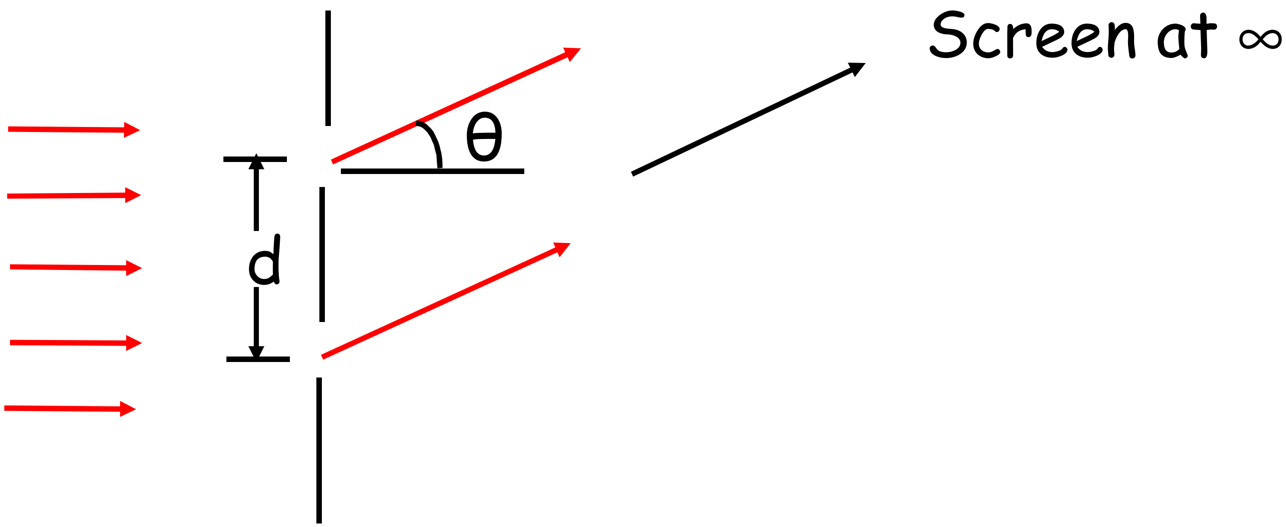
Interference of Light II

37.3, 38.4

- Intensity of double-slit pattern
- Three or more slits

Practice: Chapter 37,
Objective Question 8
Conceptual Questions 5, 6
Problems 21, 22, 23, 27, 29, 58

Young's Double Slit Experiment



Path difference $\Delta r = d \sin \theta$

zero intensity at $d \sin\theta = (m \pm ½)\, \lambda, \quad m = 0, \pm 1, \pm 2, \ldots$

max. intensity at $d \sin\theta = m\, \lambda, \quad m = 0, \pm 1, \pm 2, \ldots$

Questions:

-what is the intensity of the double-slit pattern at an *arbitrary* position?

-What about three slits? four? five?

Find the ***intensity*** *of the double-slit interference pattern as a function of position on the screen.*

Two waves: $E_1 = E_0 \sin(\omega t)$

$$E_2 = E_0 \sin(\omega t + \phi)$$

where $\phi = 2\pi (d \sin\theta)/\lambda$, and θ gives the position.

Steps: Find resultant *amplitude*, E_R;
then intensities obey

$$\frac{I_R}{I_0} = \left(\frac{E_R}{E_0}\right)^2$$

where I_0 is the intensity of each individual wave.

Trigonometry:

sin a + sin b = 2 cos [(a-b)/2] sin [(a+b)/2]

$$E_1 + E_2 = E_0\,[\sin(\omega t) + \sin(\omega t + \phi)]$$
$$= \underbrace{2\,E_0 \cos(\phi/2)}\,\sin(\omega t + \phi/2)]$$

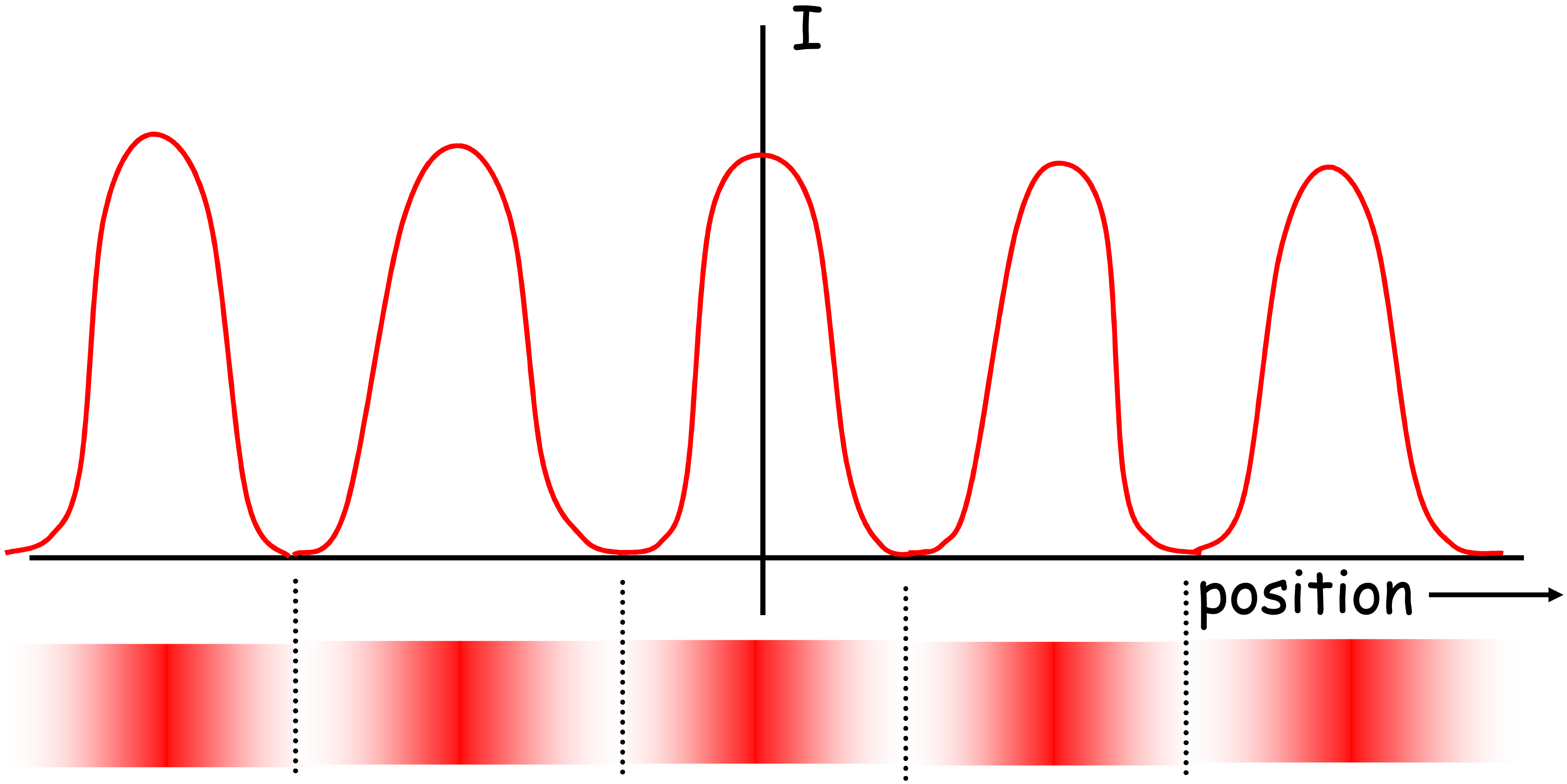
Resultant amplitude $E_R = 2\,E_0 \cos(\phi/2)$

Resultant intensity,

$$I_R = 4 I_0 \cos^2\left(\tfrac{1}{2}\phi\right)$$

$$\text{with } \phi = 2\pi \frac{\Delta r}{\lambda} = 2\pi \frac{d \sin\theta}{\lambda}$$

$$I_R \propto \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)$$ (a function of position θ on the screen)



- fringes are wide, with fuzzy edges

- equally spaced (in sin θ)

- equal brightness (but we have ignored diffraction)

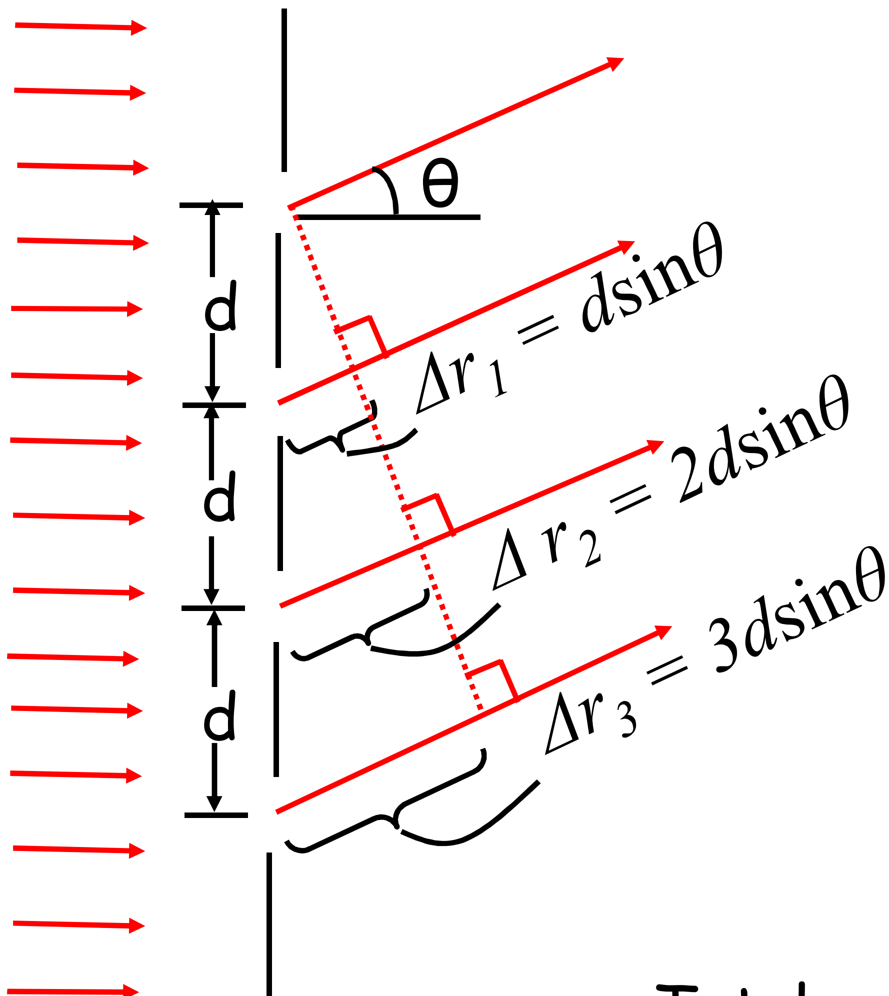
Quiz

With only one slit open, the intensity in the centre of the screen is 100 W/m^2. With both (identical) slits open together, the intensity at locations of constructive interference will be

a) zero
b) 100 W/m^2
c) 200 W/m^2
d) 400 W/m^2

How does this compare with shining two laser pointers on the same spot?

Several Equally-Spaced Slits



$$\phi = 2\pi \frac{d \sin \theta}{\lambda}$$

$$E_1 = E_0 \sin(\omega t)$$

$$E_2 = E_0 \sin(\omega t + \phi)$$

$$E_3 = E_0 \sin(\omega t + 2\phi)$$

$$E_4 = E_0 \sin(\omega t + 3\phi)$$

$$\vdots$$

Total, $E = E_R \sin(\omega t + \phi_R)$

Example: 3 Slits

The total field is

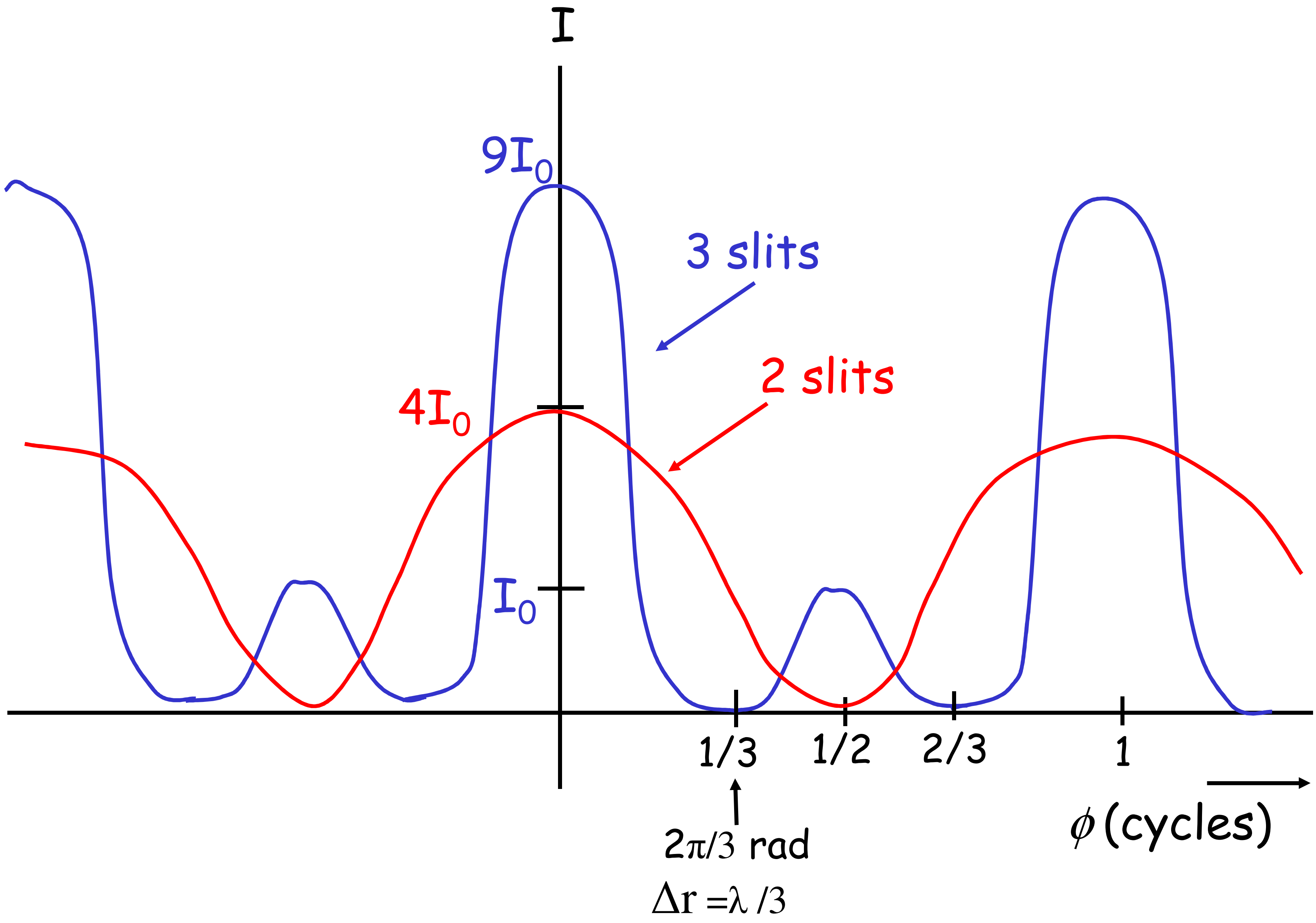
$$E_R \sin(\omega t + \phi_R) = E_0\{\sin(\omega t) + \sin(\omega t + \phi) + \sin(\omega t + 2\phi)\}$$

where $\phi = 2\pi(d \sin\theta)/\lambda$

- When $d \sin\theta = 0, \lambda, 2\lambda, \ldots,\quad E_R = 3E_0$ (maximum value); so the *brightest* fringes are in the *same locations as with two slits*.

- There are minima (zero intensity) at $\phi = 120°$ and $\phi = 240°$ (where $d \sin\theta = \lambda/3, 2\lambda/3$); to see this, add the first and third term above).

- At $\phi = 180°$ (where $d \sin\theta = \lambda/2$) there is a small *maximum*: $E_R = E_0$



I
9I0
3 slits
2 slits
4I0
I0
1/3
1/2
2/3
1
2π/3 rad
Δr =λ /3
φ (cycles)

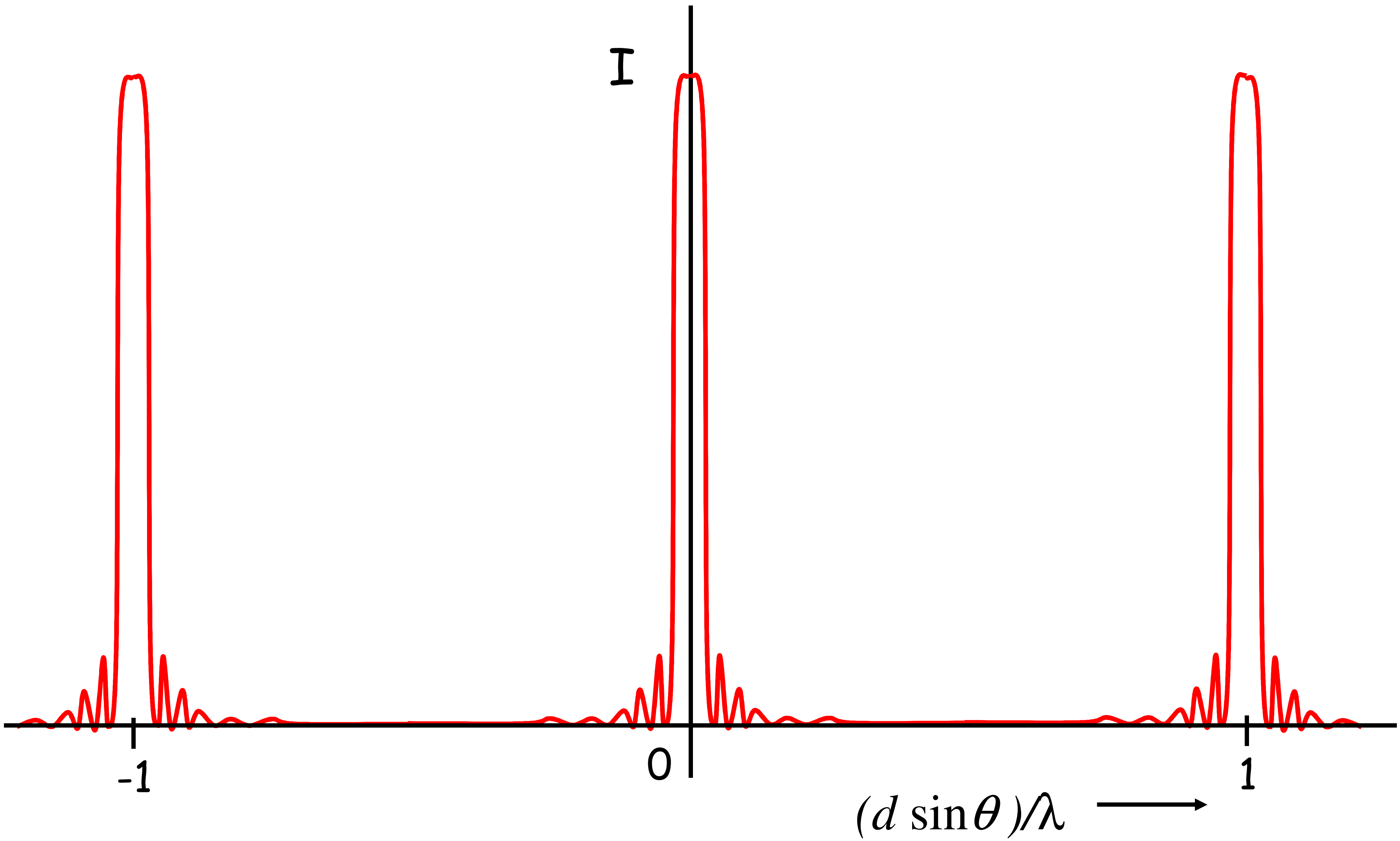
Differences between 2-slit and 3-slit patterns:

3 Slits:

- Main peaks become ***narrower*** & ***brighter***
- One "extra" *faint* peak in between
- Main peaks ($\phi = m \times 2\pi$) are in the same places

As the number of slits becomes large, the main interference fringes become even narrower and brighter; the additional faint peaks become less and less noticeable in comparison.

Many Slits ("diffraction grating")



Quiz

Suppose the multiple slits from the previous slide were illuminated with white light, instead of red. What would the pattern on the screen look like?

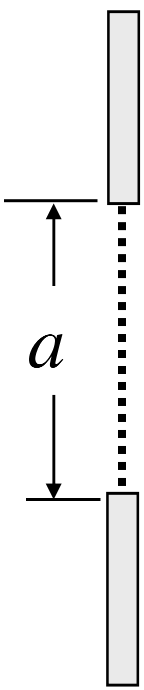
A) The three bright lines would be white, with a little colour at the edges, but otherwise the same
B) Each of the three lines would spread into a wide coloured spectrum
C) Two of the three lines would spread into a wide coloured spectrum, with the centre one narrow and white

*What if a **double** slit were illuminated with white light?*

A single wide slit:

So far we have added more slits, keeping d constant; the total width of the apparatus is Nd*, and grows as* N*.*

Consider instead a single slit of width a*, divided into* N *narrow slits separated by* $d = a / N$*, so that* d *decreases as* N *increases.*



In the limit as $N \to \infty$ and $d \to 0$, only the central main peak and the smaller side peaks remain; the widths of these peaks *do not* go to zero when N increases and d simultaneously decreases (next lecture).