Electric Fields II

- Electric field produced by point charges
- Continuous charge distributions

Text sections 23.4, 23.5

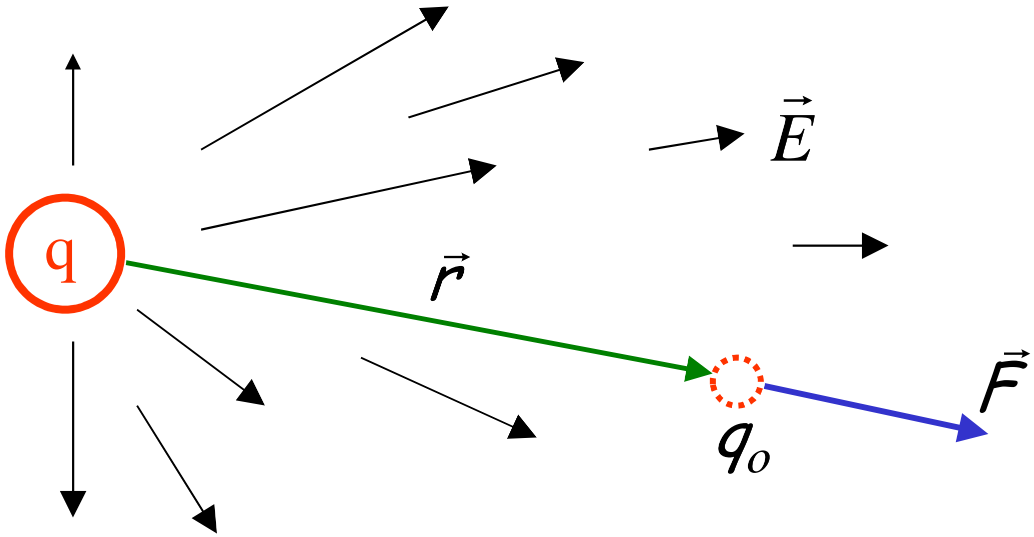
Practice: Chapter 23,
Objective Questions 11, 13, 14
Problems 23, 27, 29, 39, 41, 45

Define:

$$\vec{E} = \frac{\vec{F}}{q_o}$$

$\vec{F}$ is the force exerted <u>on</u> q_o <u>by</u> $\vec{E}$.

Field Produced by a Point Charge:



Coulomb: $\vec{F} = k_e \frac{q q_o}{r^2} \hat{r}$

But also: $\vec{F} = q_o \vec{E}$

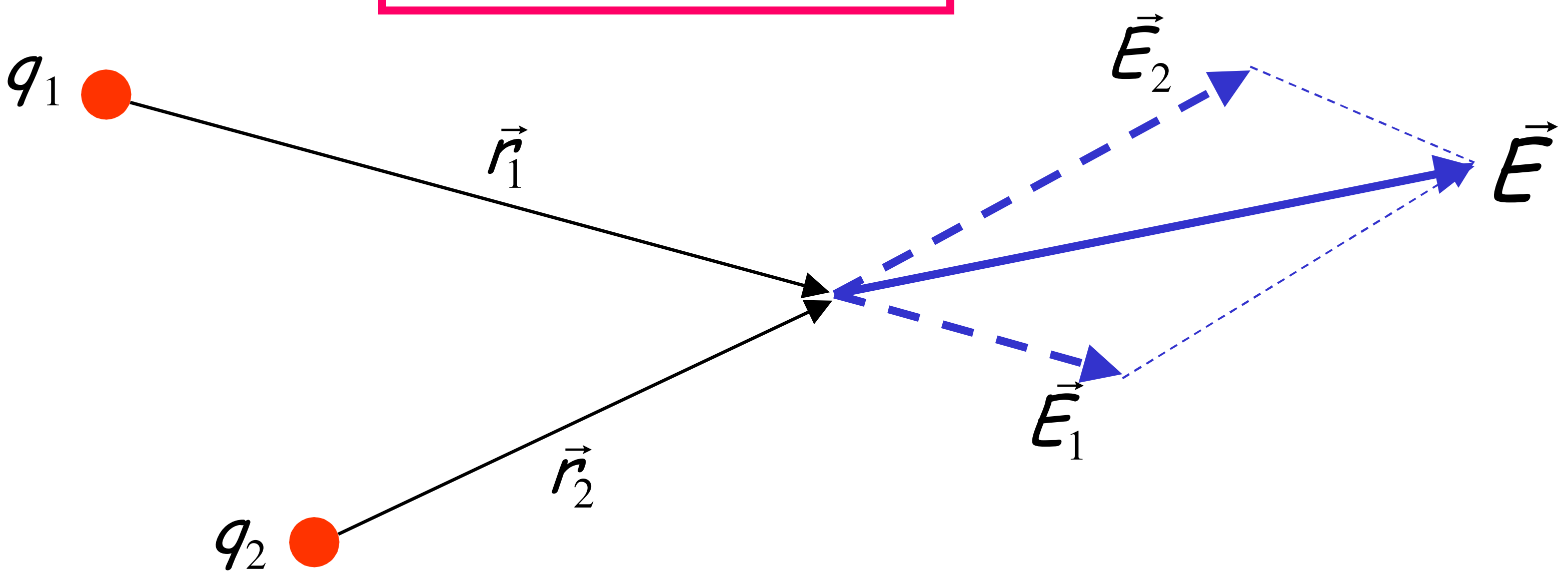
$$\Rightarrow \quad \boxed{\vec{E} = k_e \frac{q}{r^2} \hat{r}}$$

Field produced by a point charge q

Several Charges:

$\vec{E}$ is the (vector) sum of the fields produced by the individual charges:

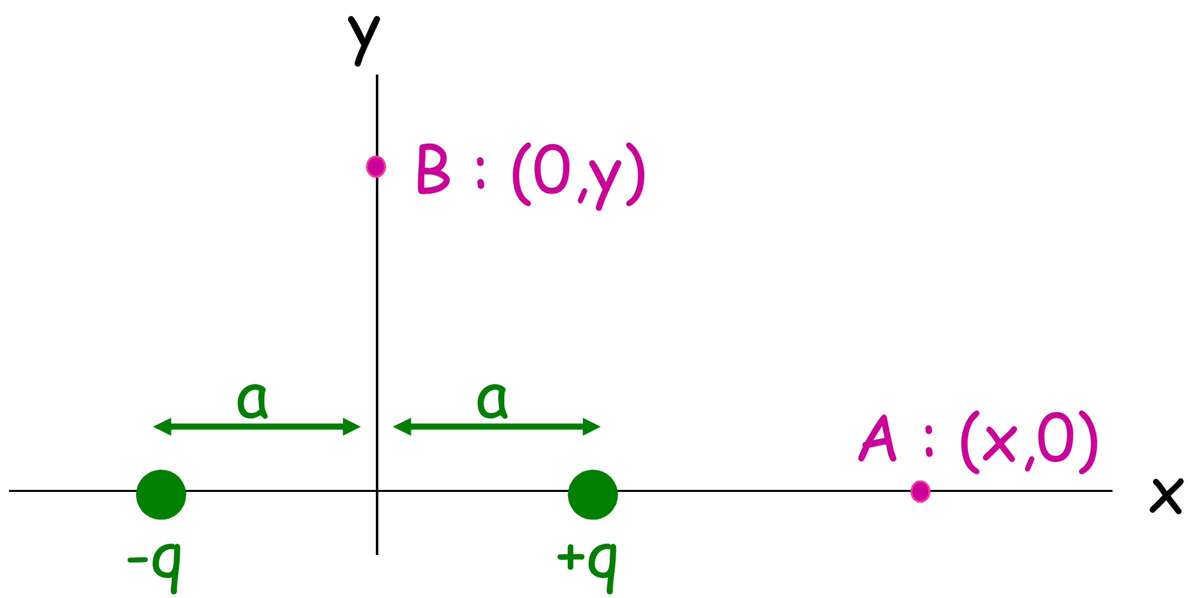
$$\vec{E} = \sum_i k_e \frac{q_i}{r_i^{\ 2}} \hat{r}_i$$



Example: Dipole

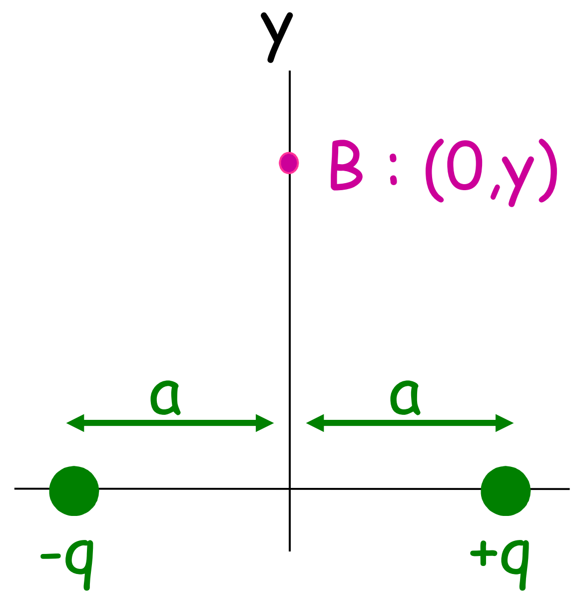
An *electric dipole* is a pair of equal and opposite charges placed a short distance (2a in the diagram) apart.

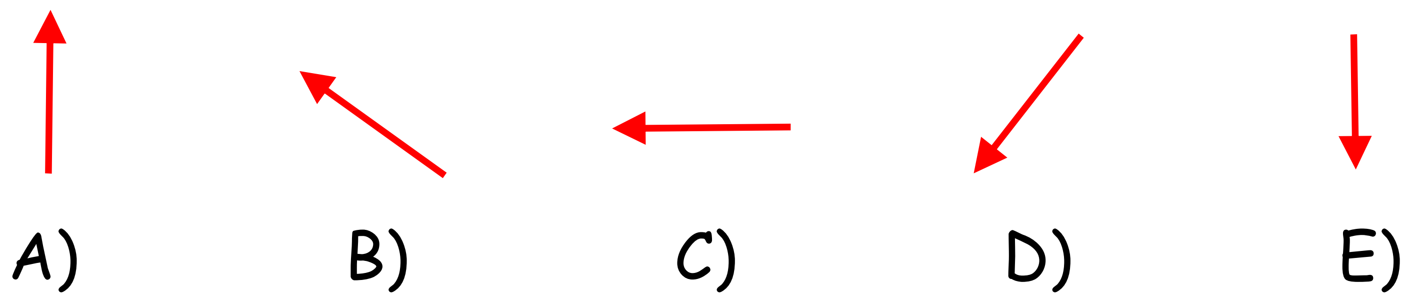
Derive an expression for $\vec{E}$ at point B.

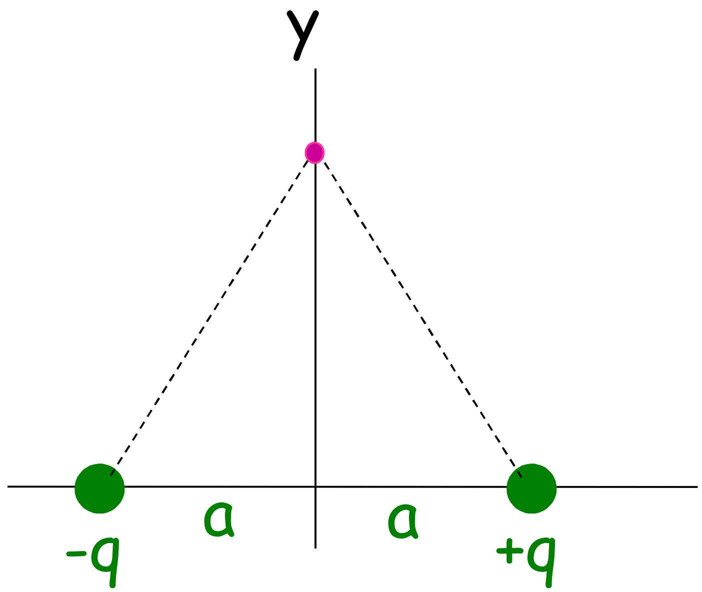


QUIZ

What will be the direction of the field at B?






y
a
a
-q
+q

Answer: $\mathbf{E} =$

where $r = (y^2 + a^2)^{1/2}$

Note... $|\vec{E}| \propto \frac{1}{r^3}$ (dipole)

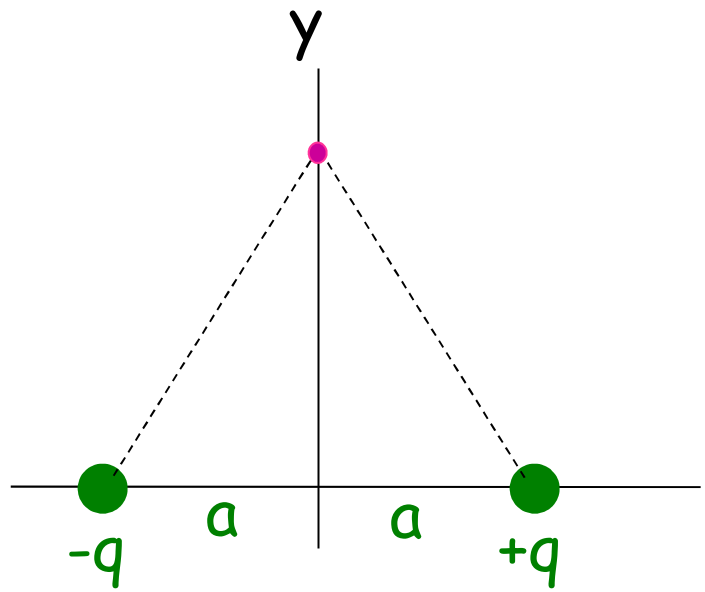
A note on "style": the final result is given in terms of the variables in the original problem (a, q, and y).

For fun: find E at point A, and show that it is *approximately* proportional to x^{-3}, at large distance x.

QUIZ

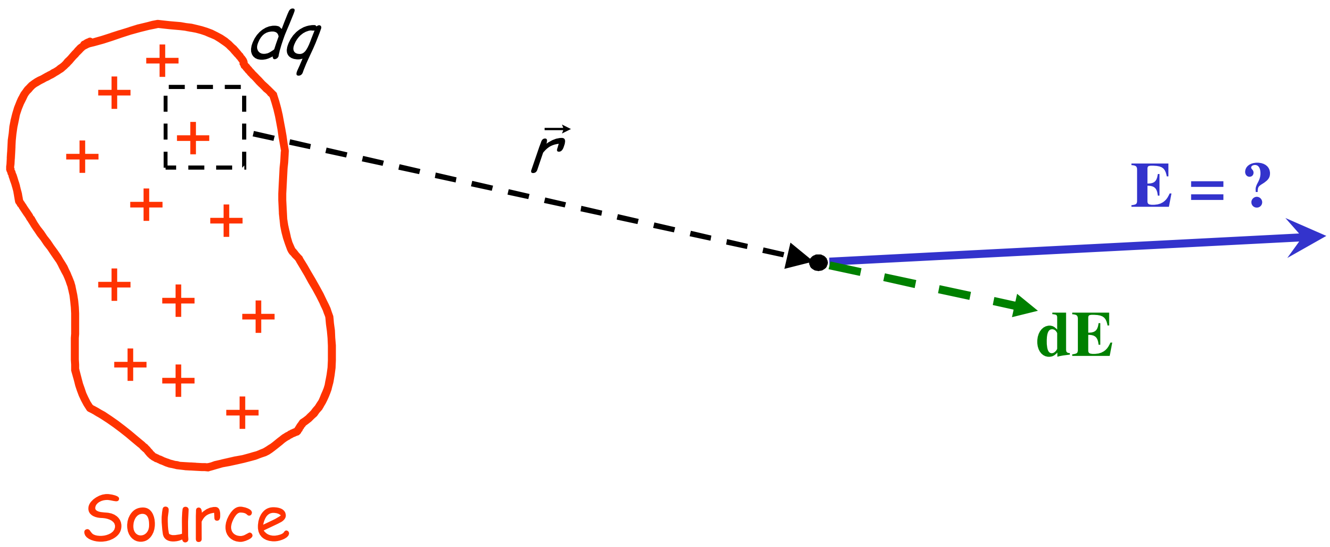
The electric field is also defined as force/charge.

For the field we have just calculated, the "charge" in this denominator should be



A) -q
B) +q
C) zero
D) None of the above

Continuous Charge Distributions



- Cut source into small ("infinitesimal") charges dq
- Each produces

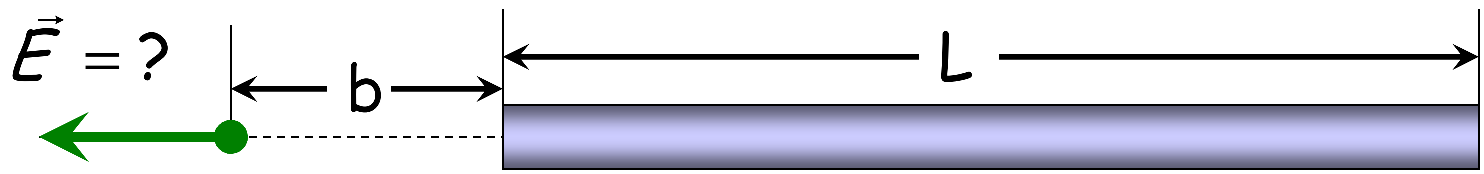
$$\mathbf{dE} = k_e \frac{(dq)}{r^2} \hat{\mathbf{r}}$$

Total,

$$\mathbf{E} = \int_{\text{source}} k_e \frac{dq}{r^2} \hat{\mathbf{r}}$$

Example: Uniformly-Charged Thin Rod

(length L, charge Q)



Charge/Length = "Linear Charge Density" λ
= constant = Q/L

$$\mathbf{E} = \int_{\text{rod}} \mathbf{dE} = \int_{\text{rod}} k_e \frac{dq}{r^2}\,\hat{\mathbf{r}}$$

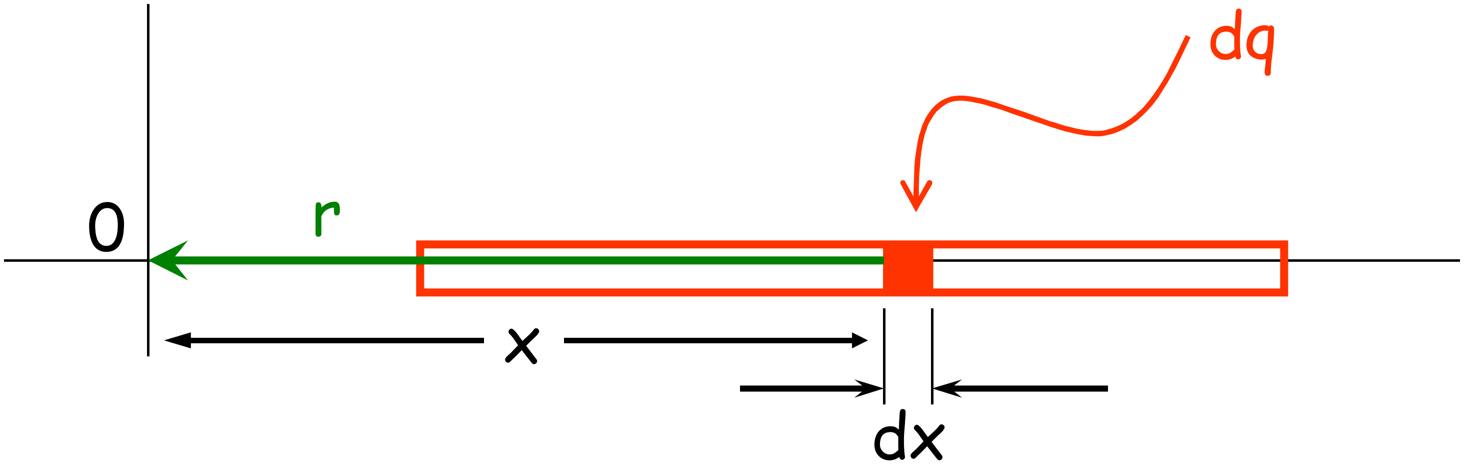
Steps:

- Put a coordinate system on the diagram
- Draw an infintesimal element dq
- Choose an integration variable (*e.g.*, x)
- Write r and any other variables in terms of x
- Write dq in terms of dx
- Put limits on the integral
- Do the integral or look it up in tables.

Result:

$$\mathbf{E} = \frac{k_e Q}{(b+L)b}(-\hat{\mathbf{i}})$$

Solution:



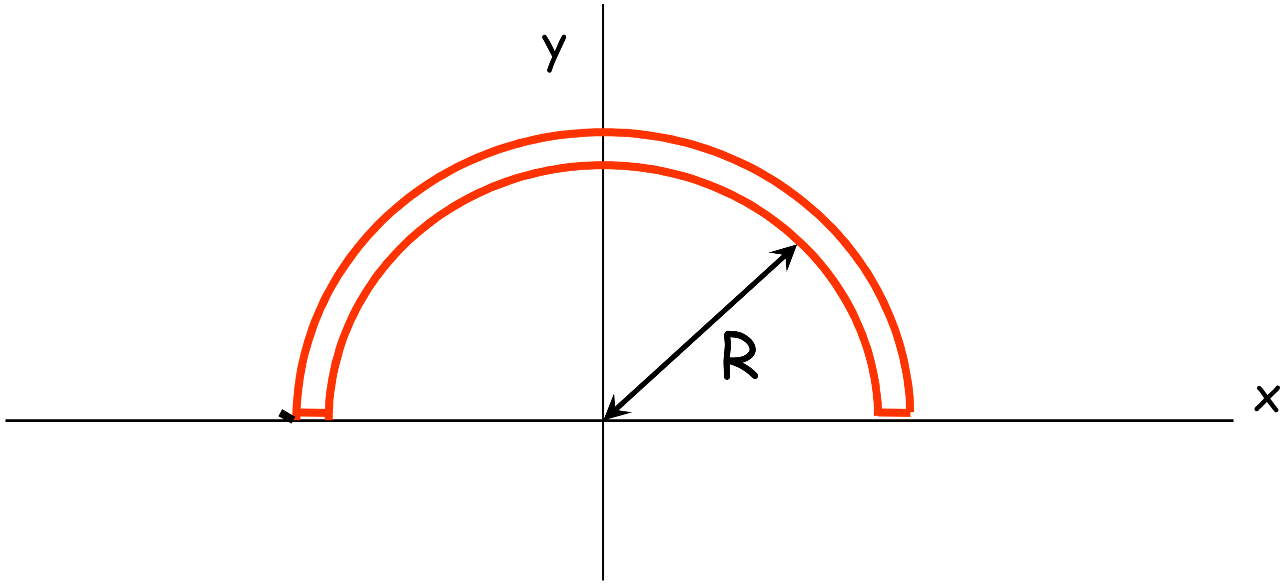
dq: charge on piece between "x" and "x+dx"

In 2D problems, integrate components separately:

$$E_x = \int dE_x = \int k \frac{dq}{r^2} \underbrace{(\cos\theta)}_{\text{x-component of } \hat{\mathbf{r}}}$$

$$E_y = \int dE_y = \ldots\ldots\ldots$$

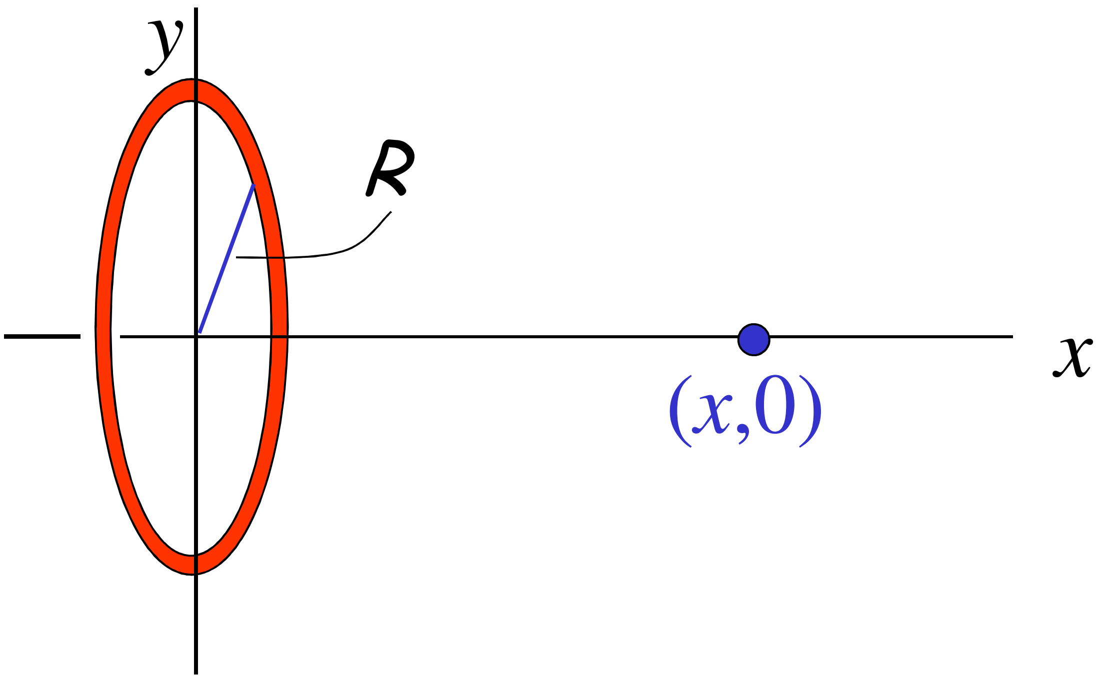
Bonus Example: Uniformly-Charged Semicircle



Charge/unit length, λ , is uniform

Find: $\vec{E}$ at origin

Exercise: Uniformly-Charged Ring



Total charge Q, uniform charge/unit length, radius R

Find: **E** at any point (x, 0) on the axis of the ring

Summary

- Field of several point charges q_i:

$$\vec{E} = \sum_i k_e \frac{q_i}{r_i^2}\hat{r}_i$$

- Field of continuous charge distribution:

$$\vec{E} = \int k_e \frac{dq}{r^2}\hat{r}$$