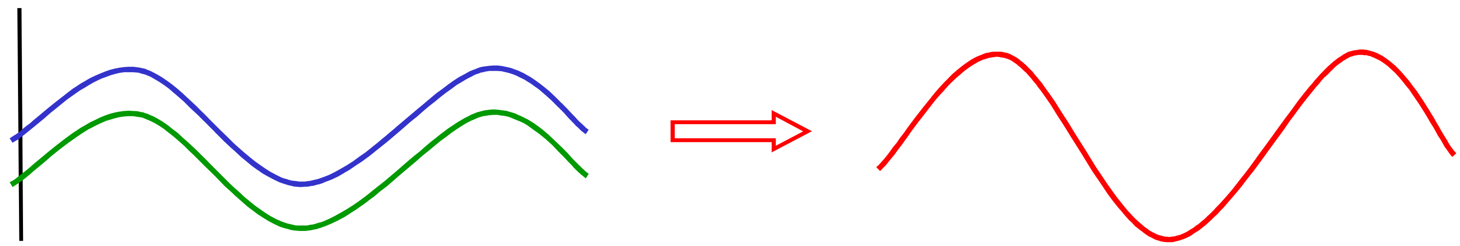
Beats
Interference of waves, in 3D

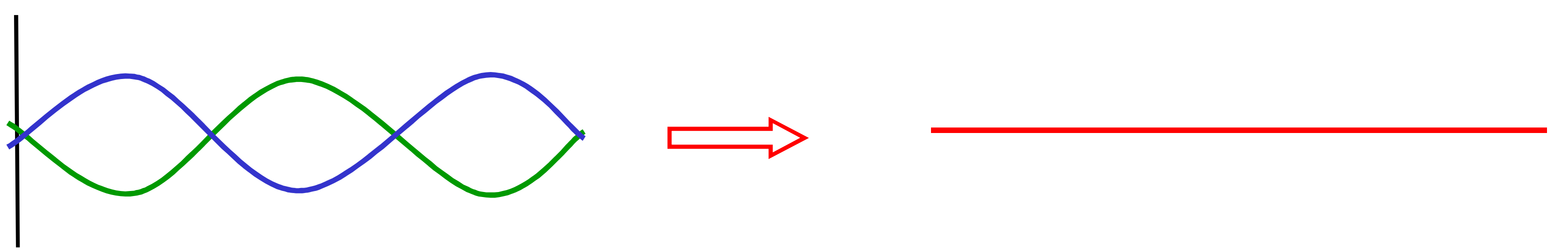
Text sections 18.7; 18.1 again

Practice: Chapter 18,
Objective Questions 1, 6, 7
Conceptual Questions 3, 6
Problems 8, 13, 57



"Constructive interference:"
phase difference =0, 2π, 4π, ...

$$A_R = A_1 + A_2$$



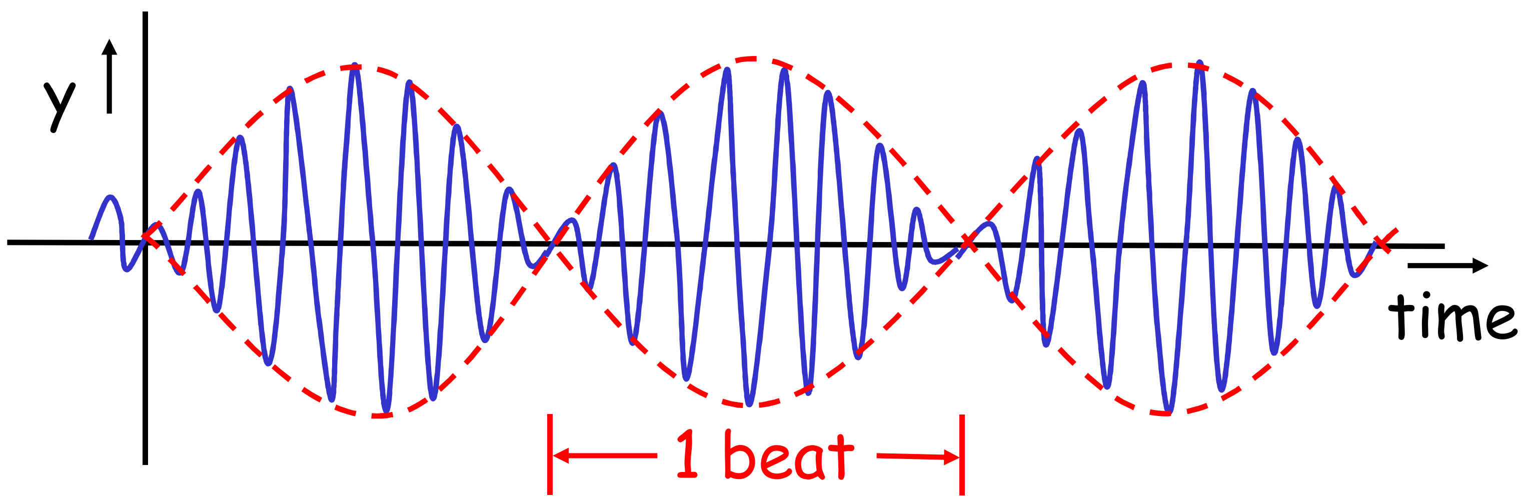
"Destructive interference:"
phase difference = π, 3π, 5π,...

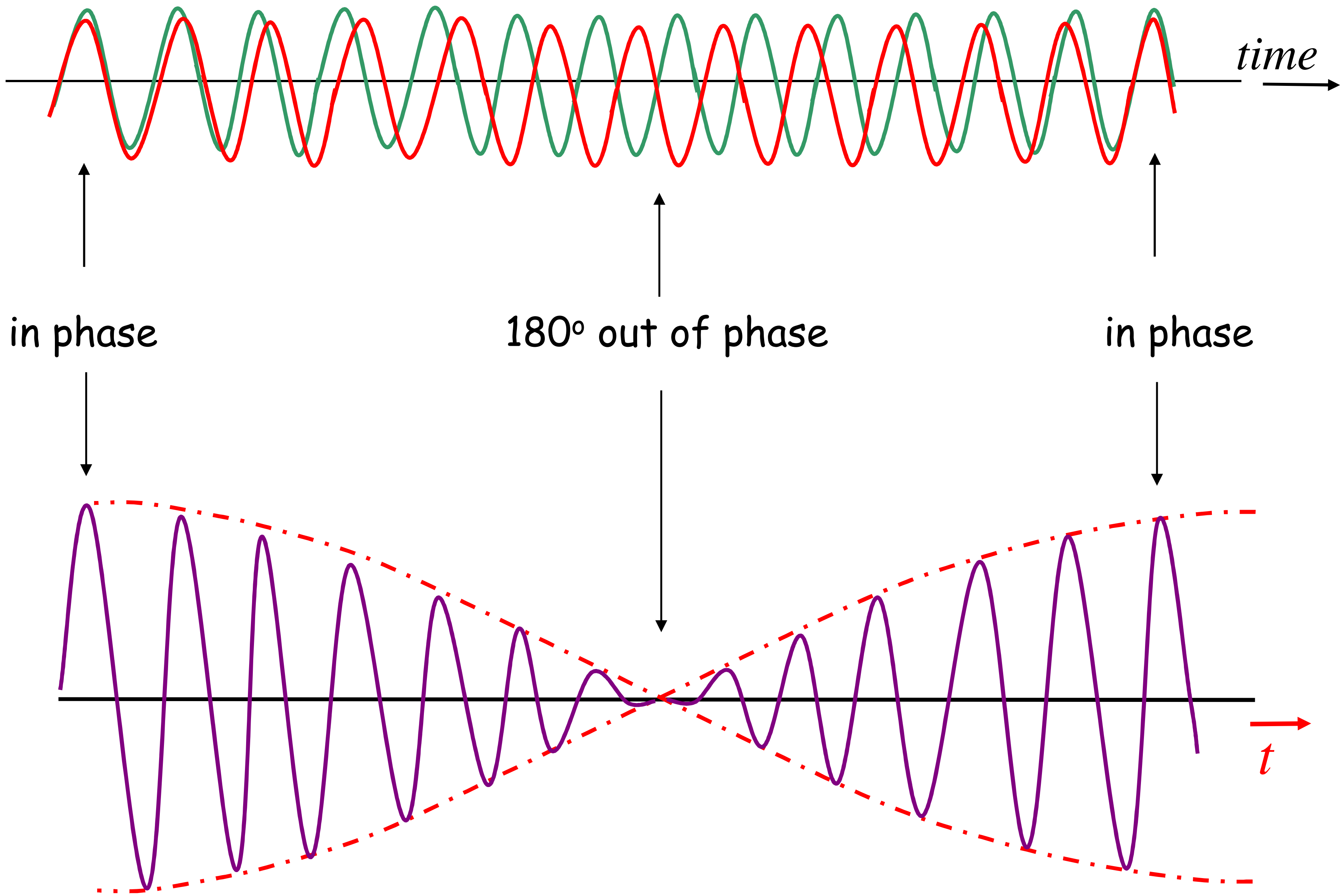
$$A_R = |A_1 - A_2|$$

Beats

Two waves of different frequencies arriving together produce a fluctuation in power or amplitude.

Since the frequencies are different, the two vibrations drift in and out of phase with each other, causing the total amplitude to vary with time.





time
in phase
180o out of phase
in phase
t

The math:

Same amplitudes, different frequencies:

$$y_1 = A_0 \cos(\omega_1 t) \qquad y_2 = A_0 \cos(\omega_2 t)$$

Trigonometry:

cos a + cos b = 2 cos [(a-b)/2] cos [(a+b)/2]

Result:

$$y = y_1 + y_2$$

$$= \underbrace{2 A_0 \cos\left(\frac{\omega_1 - \omega_2}{2} t\right)}_{\text{slowly-varying amplitude}} \underbrace{\cos\left(\frac{\omega_1 + \omega_2}{2} t\right)}_{\text{SHM at average frequency}}$$

slowly-varying amplitude

SHM at <u>average</u> frequency

Note, maximum power when "amplitude" part is equal to **either** $+2A$ **or** $-2A$

⇨ <u>2</u> beats per cycle of $\cos\left(\frac{\omega_1 - \omega_2}{2}t\right)$

⇨ # beats/second = $2 \times \left(\frac{f_1 - f_2}{2}\right)$

The beat frequency (number of beats per second) is equal to the difference between the frequencies:

$$f_b = \left| f_1 - f_2 \right|$$

Quiz

You drive past a radar trap at one ten-millionth the speed of light. The radar set sends out a 2GHz beam, and picks up a reflection from the back of your car which is lower in frequency by 0.2 parts per million. When the two signals are mixed in the radar set, how many beats per second are there?

A) 2.0000004 billion
B) 1.9999996 billion
C) 100
D) 200
E) 400

Interference in Space

2 waves, of the *same frequency*, arrive out of phase.

Eg. $y_1 = A_0 sin\ \omega t$, $y_2 = A_0 sin\ (\omega t + \phi)$

Then

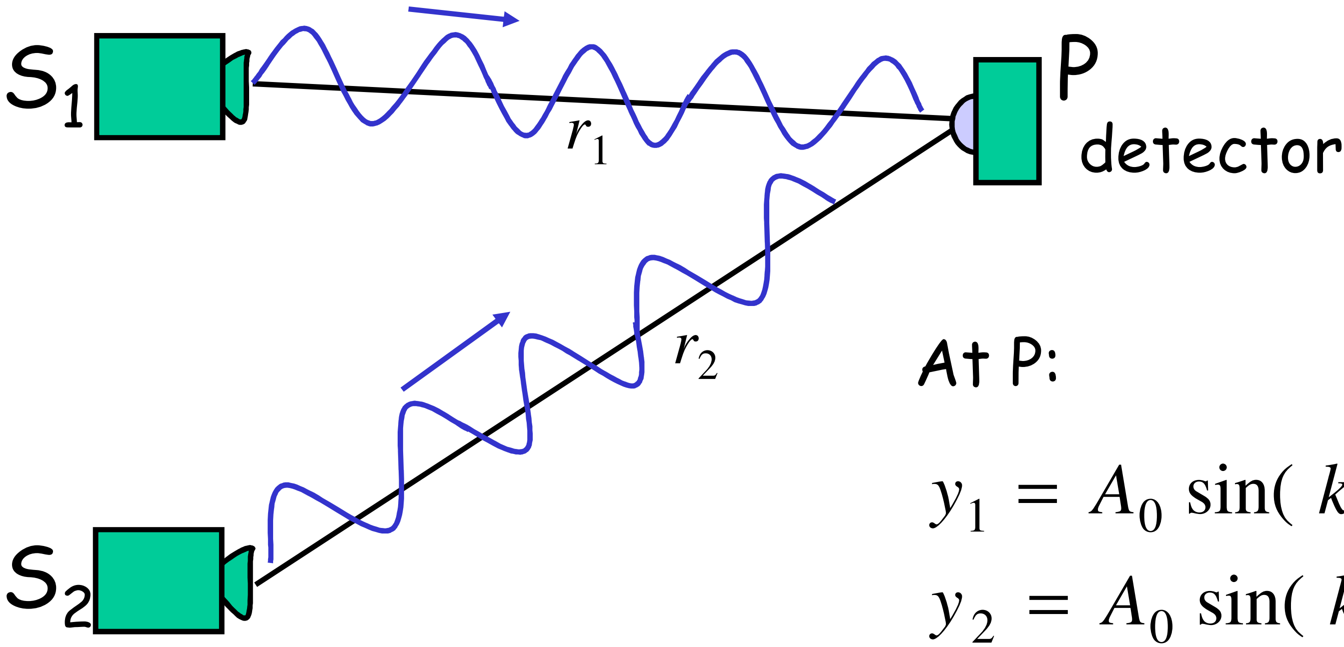
$$y_R = A_R \sin(\omega t + \phi_R),$$

and the resultant amplitude is $A_R = 2A_0 \cos(\frac{1}{2}\phi)$.

*Identical waves which travel different distances will arrive out of phase and will interfere, so that the resultant amplitude varies with **location.***

Example:

Two sources, in phase; waves arrive by different paths:



At P:

$$y_1 = A_0 \sin(kr_1 - \omega t)$$

$$y_2 = A_0 \sin(kr_2 - \omega t)$$

Phase *difference* :

$$(kr_1 - \omega t) - (kr_2 - \omega t) = k(r_1 - r_2) \equiv k\Delta r$$

Define

$$\phi \equiv k\Delta r = 2\pi \frac{\Delta r}{\lambda} \text{ radians}$$

$$\left(\text{or } \frac{\Delta r}{\lambda} \text{ cycles}\right)$$

Then, at detector:

$$y_1 = A_0 \sin(\omega t)$$

(pick starting time so initial phase is zero here)

$$y_2 = A_0 \sin(\omega t + \phi)$$

$$\Rightarrow y_R = (2A_0 \cos\frac{\phi}{2}) \sin(\omega t + \frac{\phi}{2})$$

Intensity

For waves which spread out in 3 dimensions, define

I ≡ Power per unit area *Units: W / m^2*

(the area is measured perpendicular to the wave velocity)

Intensity ∝ (amplitude)^2

Two sources, amplitude A_o, intensity I_o ,

phase difference ϕ

$$\Rightarrow A_R = 2A_o \cos(\tfrac{1}{2}\phi)$$

$$I_R = 4I_o \cos^2(\tfrac{1}{2}\phi)$$

Notes:

1) Maximum I_R is $4 \times I_O$

2) Maxima when $\frac{1}{2}\phi = 0,\ \pm\pi,\ \pm 2\pi,\ \pm 3\pi,\ \ldots$

$$\phi = 0,\ \pm 2\pi,\ \pm 4\pi,\ \pm 6\pi,\ \ldots$$

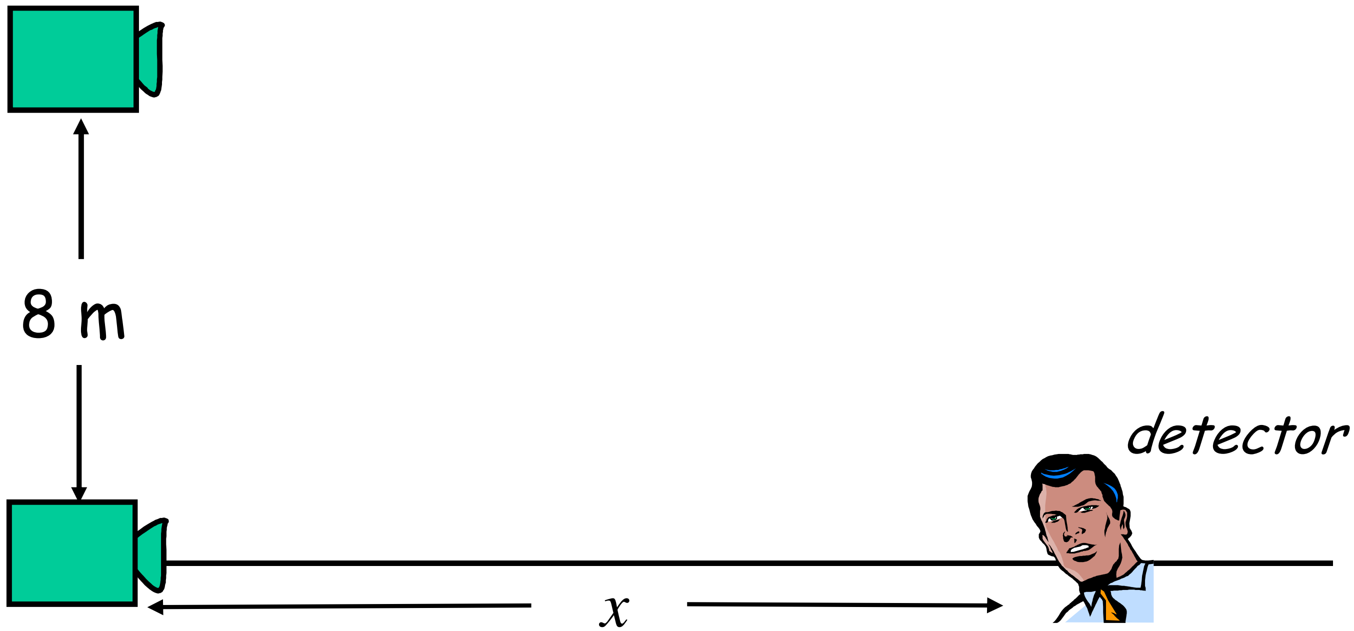
$$\Delta r = 0,\ \pm\lambda,\ \pm 2\lambda, \ldots \quad \textit{(constructive interference)}$$

Minima (zero intensity) when

$$\phi = \pm\pi,\ \pm 3\pi,\ \pm 5\pi,\ \ldots$$

$$\Delta r = \pm\lambda/2,\ \pm 3\lambda/2,\ \pm 5\lambda/2, \ldots \ \textit{(destructive interference)}$$

4) These rules assume sources are in phase.

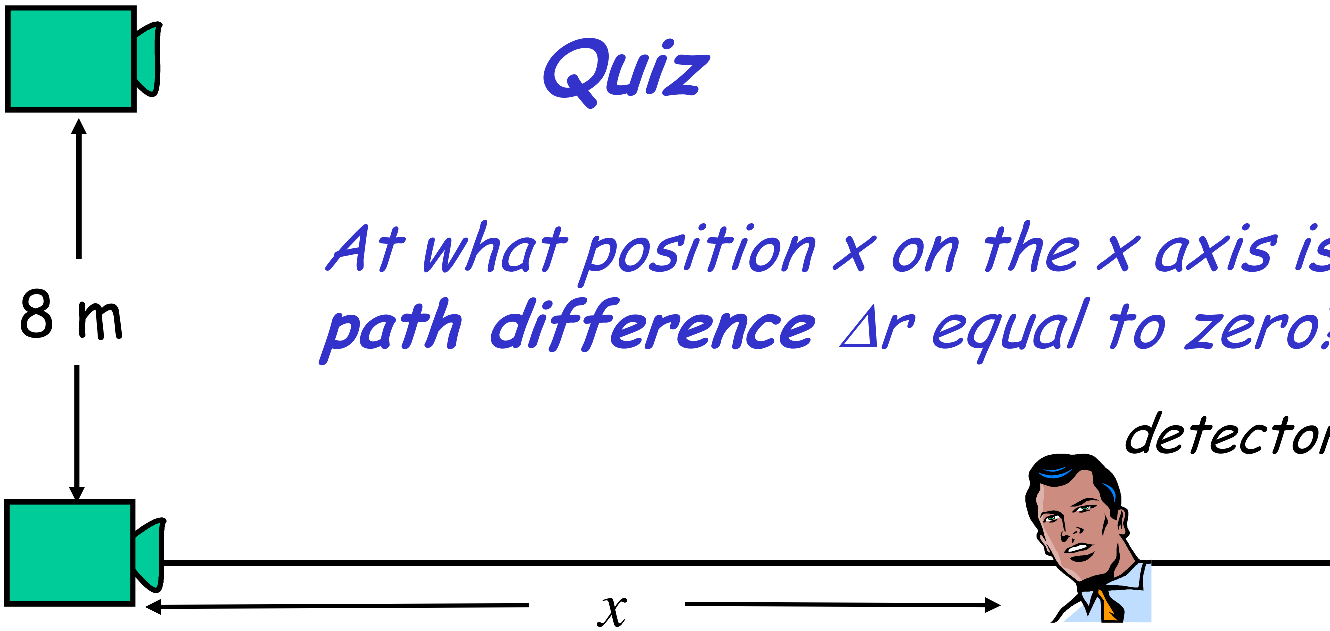


2 speakers, in phase; f = 170 Hz (so λ = 2.0 m)

As you move along the x axis, where is the sound intensity

a) a minimum (compared to nearby points)?
b) a maximum (compared to nearby points)?

Quiz

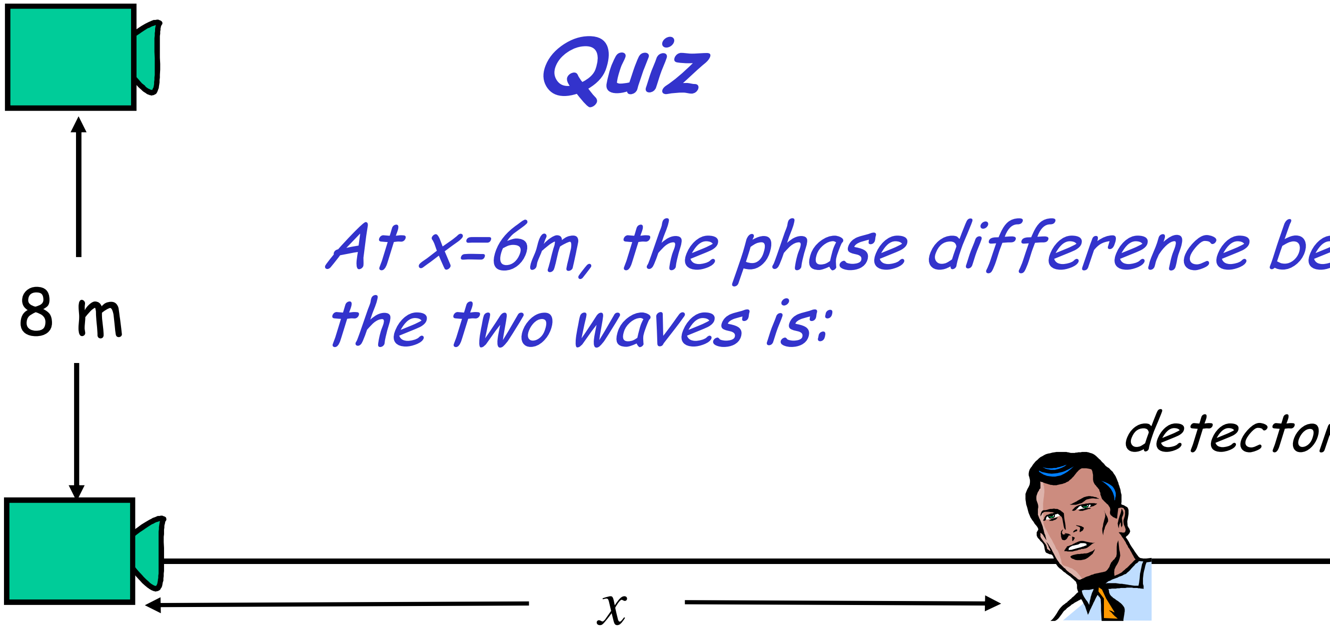


*At what position x on the x axis is the **path difference** Δr equal to zero?*

A) 0
B) 4m
C) 8m
D) 16m
E) ∞

Quiz

At x=6m, the phase difference between the two waves is:



(2 speakers, in phase; f = 170 Hz, λ = 2.0 m)

A) π
B) 2π
C) 3π
D) 4π
E) 6π