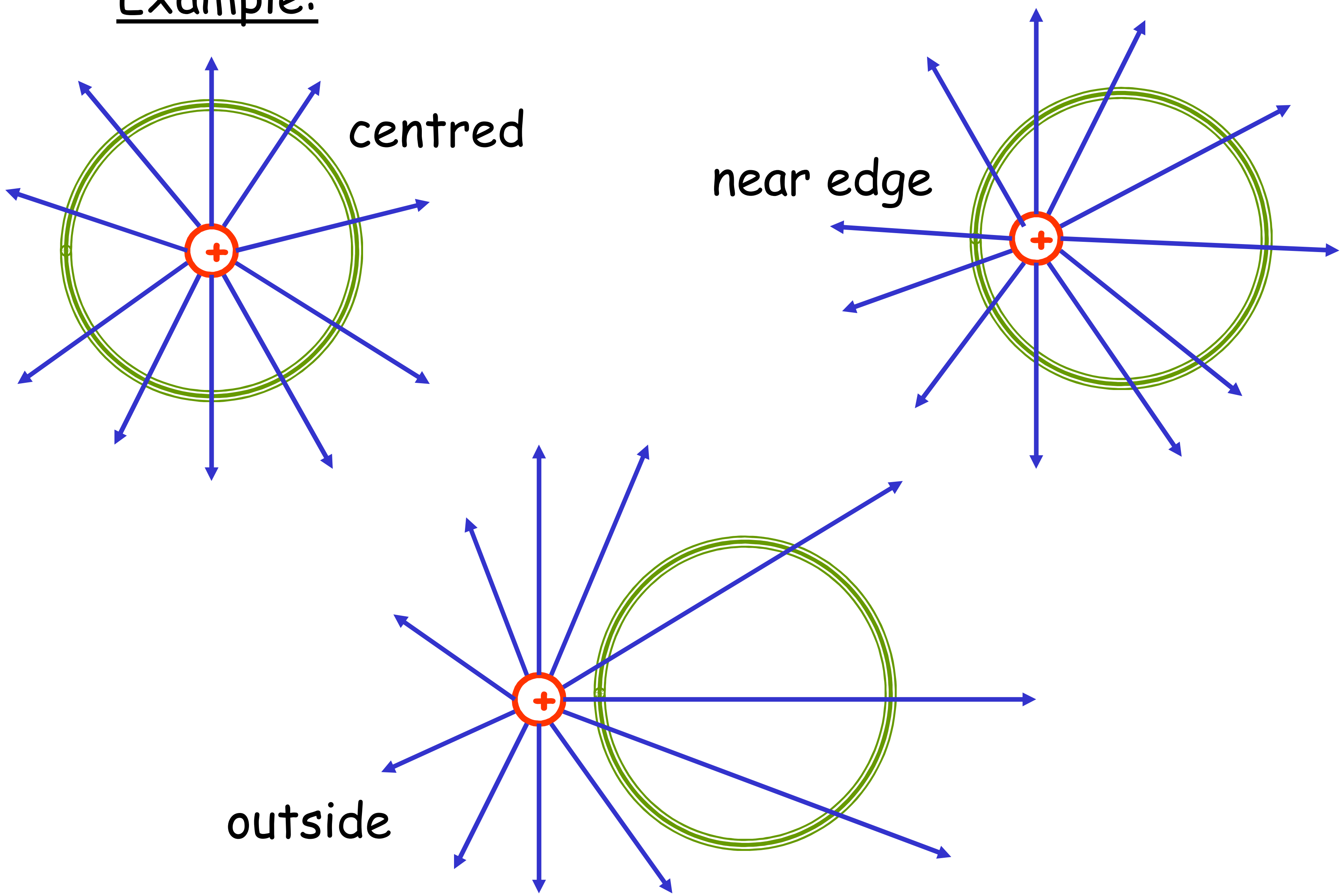
$$\text{and}\ \ k_e = \frac{1}{4\pi\varepsilon_o},\quad \varepsilon_o = \frac{1}{4\pi\, k_e}$$

Or…

$$\oint_{\substack{Closed \\ Surface}} \mathbf{E} \cdot \mathbf{dA} = \frac{Q_{enclosed}}{\varepsilon_o}$$

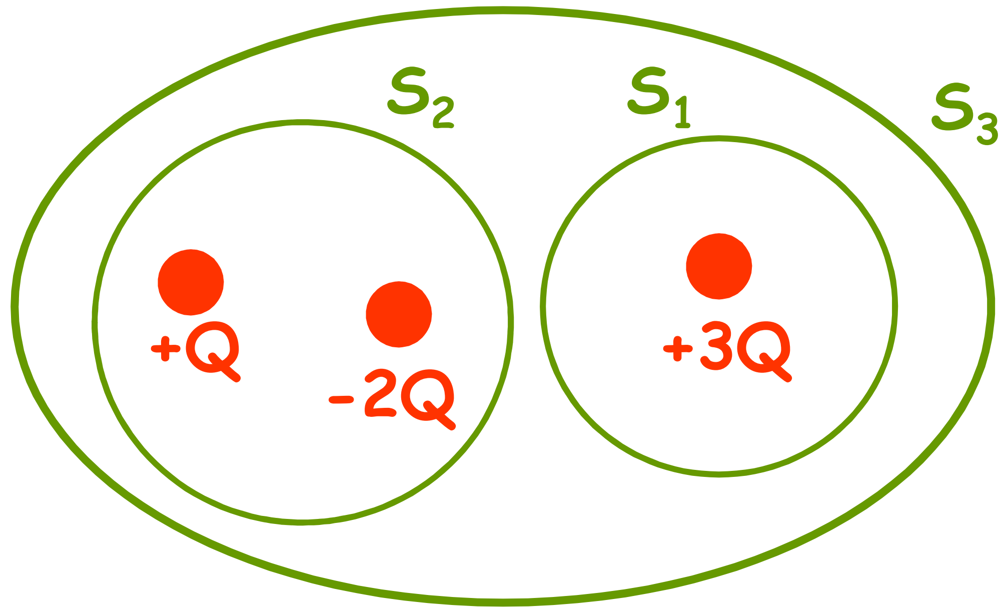
The **closed** "gaussian surface" is arbitrary (and usually exists only in your imagination).

Example:



Quiz

Let Φ_1, Φ_2, Φ_3, be the fluxes through the 3 closed surfaces. Then,



A) $|\Phi_1| > |\Phi_2| > |\Phi_3|$
B) $|\Phi_1| > |\Phi_3| > |\Phi_2|$
C) $|\Phi_3| > |\Phi_2| > |\Phi_1|$
D) $|\Phi_2| > |\Phi_1| > |\Phi_3|$
E) none of the above is true

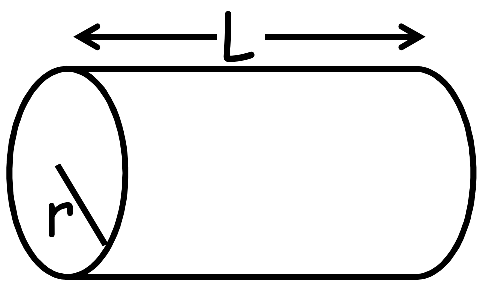
To prepare for next the lecture,
review geometry, and answer:

<u>What is</u>

a) The volume of a sphere of radius r?

b) The surface area of a sphere of radius r?

c) The volume of the cylinder?



d) The <u>total</u> surface area of the cylinder?

Summary

Gauss's Law: The total outward electric flux through any *closed* surface is proportional to the net charge enclosed.

- Gauss's Law is a consequence of Coulomb's Law (but is more general)