Wave Motion (II)

Text sections 16.2 – 16.3, 16.6

- Sinusoidal waves

Practice: Chapter 16,
Objective Questions 1, 3, 8
Conceptual Question 5
Problems 9, 11, 13, 27

Non-dispersive waves:

$$y(x,t) = f(x \pm vt)$$

+ sign: wave travels towards −x
- sign: wave travels towards +x

f is any (smooth) function of <u>one</u> variable.

eg. $f(x) = A \sin(kx)$

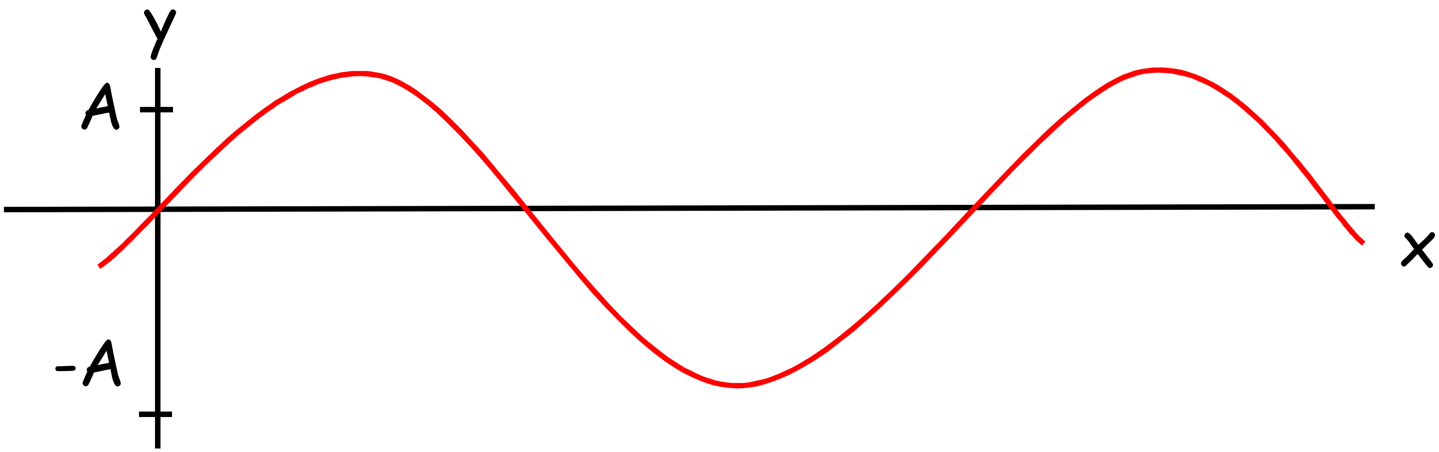
Sine Waves

For a wave travelling in the $+x$ direction, the displacement y is given by

$$y\,(x,t) = f\,(x - vt)$$

For sinusoidal waves, the shape is a sine function, *eg.*,

$$f(x) = y(x,0) = A\sin(kx)$$ (A and k are constants)



Then $y\,(x,t) = f(x - vt) = A\sin[k(x - vt)]$

or $y\,(x,t) = A\sin\,(kx - \omega\,t)$ with $\omega = kv$

The displacement of a particle at a fixed location x is a sinusoidal function of time – i.e., ***simple harmonic motion:***

$$y = A \sin (kx - \omega t) = A \sin [\text{ constant} - \omega t]$$

*The "angular frequency" of the particle motion is ω; the initial phase is kx (**different** for different particles).*

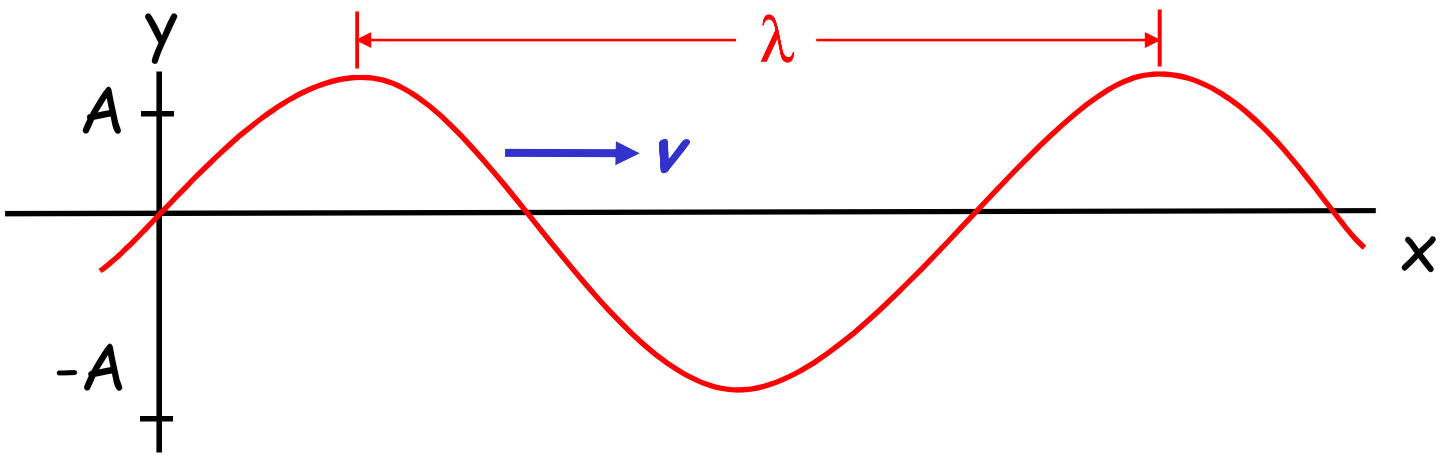
Review: Recall from last term, SHM is described by functions of the form $y(t) = A \cos(\omega t + \phi) = A \sin(\pi/2 - \phi - \omega t)$, *etc., with*

$$\omega = 2\pi f$$

"angular frequency"
radians/sec

frequency:
cycles/sec (=hertz)

Sine wave: $y\,(x,t) = A\sin(kx - \omega t)$



λ ("lambda") is the wavelength (length of one complete wave); and so (kx) must increase by 2π radians (one complete cycle) when x increases by λ. So $k\lambda = 2\pi$, or

$$k = 2\pi / \lambda$$

The most general form of sine wave is $y = A\sin(kx \pm \omega t + \phi)$

amplitude

"phase"

$$y(x,t) = A\,sin\,(kx \pm \omega t + \phi)$$

angular wavenumber
$k = 2\pi / \lambda$

angular frequency
$\omega = 2\pi f$

phase constant ϕ

The wave speed is v = 1 wavelength / 1 period, so

$$v = f\lambda = \omega/k$$

Particle Velocities

particle displacement, $y\,(x,t)$

particle velocity, $v_y = dy/dt$ (x held constant)

(Note that v_y is not the wave speed v !)

Acceleration, $a_y = \dfrac{dv_y}{dt} = \dfrac{d^2y}{dt^2}$

(actually, these are partial derivatives: $\dfrac{\partial^2 y}{\partial t^2}$, etc.)

"Standard" sine wave:

$$y = A\sin(kx \pm \omega t + \varphi)$$

$$v_y = \frac{dy}{dt} = \pm \omega A\cos(kx \pm \omega t + \varphi)$$

$$a_y = \frac{dv_y}{dt} = -\omega^2 A\sin(kx \pm \omega t + \varphi)$$

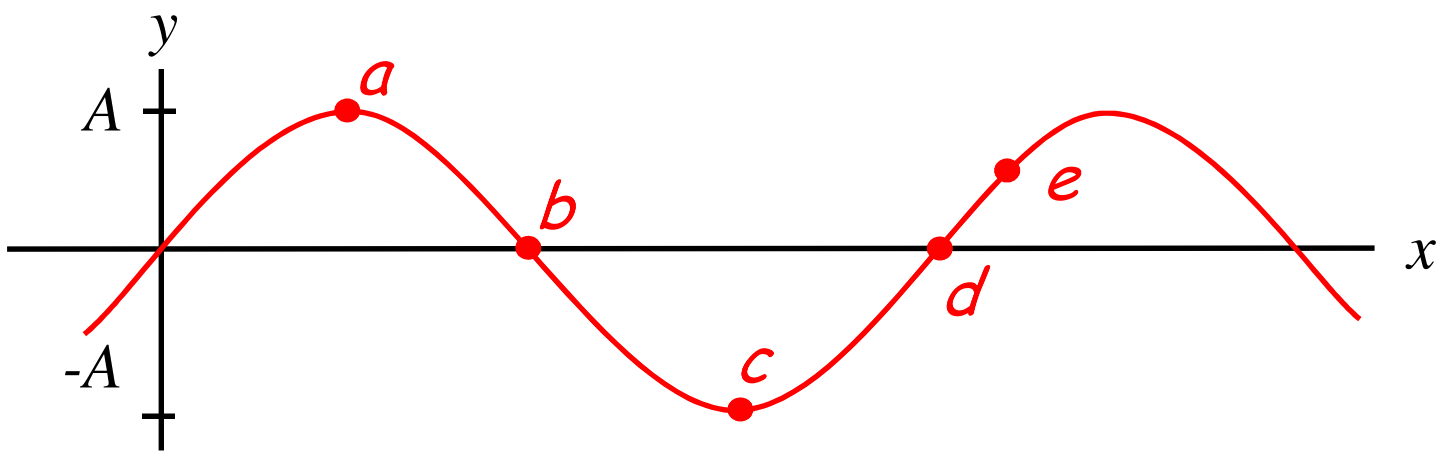
$$= -\omega^2 y$$

maximum displacement, $y_{\max} = A$

maximum velocity, $v_{\max} = \omega A$

maximum acceleration, $a_{\max} = \omega^2 A$

Quiz



Shown is a picture of a wave, $y = A \sin(kx - \omega t)$, at time $t = 0$.

i) Which particle moves according to $y = A \cos(\omega t)$?

ii) Which particle moves according to $y = A \sin(\omega t)$?

iii) If $y_e(t) = A \cos(\omega t + \phi_e)$ for particle e, what is ϕ_e ?

Wave Velocity

The wave velocity is determined by the properties of the medium; for example,

1) Transverse waves on a string:

$$v_{\text{wave}} = \sqrt{\frac{\text{tension}}{\text{mass/unit length}}} = \sqrt{\frac{F_{\text{T}}}{\mu}}$$

(proof from Newton's second law)

2) Electromagnetic wave (light, radio, etc.)

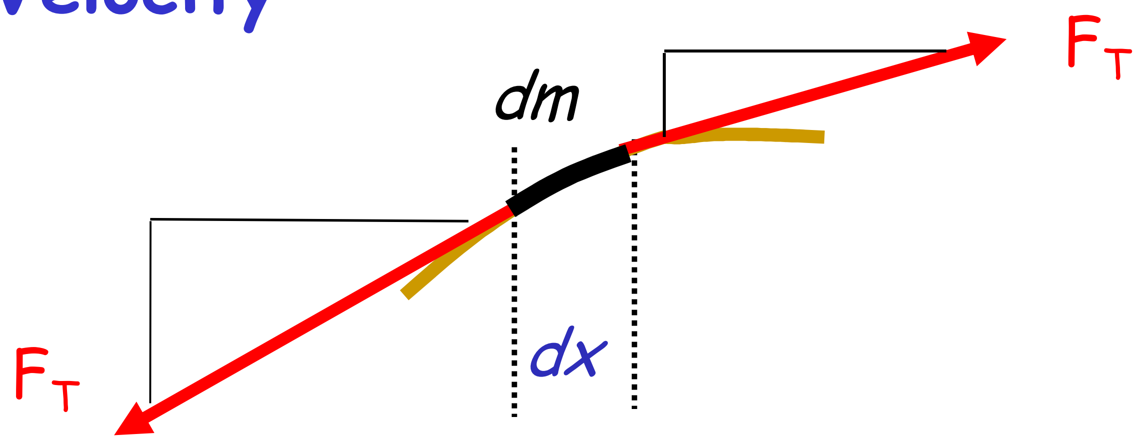
$$v = \sqrt{\frac{1}{\mu_{o}\varepsilon_{o}}} = c$$

(proof from Maxwell's Equations for E-M fields)

Exercise

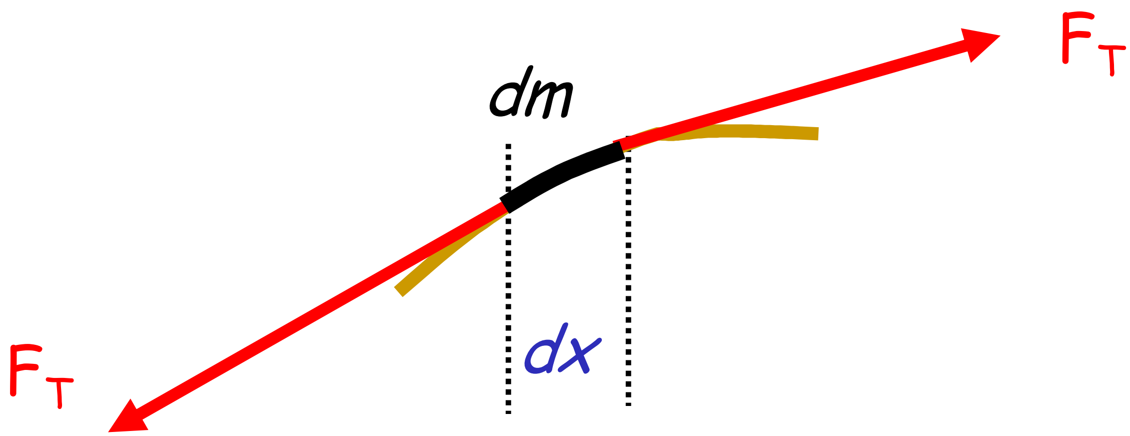
What are ω and k for a 99.7 MHz FM radio wave?

Wave Velocity



Exercise: show that the net vertical force on the short element of rope (of length dx) is, approximately, given by

$$dF_{net} = F_T \frac{\partial^2 y}{\partial x^2} dx$$



Newton's second law is

$$dF_{net} = F_T \frac{\partial^2 y}{\partial x^2} dx = (dm)a = (\mu dx)\frac{\partial^2 y}{\partial t^2}$$

where μ is the linear mass density.

Wave equation

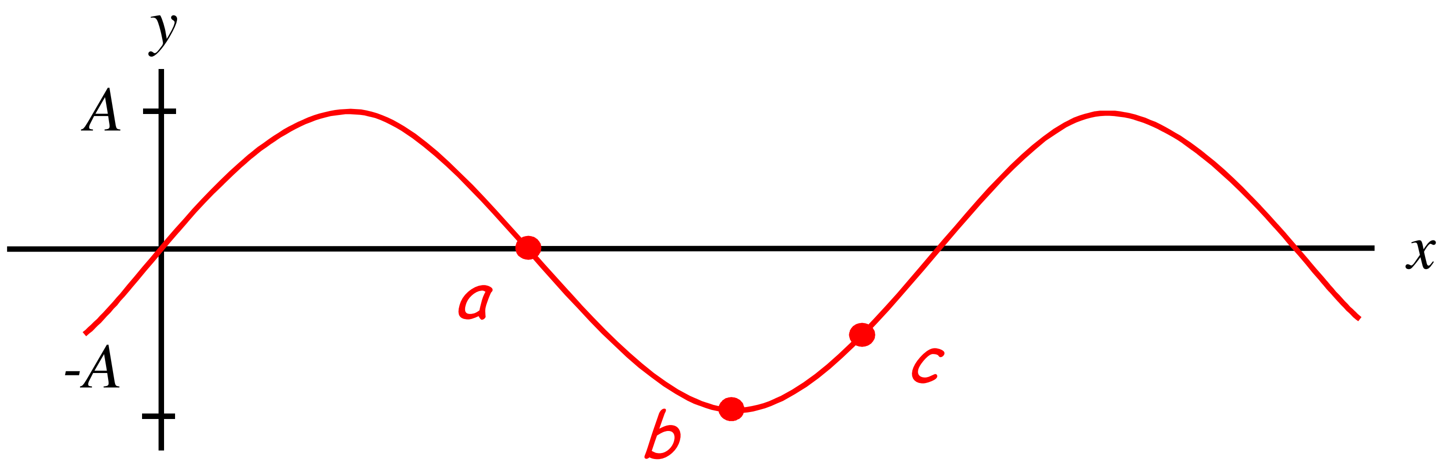
Rearranging, we get the **wave equation:**

$$\frac{\partial^2 y}{\partial t^2} = \left(\frac{F_T}{\mu}\right)\frac{\partial^2 y}{\partial x^2}$$

Exercise: apply this to our standard sine-wave expression for y(x,t), to show that

$$\frac{F_T}{\mu} = \frac{\omega^2}{k^2} = v_{wave}^2$$

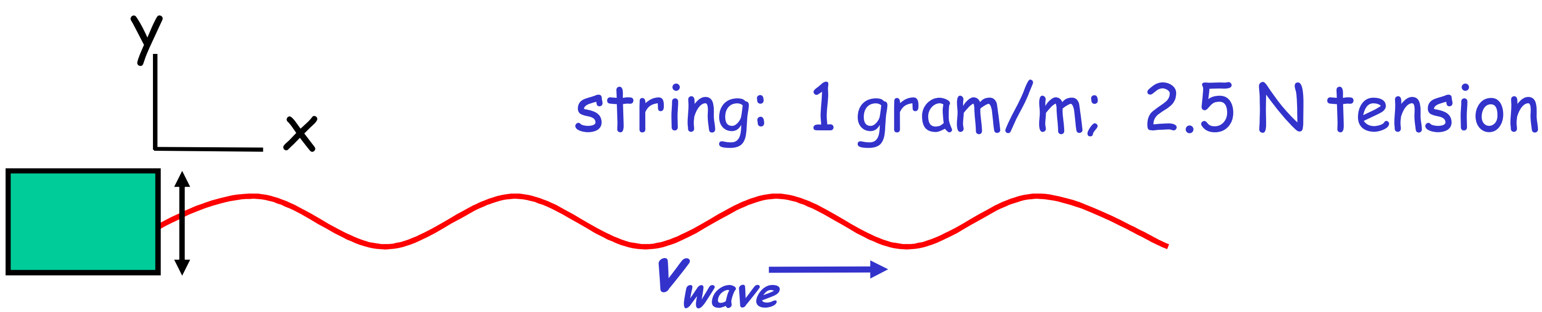
Quiz



Which particle has the largest particle acceleration at this moment?

A) A
B) B
C) C
D) they all have a=0 for a nondispersive wave.

Example



Oscillator:
50 Hz, amplitude 5 mm

Find: $y\ (x,\ t)$

$v_y\ (x,\ t)$ and maximum speed

$a_y\ (x,\ t)$ and maximum acceleration