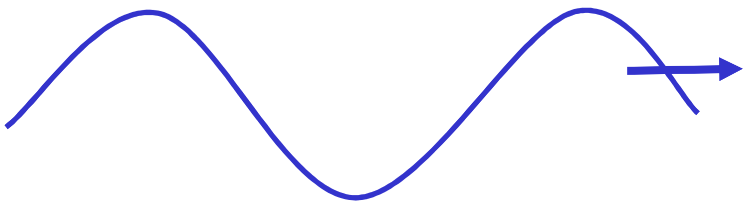
Standing Waves

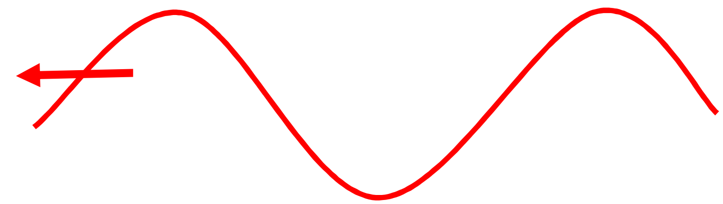
Text sections 18.2-18.4

Practice: Chapter 18,
Objective Questions 2, 5
Conceptual Questions 1, 4, 8
Problems 17, 19, 21, 33, 41

Sine Waves In Opposite Directions:



$y_1 = A_o \sin(kx - \omega t)$



$y_2 = A_o \sin(kx + \omega t)$

Total displacement, $y(x,t) = y_1 + y_2$

Trigonometry: $\sin a + \sin b = 2 \sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$

$(kx + \omega t) \rightarrow "a"$

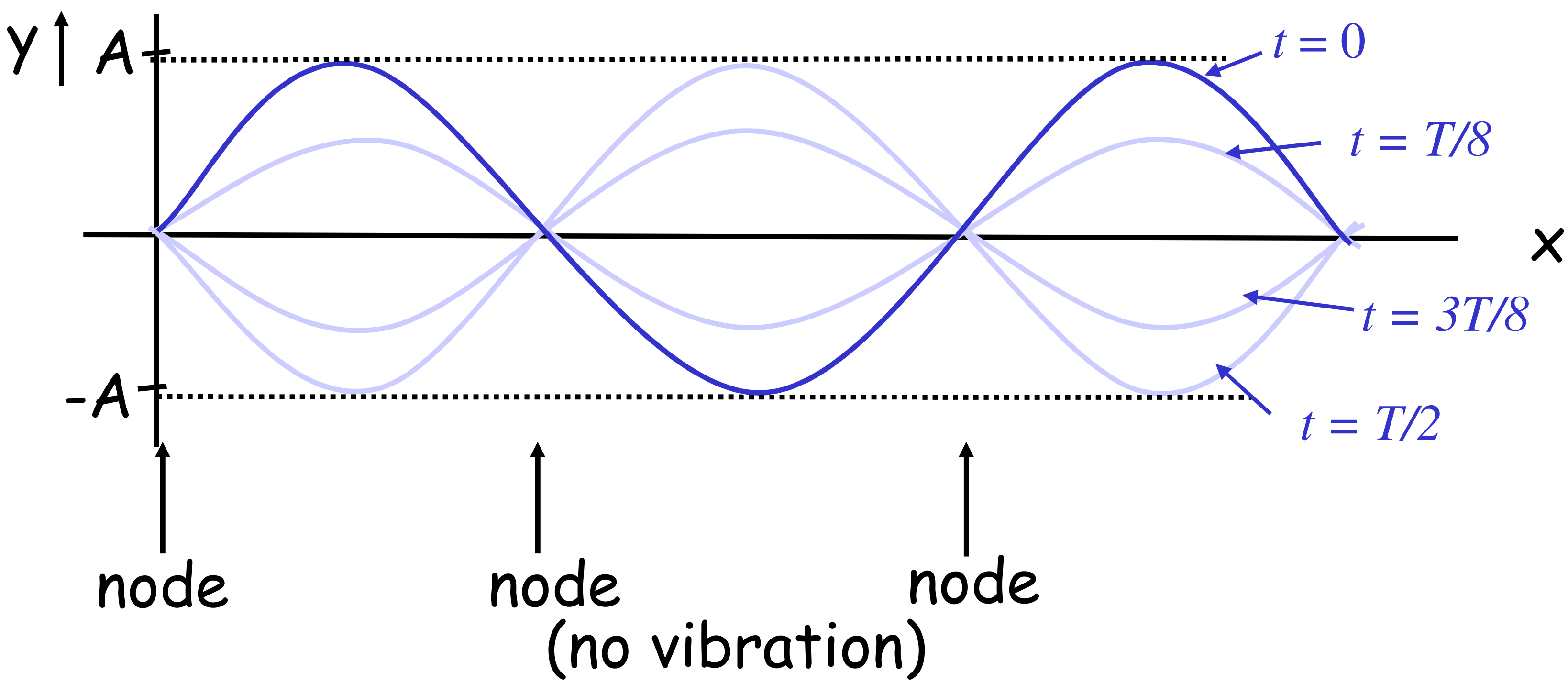
$(kx - \omega t) \rightarrow "b"$

Then: $y(x,t) = 2A_0 \sin kx \cos \omega t$

$$y(x,t) = (A\sin kx)\cos\omega t$$

(where $A = 2A_o$)

*The particle motions are simple harmonic oscillations which are all **in phase** (or ½ cycle out of phase) with each other, but with different amplitudes.*



Nodes are positions where the amplitude is zero:

at $kx = 0 \qquad (x = 0)$

$kx = \pi \qquad (x = \lambda/2)$

$kx = 2\pi \qquad (x = \lambda)$,

etc.

i.e., **Nodes are ½ wavelength apart.**

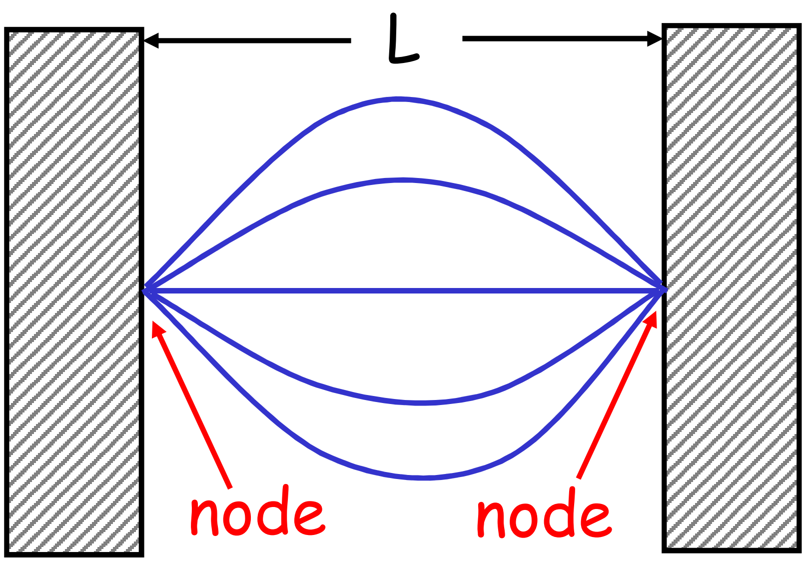
Antinodes (maximum amplitude) are halfway between nodes.

Practical Setup: Fix the ends, use reflections.

We can think of travelling waves reflecting back and forth from the boundaries, and creating a standing wave. The resulting standing wave ***must have a node at each fixed end.*** *Only certain wavelengths can meet this condition, so only certain particular frequencies of standing wave will be possible.*

$$f\lambda = v = \sqrt{\frac{F_T}{\mu}}$$

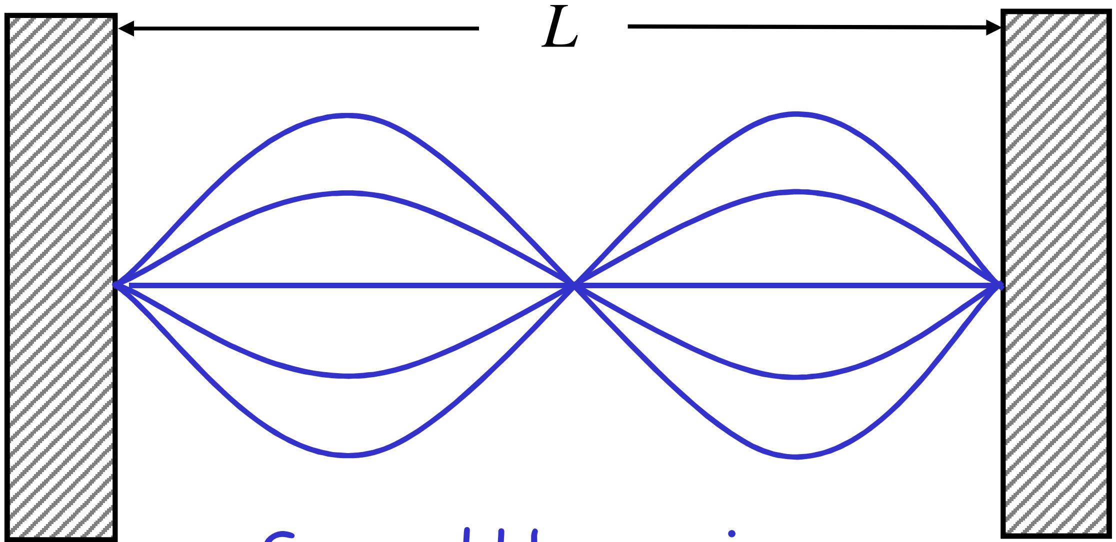
example:



$$\tfrac{1}{2}\lambda_1 = L \Rightarrow \quad \lambda_1 = 2L$$

$$f_1 = \frac{v}{2L}$$

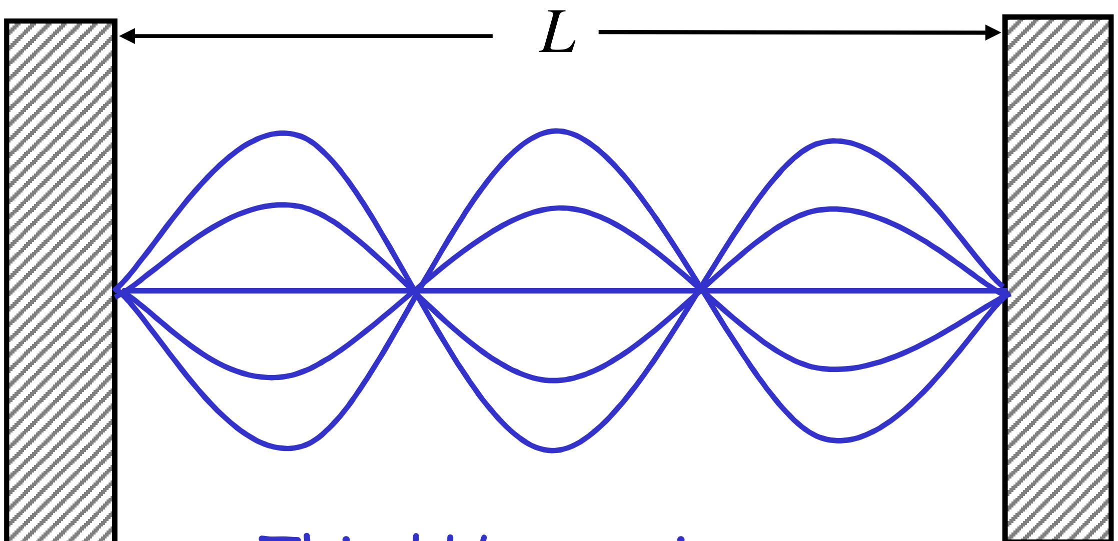
("fundamental mode")



Second Harmonic

$$\lambda_2 = L$$

$$f_2 = \frac{v}{L} = 2f_1$$



Third Harmonic . . . .

$$\tfrac{3}{2}\lambda_3 = L$$

$$\lambda_3 = \tfrac{2}{3}L$$

$$f_3 = \frac{v}{\lambda_3} = 3\frac{v}{2L}$$

$$= 3f_1$$

In this case (a one-dimensional wave, on a string with both ends fixed) the possible standing-wave frequencies are multiples of the fundamental:

$$f_1, 2f_1, 3f_2, \text{ etc.}$$

This pattern of frequencies depends on the shape of the medium, and the nature of the boundary (fixed end or free end, etc.).

Other Boundary Conditions

What standing-wave frequencies do we get if there is

*-an **antinode** at each end?*

-an antinode at one end, and a node at the other?

Quiz

If there is an antinode at each end of a medium of length L, the longest standing-wave wavelength will be:

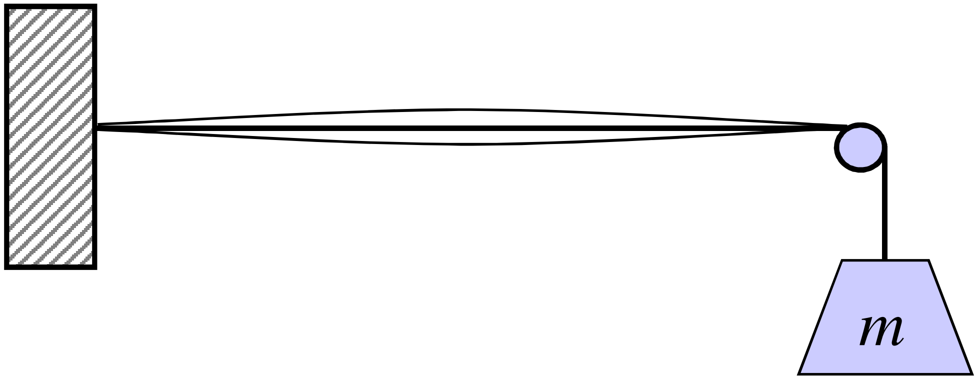
A) $\frac{1}{4}$ L
B) $\frac{1}{2}$ L
C) L
D) 2L
E) 4L

Quiz

If there is an antinode at one end, and a node at the other, the longest standing-wave wavelength will be:

A) $\frac{1}{4}$ L
B) $\frac{1}{2}$ L
C) L
D) 2L
E) 4L

Quiz

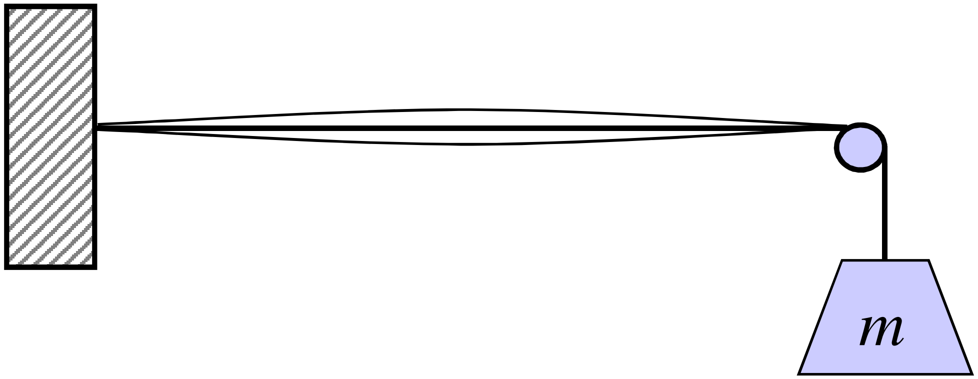


When the mass m is doubled, what happens to

a) the wavelength, and
b) the frequency

of the fundamental standing-wave mode?

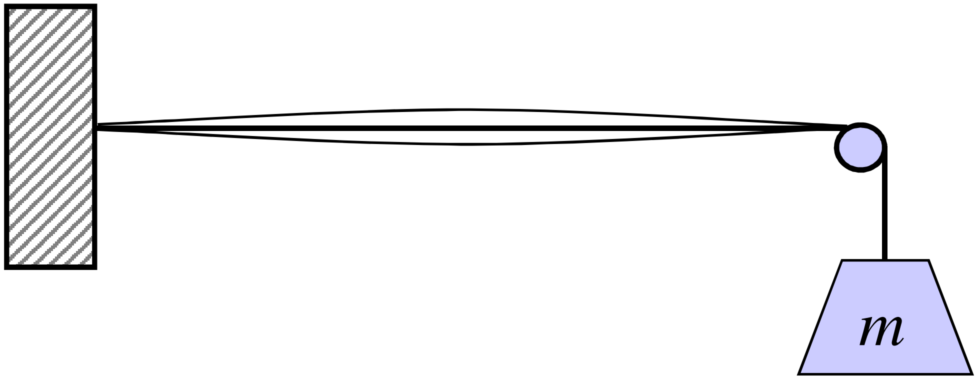
Quiz



When the mass m is doubled, the wavelength of the fundamental standing-wave mode

A) is unchanged
B) increases
C) decreases

Quiz



When the mass m is doubled, the frequency of the fundamental standing-wave mode

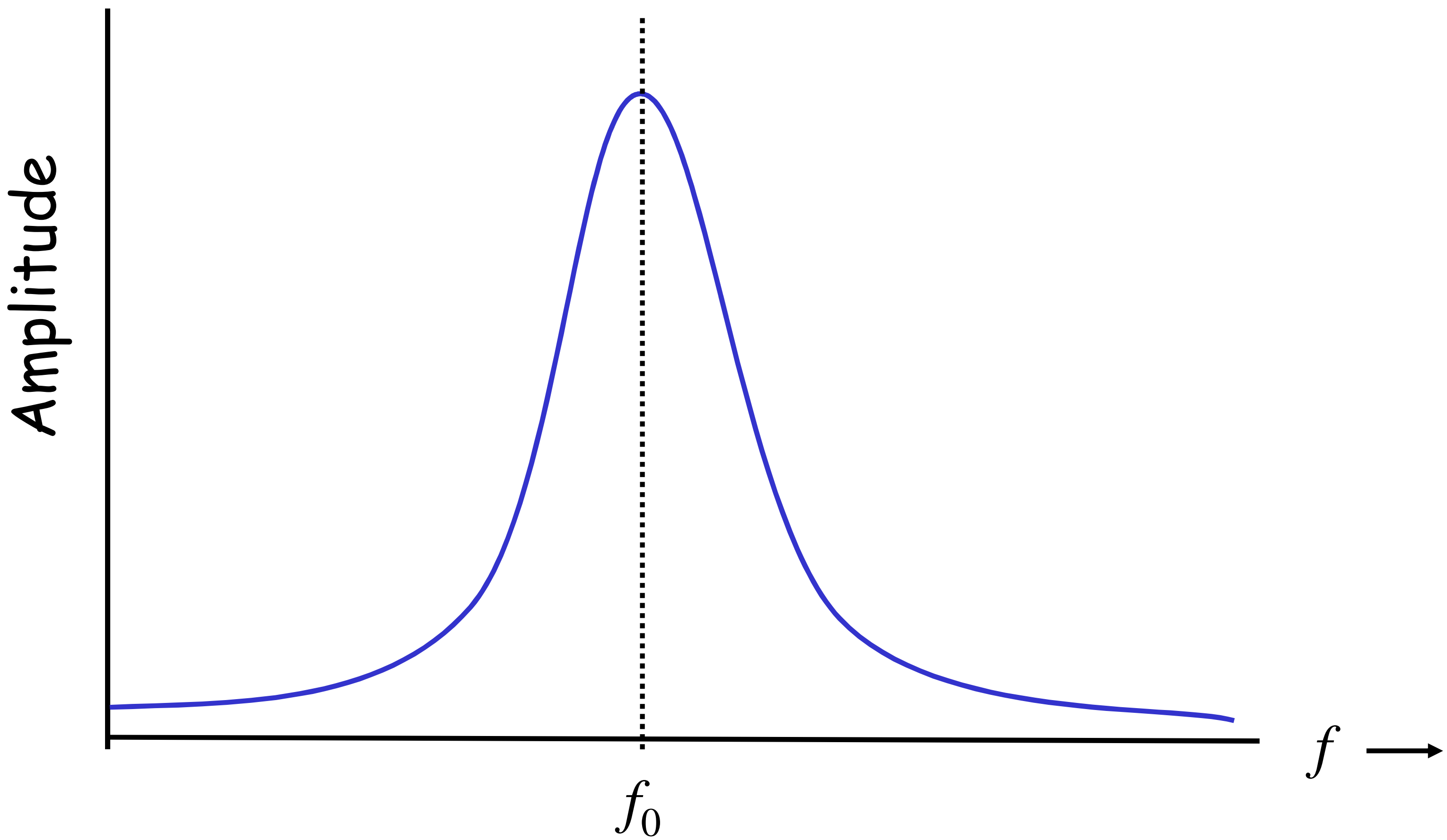
A) is unchanged
B) changes by a factor of $\sqrt{2}$
C) changes by a factor of 2
D) changes by a factor of 4

Resonance

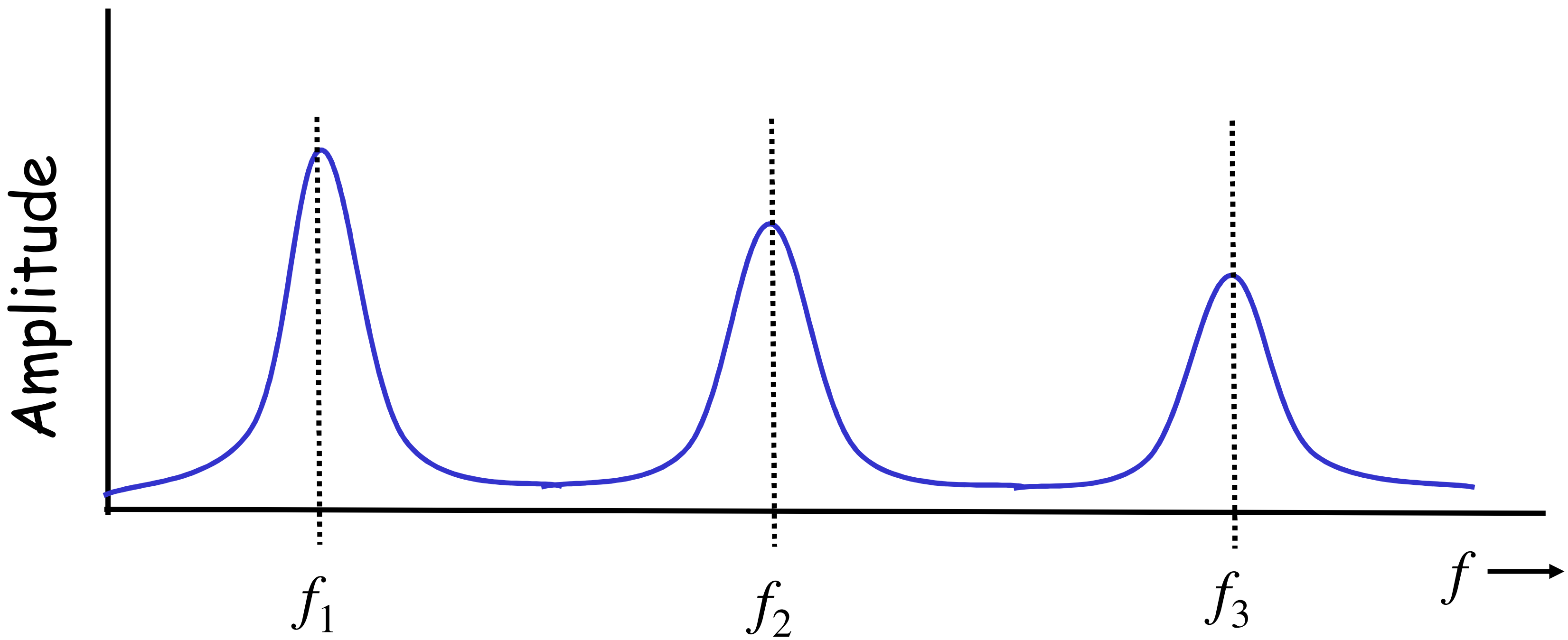
What happens if you try to vibrate the string at the "wrong" frequency?

Compare this with a simple harmonic oscillator (e.g., a mass on a spring), which can oscillate at some "natural frequency" f_0. *If we apply an external periodic force at frequency* f, *the mass will oscillate at the frequency* f *of the external force, but the amplitude will be small unless* $f \approx f_o$.

Driven oscillator (mass on a spring)

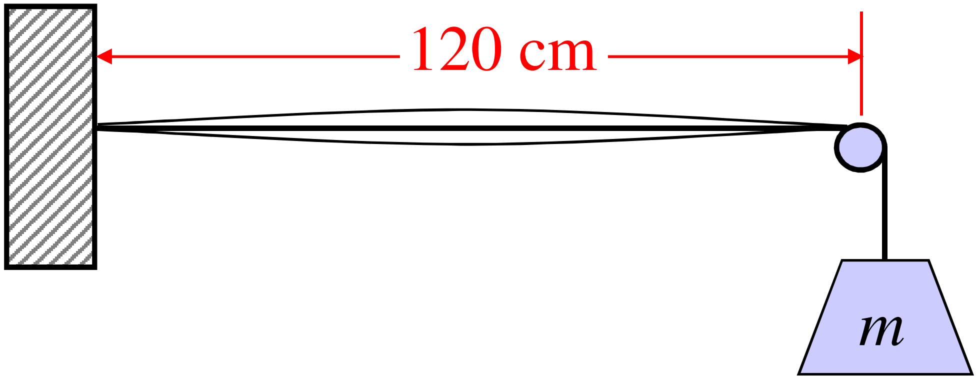


String Fixed at Both Ends



Each mode has its own “resonant frequency”.

Example



a) m = 150g, f_1 = 30 Hz. Find μ (mass per unit length)

(Answer: 0.28 grams/metre)

b) Find m needed to give f_2 = 30 Hz

c) m = 150g. Find f_1 for a string *twice as thick,* made of the same material.