Limits on Resolution

(38.3)

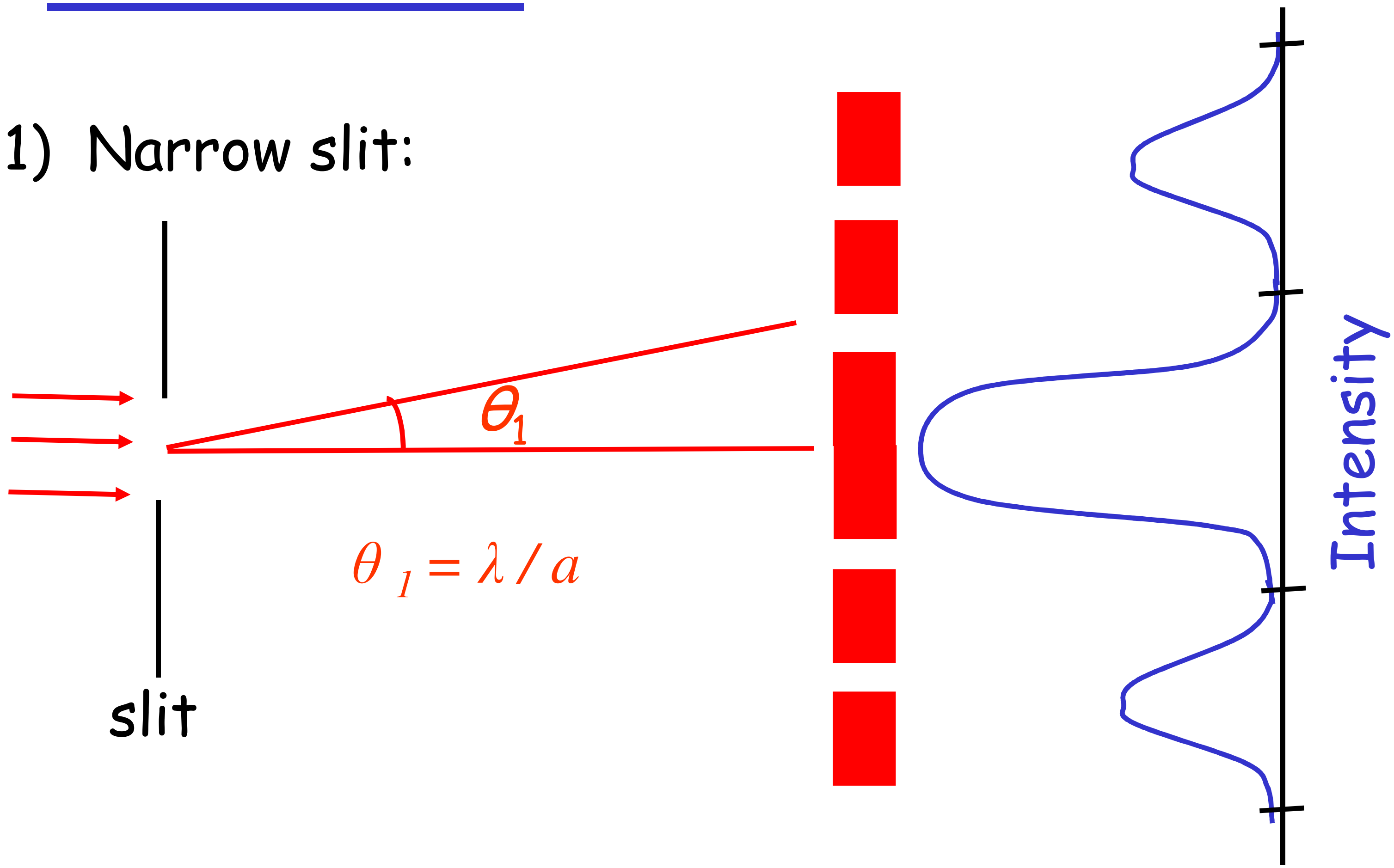
- angular resolution of telescopes

- resolution of microscopes

Practice: Chapter 38,
Objective Questions 8, 12
Problems 14, 15, 17, 19

Diffraction

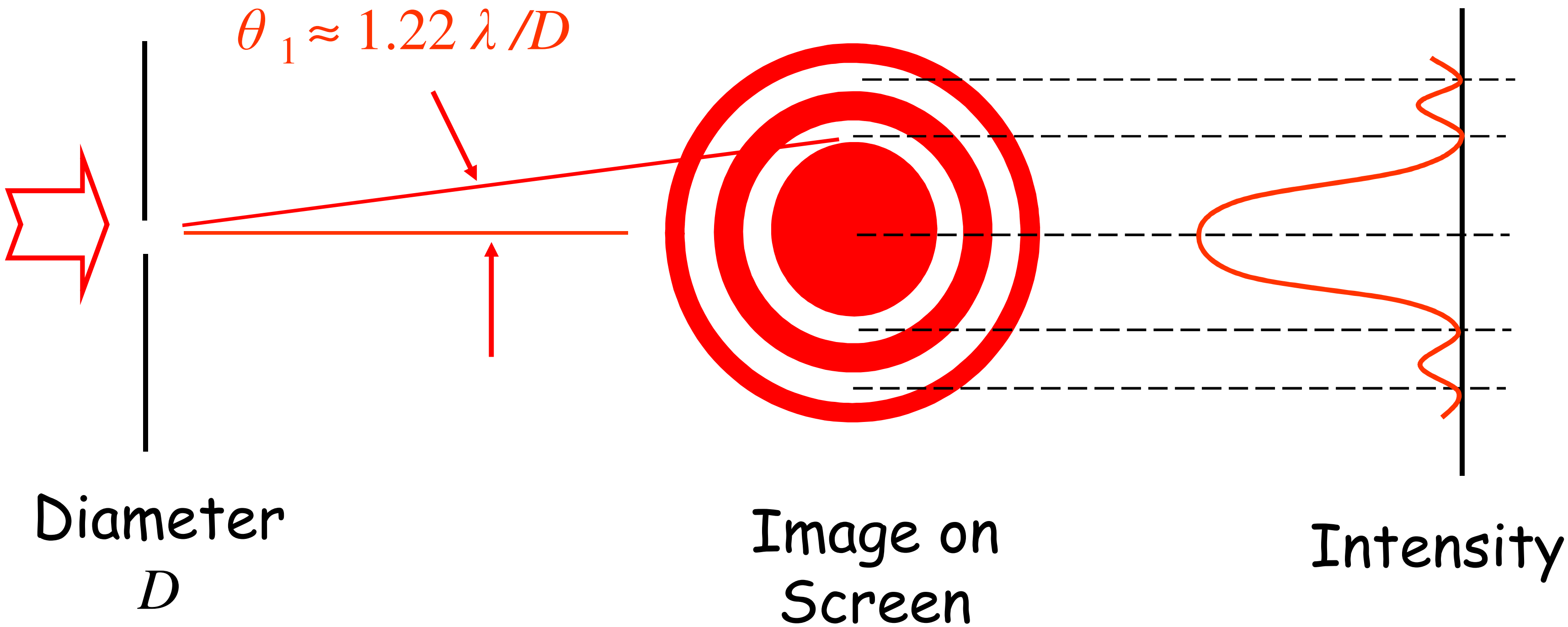
1) Narrow slit:



"angular width" of central peak is

$2 \ (\lambda / a)$ (radians).

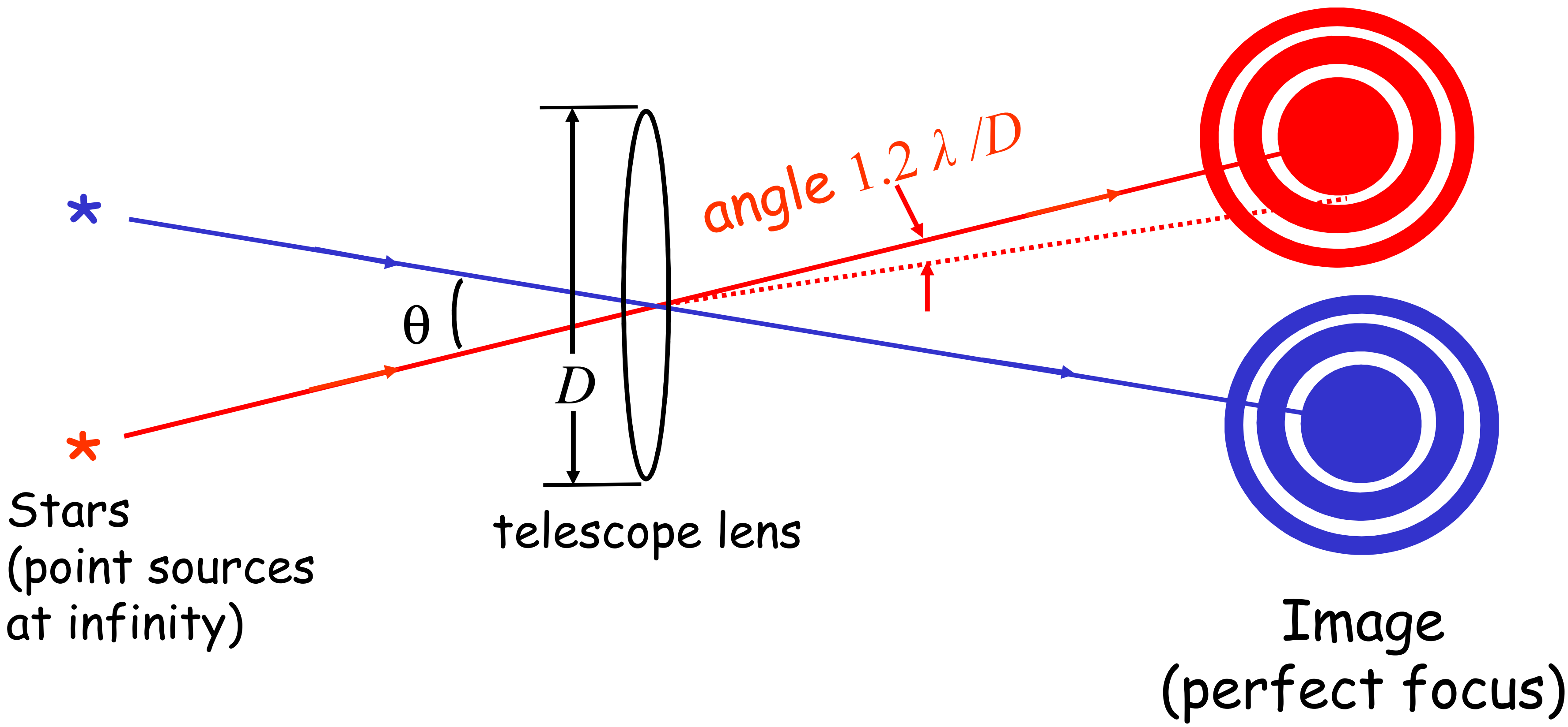
Diffraction through a **circular** aperture:

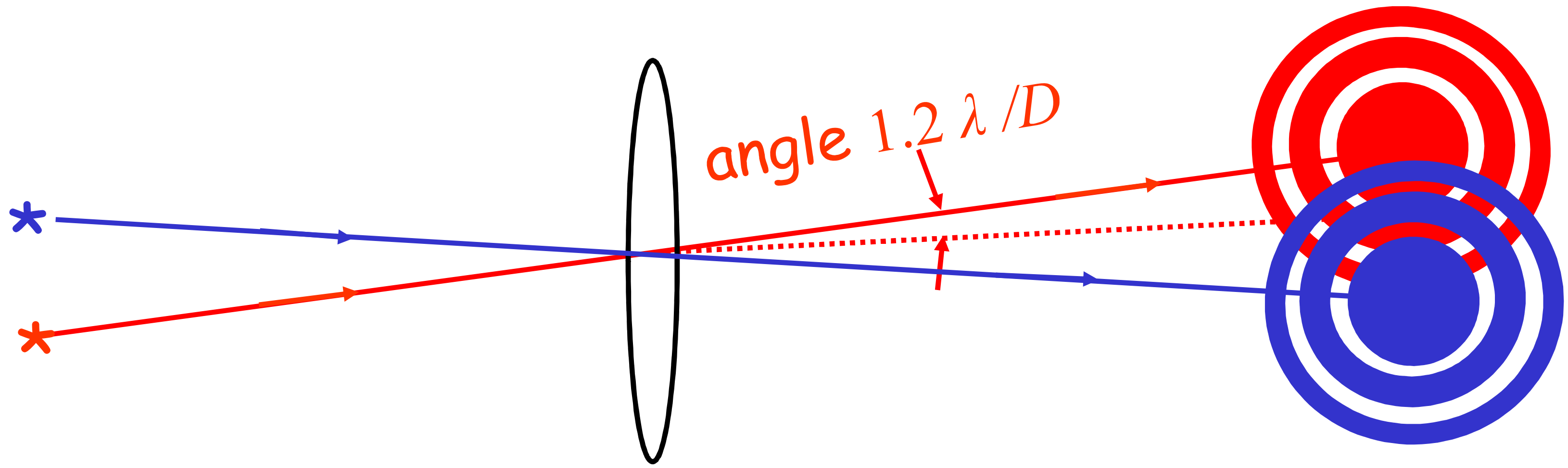


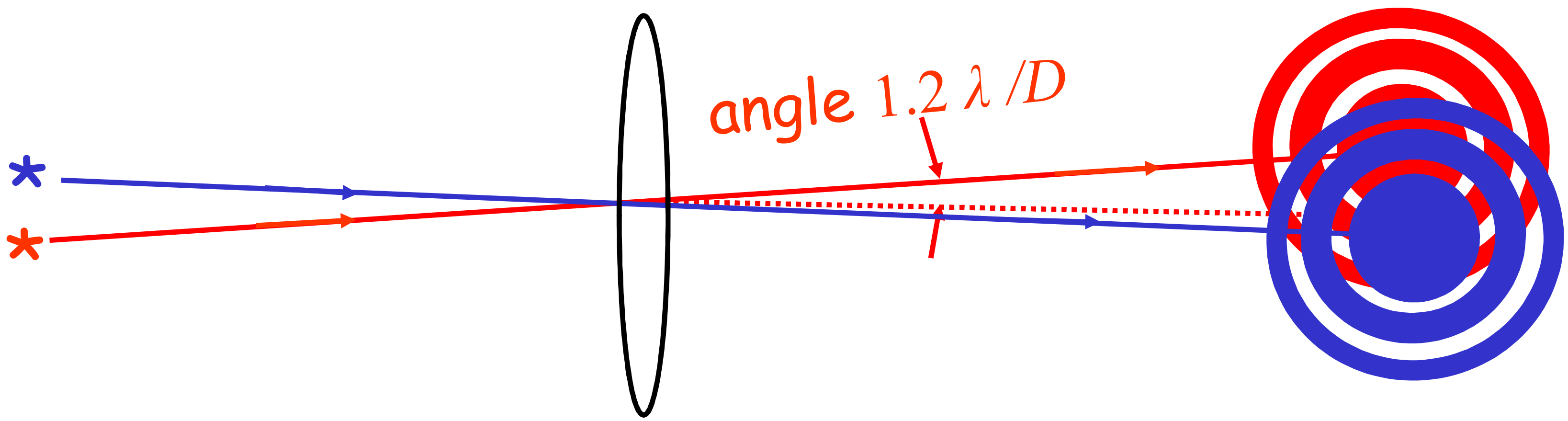
The angle θ_1 from the centre to first dark ring ("angular radius" of central spot) is about $1.22\,\lambda/D$ radians.

Resolution of a Telescope

A "perfect" telescope only magnifies the <u>diffraction pattern</u> of the circular "hole" it looks through.

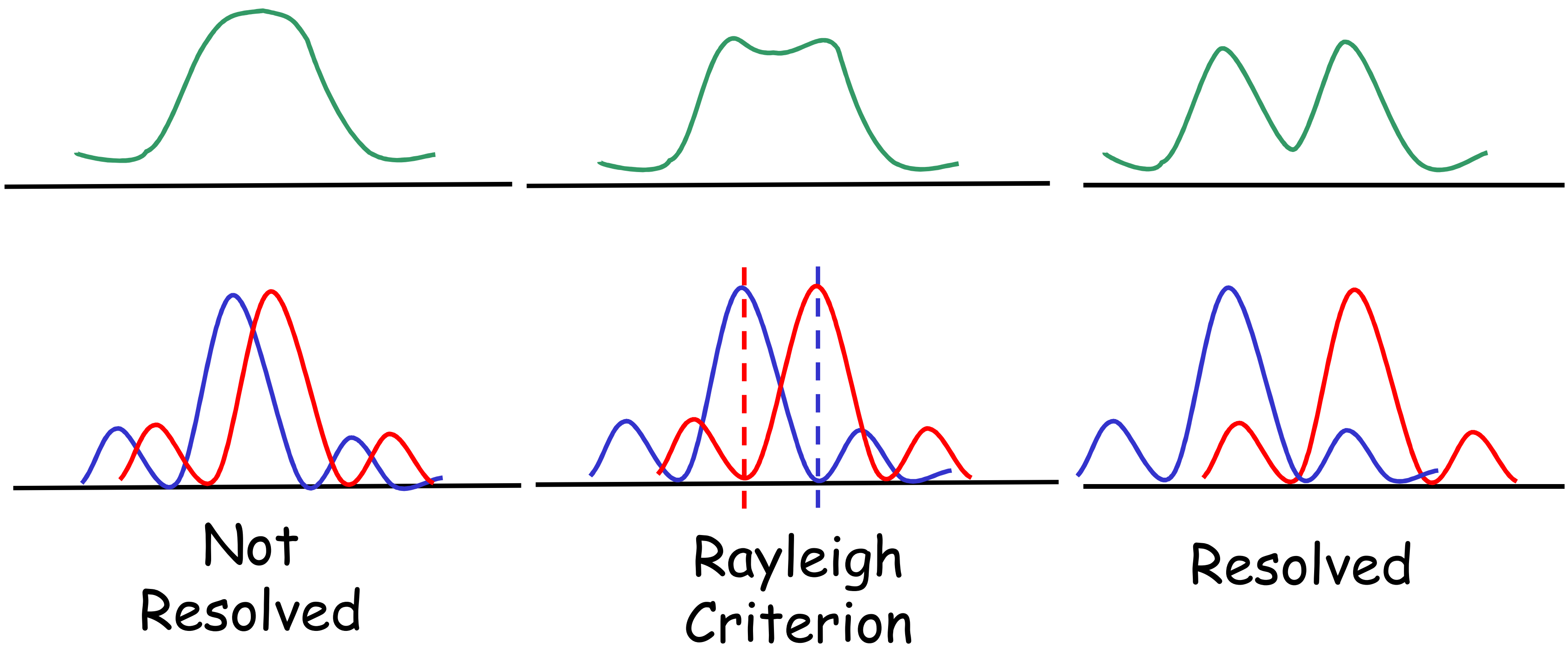



angle 1.2 λ /D

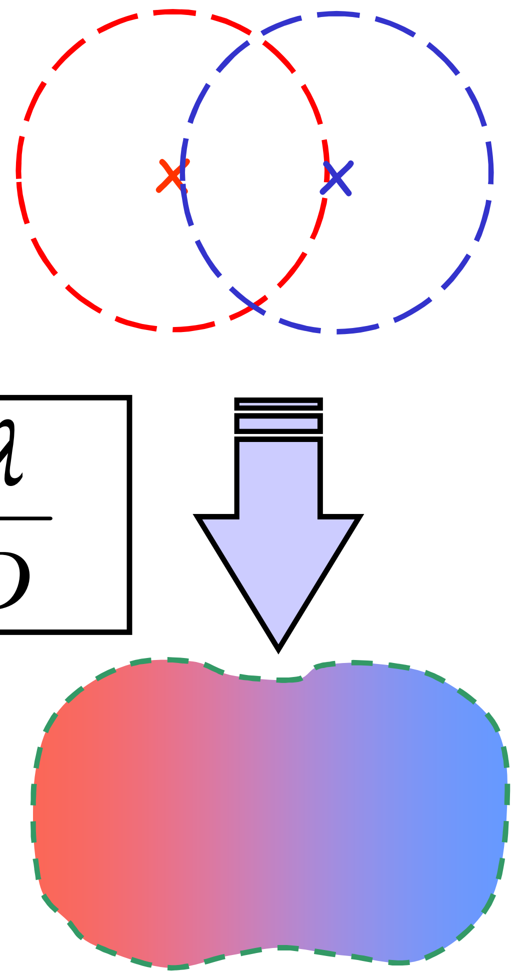

angle 1.2 λ /D

Rayleigh Criterion

Images are just resolved when the centre of one pattern overlaps the first dark line of the other pattern.



"Just resolved"
(or "just **not** resolved")

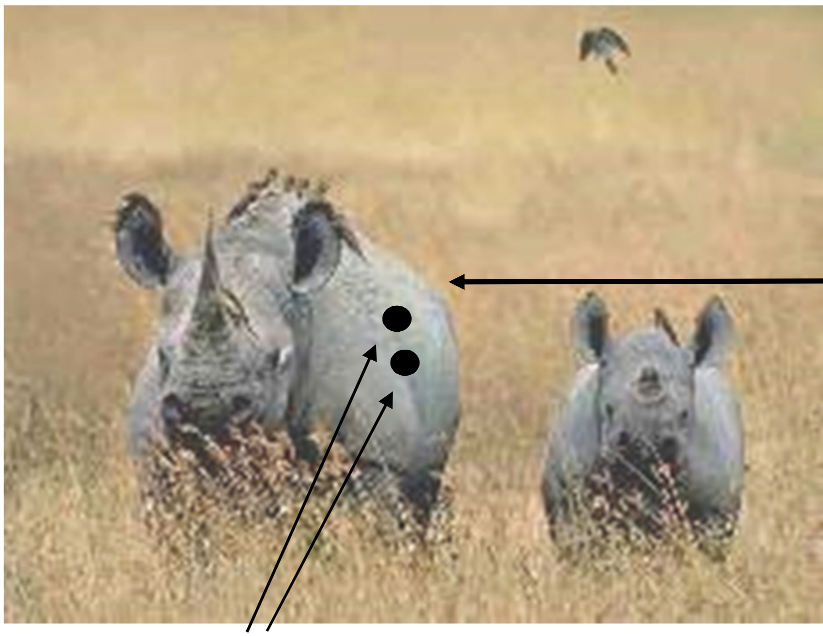


$$\text{Resolved} \quad \text{if} \;\; \theta \geq 1.22\,\frac{\lambda}{D}$$

Quiz:

Compared to a visible-light telescope, an infrared telescope, for the same angular resolution, would need a mirror diameter that is:

A) larger
B) smaller
C) the same



fleas, s = 2mm apart

R = 200 m

Find: Minimum lens diameter
to resolve fleas.
($\lambda \approx$ 500nm = 1/2000 mm)

Quiz:

If you moved the camera closer to the rhinoceros, the angular separation of the fleas would:

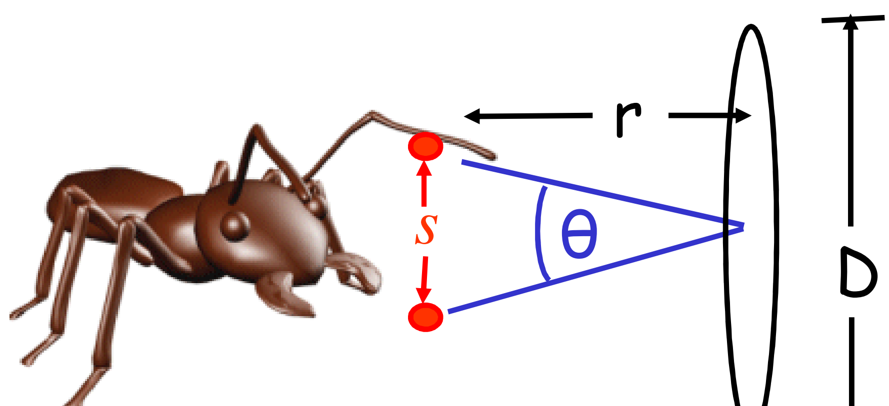
A) increase
B) decrease
C) stay the same

What telescope diameter would you need to get
a resolution of 1 arc minute?
of 1 arc second?

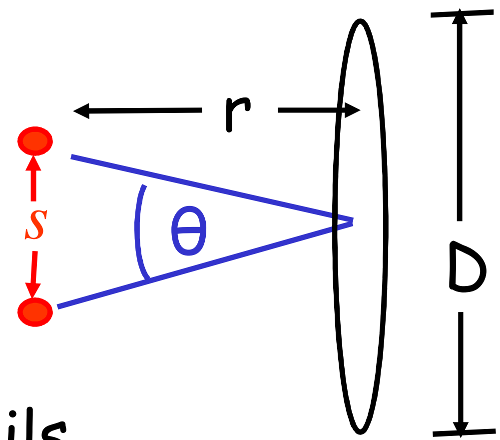
Answers: about 2.5 mm and 15 cm

Resolution of microscopes:

We are interested in the **linear size** *of the smallest details that can be resolved. If the object is placed close to the lens, the angular size of these details increases (and so they are more easily resolved).*
However, when the microscope lens is close to the sample, the simple Fraunhofer diffraction theory breaks down. But we can get an approximate *limit on the resolution by assuming the maximum* ***useful*** *lens diameter is about equal to the distance between the lens and the sample:*



Assume $r \approx D$



minimum resolved angle,

$$\theta_{\min} \approx 1.2\,\lambda/D$$

size $s_{\min}$ of smallest resolved details,

$$s_{\min} = r\,\theta_{\min}$$

We assume $r \approx D$; then $s_{\min} \approx 1.2\,\lambda$

This is only approximate. A complete calculation gives $s_{\min}$ typically about $\frac{3}{4}\lambda$ for a sufficiently wide lens very close to the object. The usual rule of thumb is: ***You can resolve details down to about one wavelength in size.***

For a **small** *lens* **far** *from the object, calculate the angular resolution, as for a telescope.*

Limits on Resolution - Summary

Because of diffraction, images formed by "perfect" optics are fuzzy.

Microscope: Smallest visible details are $\approx$ 1 wavelength in size.

Telescope: Angular size of finest details is $\approx 1.2\ \lambda/D$ radians.

where D = lens diameter