Make-Up Labs Next Week Only

Monday, Mar. 30 to Thursday, April 2

Make arrangements with Dr. Buntar in BSB-B117

If you have missed a lab for any reason, you must complete the lab in make-up week.

Energy; Superposition

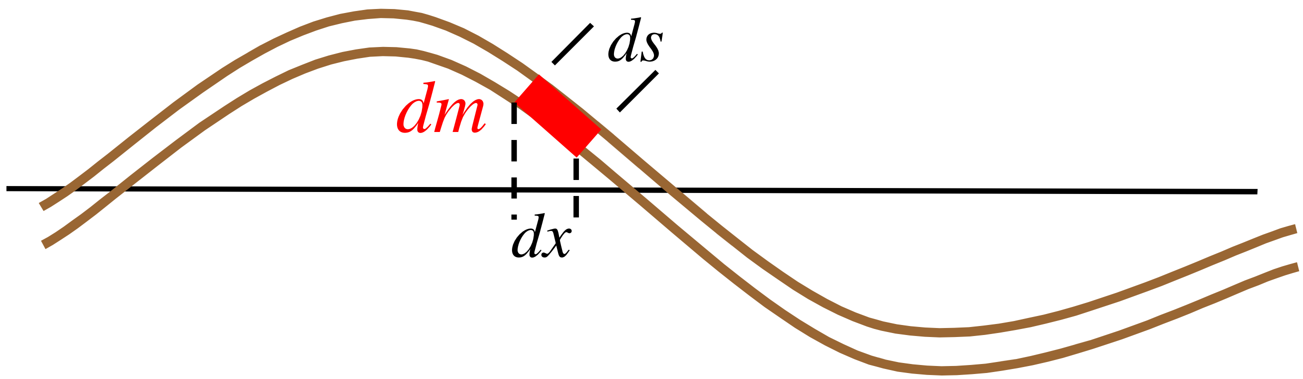
Text sections 16.5, 18.1, 8.2

Practice: Chapter 16,
Problems 32, 39, 40;
Chapter 18,
Objective Questions 4, 9, 12
Conceptual Questions 2, 9
Problems 1, 2, 4, 5, 15, 16

Energy, Power

$$\text{Energy, Power, etc.} \propto (\text{amplitude})^2$$

Stretched rope, energy/unit length:



Ignore difference between "ds", "dx"
(small A, large λ):

$$dm = \mu\, dx \quad (\mu = \text{mass/unit length})$$

The mass dm vibrates in simple harmonic motion. Its maximum kinetic energy is

$$dK_{max} = \tfrac{1}{2}(dm)(v_{max})^2$$
$$= \tfrac{1}{2}(dm)(\omega A)^2$$

*The **average** kinetic energy is half this maximum value, but there is also an equal amount of potential energy in the wave. The total energy (kinetic plus potential) is therefore*

$$dE = \tfrac{1}{2}(dm)\,\omega^2 A^2$$

To get the energy per unit length, replace the mass dm with the mass per unit length μ:

$$\boxed{\frac{Energy}{length} = \frac{1}{2}\mu\omega^2 A^2}$$

Power: Energy travels at the wave speed *v*,

So $$P = \left(\frac{\text{Energy}}{\text{length}}\right) \times v$$

waves on a string, $$P = \tfrac{1}{2}\mu\omega^2 A^2 v$$

*Both the energy density and the power transmitted are proportional to the **square** of the amplitude. This is a general property of sinusoidal waves.*

Quiz

A radio station produces oscillating electric fields of 20μV/m at your house during the day. At night, the station turns its transmitters down to half power. What is the electric-field amplitude at night?

A) 5.0μV/m
B) 10μV/m
C) 14μV/m
D) 20μV/m

Intensity

For waves which spread out in 3 dimensions, define

Intensity ≡ Power per unit area

Units: W / m^2

(the "area" is measured perpendicular to the wave velocity)

Example: Sunlight,

$I \approx 1400\ W/m^2$, above the atmosphere

$I < 1000\ W/m^2$, at sea level

For these waves (light, sound, …),

Intensity $\propto$ (amplitude)^2

Quiz

An outdoor concert produces sound waves with an an amplitude (of the motion of the air molecules) of 4mm at a distance of 50 m. What would the amplitude be at a distance of 100 m?

A) 4 mm
B) 2 mm
C) 1 mm
D) 0.5 mm
E) 0.25 mm

Principle of Superposition

2 Waves In The Same Medium:

The observed displacement $y(x,t)$ is the sum of the individual displacements:

$$y_1(x,t) + y_2(x,t) = y(x,t)$$

(for a "linear medium")

What's Special about Sine Waves?

2 waves, of the *same frequency,* arrive out of phase:

Eg. $$y_1 = A\sin(kx - \omega t)$$

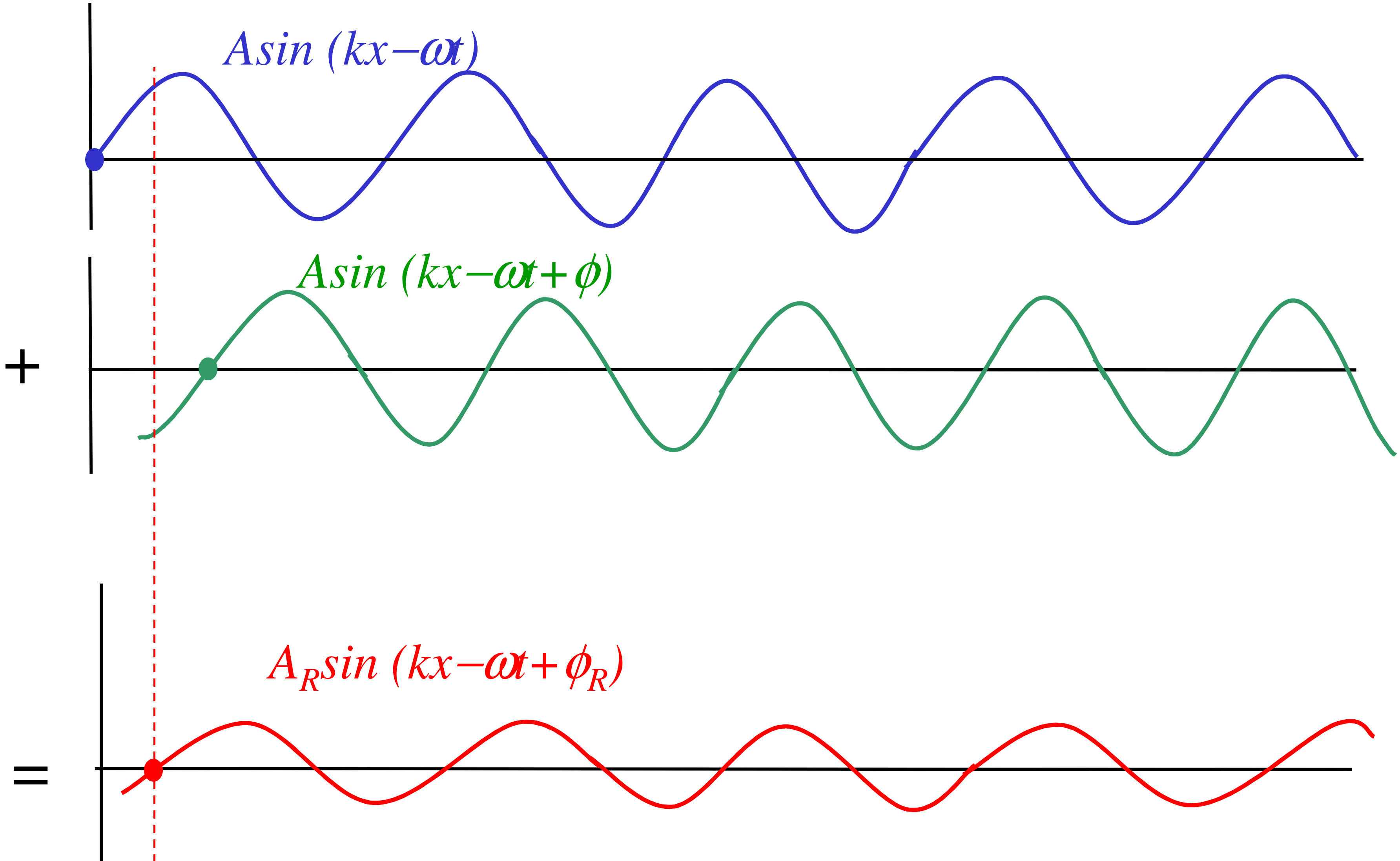
$$y_2 = A\sin(kx - \omega t + \phi)$$

Trigonometry:

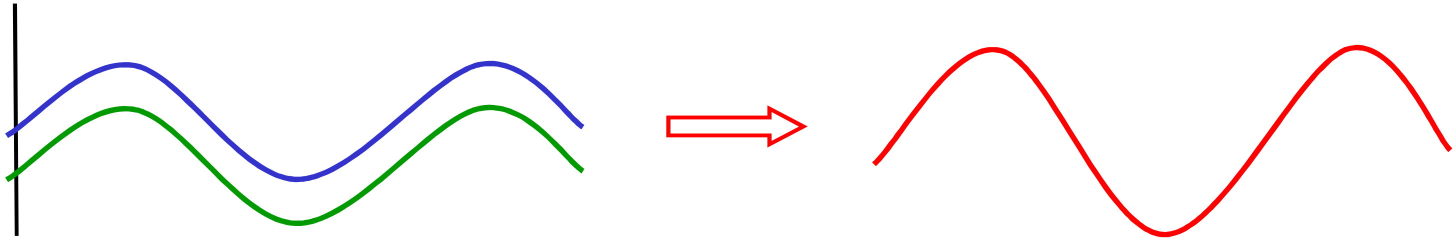
sin a + sin b = 2 cos [(a-b)/2] sin [(a+b)/2]

Result: $$y = y_1 + y_2$$

$$= \underbrace{2A\cos\left(\frac{\phi}{2}\right)}_{\text{amplitude}}\sin\left(kx - \omega t + \frac{\phi}{2}\right)$$

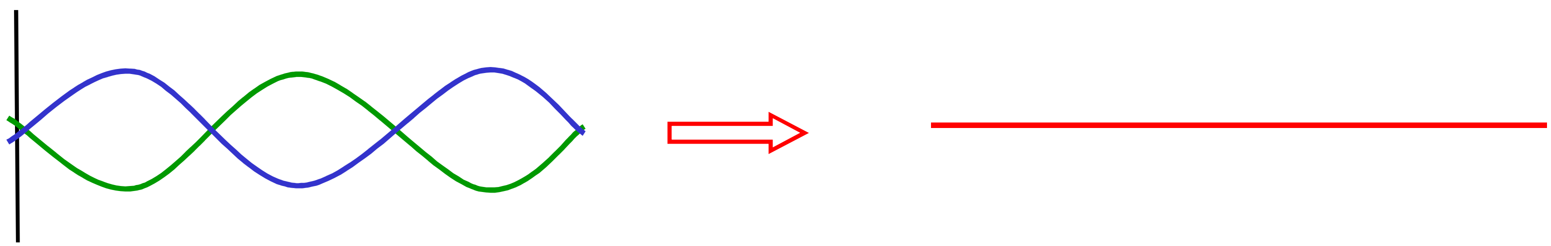


Resultant: ***Sine wave***, *same* f; A_R *depends on phase difference*



"Constructive interference:"
phase difference =0, 2π, 4π, ...

$$A_R = A_1 + A_2$$

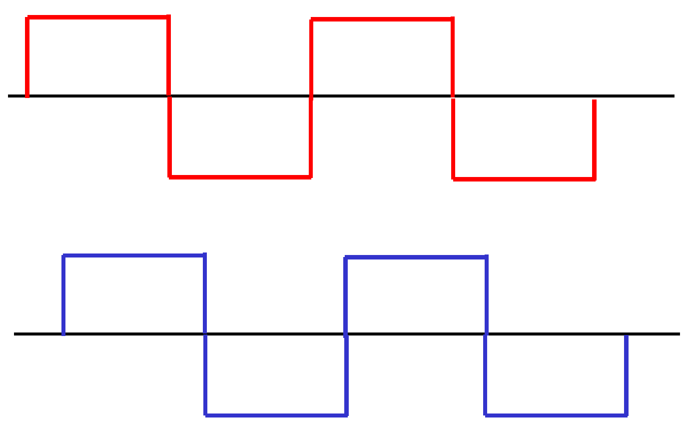


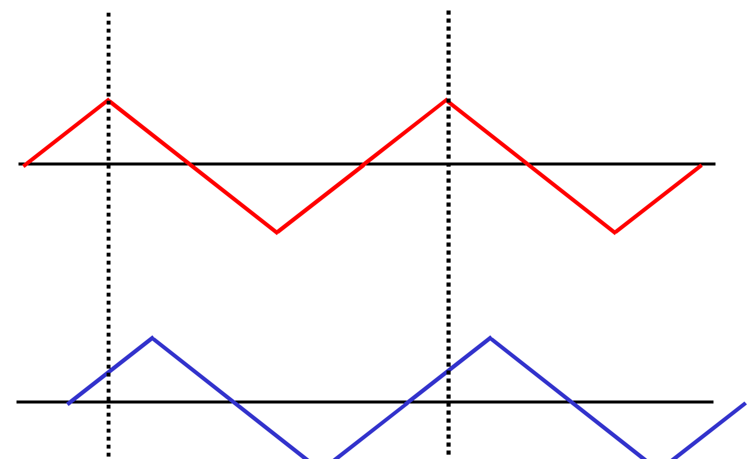
"Destructive interference:"
phase difference = π, 3π, 5π,...

$$A_R = |A_1 - A_2|$$

Exercise

What do you get if you add two identical (but out-of-phase) square or triangular waves?





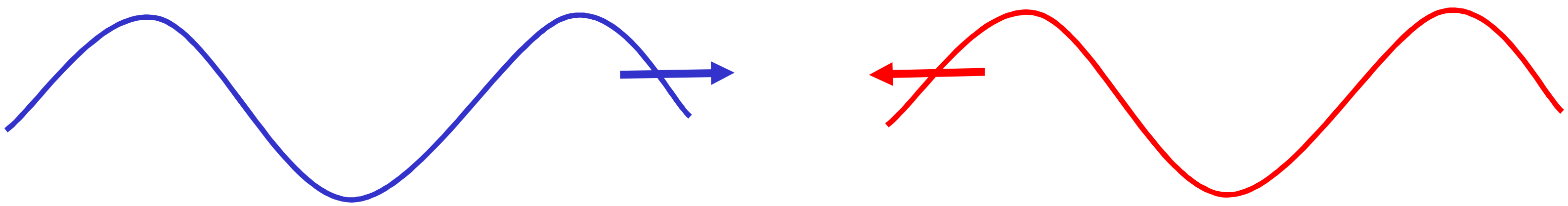
+

= ?

+

= ?

Sine Waves In Opposite Directions:



$y_1 = A_o \sin(kx - \omega t)$ $\qquad$ $y_2 = A_o \sin(kx + \omega t)$

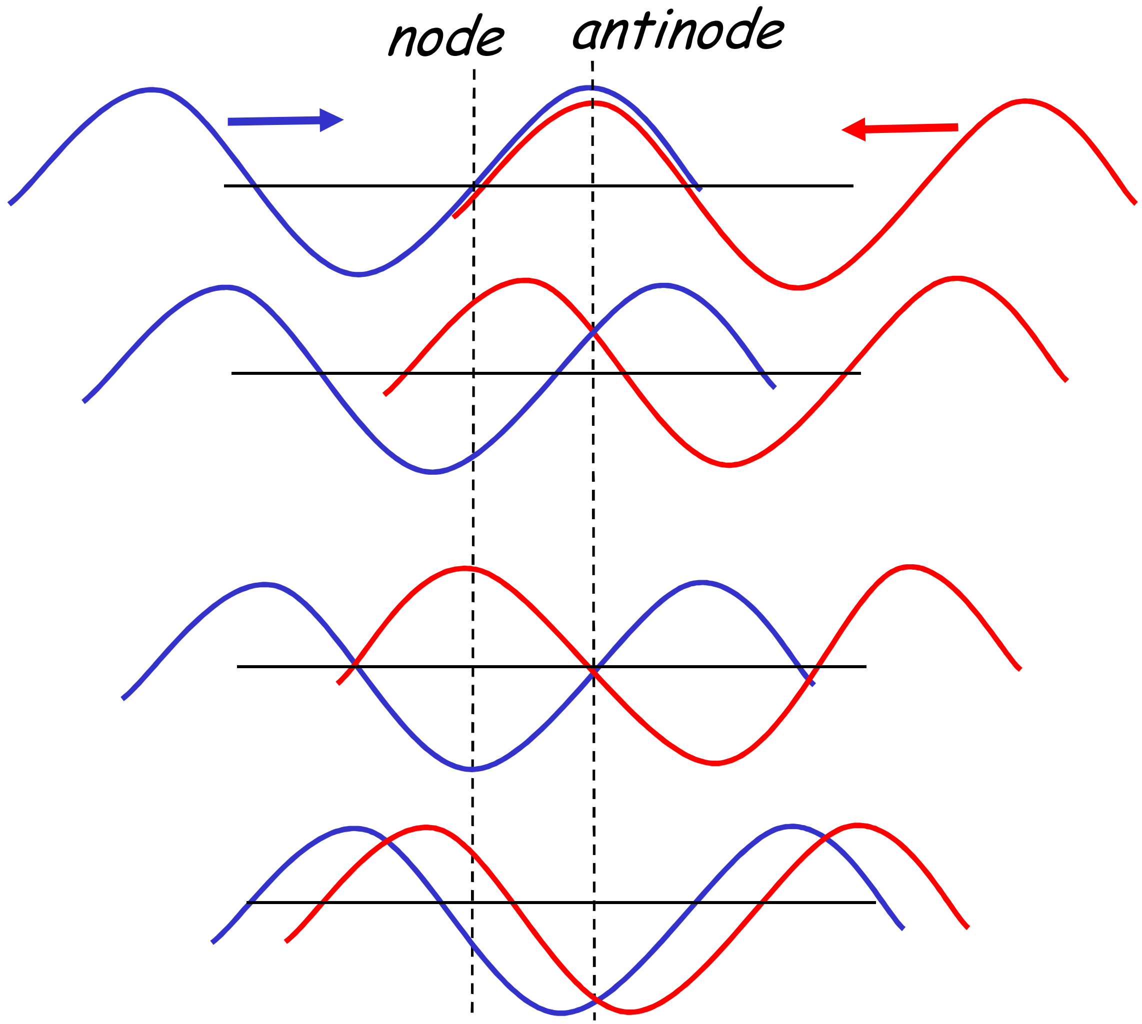
Total displacement, $y(x,t) = y_1 + y_2$ is a "standing wave".

where waves arrive in phase:

$\rightarrow$ constructive interference ("antinode")

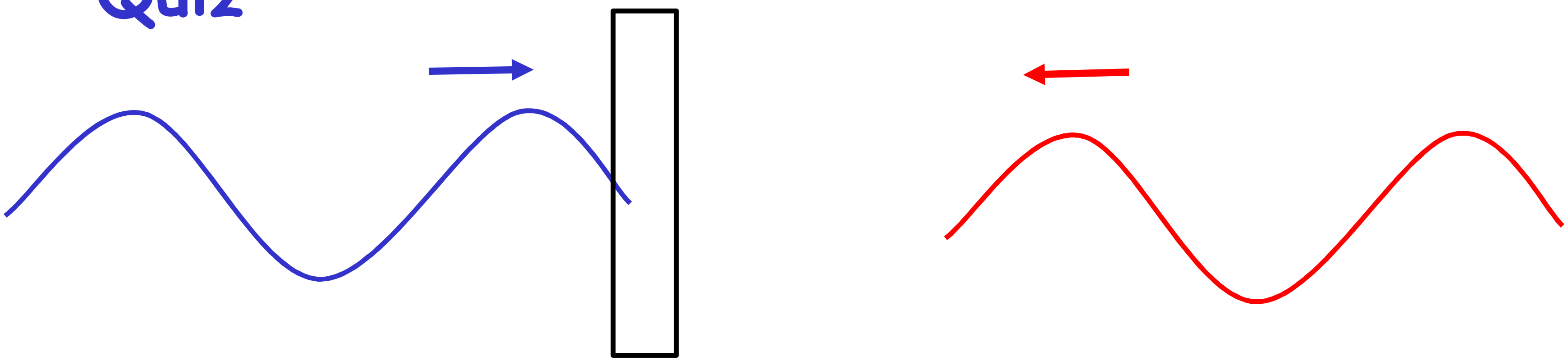
where waves arrive 180° out of phase:

$\rightarrow$ destructive interference ("node")



Antinodes form where the waves always arrive in phase ("constructive interference"); nodes form at locations where the waves are 180° (½ cycle) out of phase ("destructive interference").

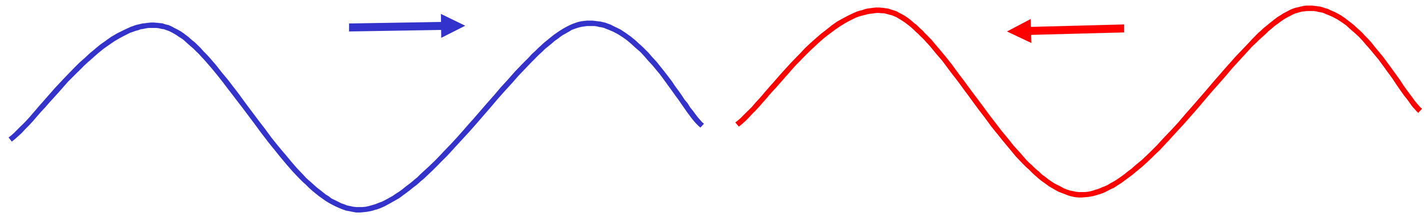
Quiz



The wave travelling from left to right is delayed by 1/10 of a period before the waves interfere. The pattern of nodes and antinodes will:

A) disappear
B) shift sideways to the left
C) shift sideways to the right

Quiz



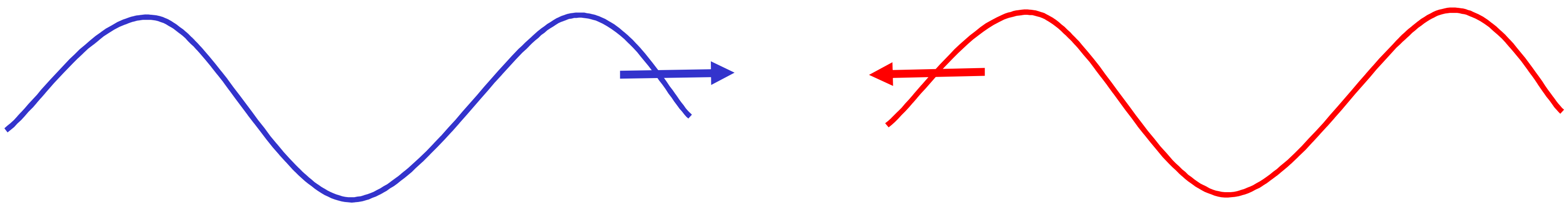
At the antinode in the middle, the two waves arrive in phase. How far away is the nearest point where the waves are ½ cycle out of phase with each other?

A) ¼ wavelength
B) ½ wavelength
C) 1 wavelength
D) 2 wavelengths
E) 4 wavelengths

Question

The energy density in a travelling wave is proportional to the square of the amplitude (e.g., for a wave on a stretched string, the energy per unit length is $(\frac{1}{2}\mu\omega^2A^2)$*). Does the energy density add up properly at each point when two travelling waves combine to form a standing wave? Does the power transmitted add up?*

Sine Waves In Opposite Directions:



$$y_1 = A_o \sin(kx - \omega t)$$

$$y_2 = A_o \sin(kx + \omega t)$$

Total displacement, $y(x,t) = y_1 + y_2$

Trigonometry: $\sin a + \sin b = 2 \sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$

$$(kx + \omega t) \rightarrow "a"$$

$$(kx - \omega t) \rightarrow "b"$$

Then: $y(x,t) = 2A_0 \sin kx \cos \omega t$