Inductance (II)

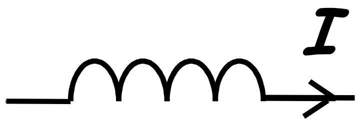
Text sections 32.2

- Inductors in circuits
- RL circuits

Practice:
Chapter 32,
Objective Questions 4, 7
Conceptual Questions 3, 4, 5
Problems 19, 21, 22, 31

Self-Inductance

$$\varepsilon_L = -L\frac{dI}{dt}$$



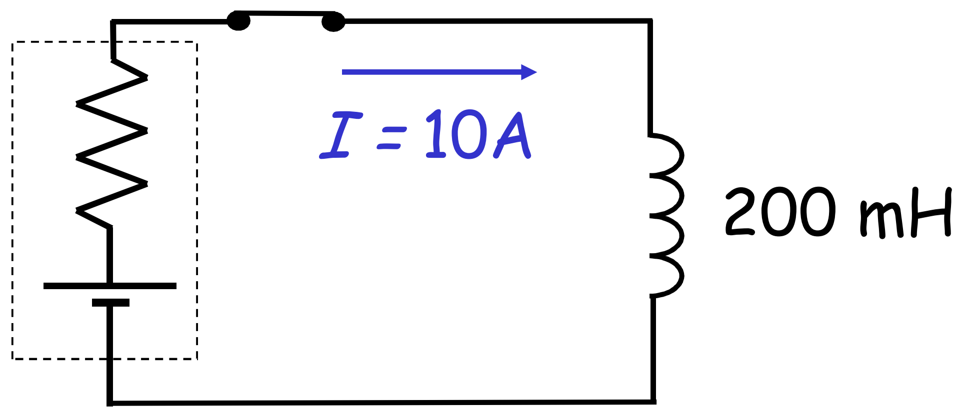
Potential energy stored in an inductor:

$$U_L = \tfrac{1}{2}LI^2$$

Energy density in a magnetic field:

$$u_B \equiv \frac{U}{\text{Volume}} = \frac{B^2}{2\mu_o}$$

Example:

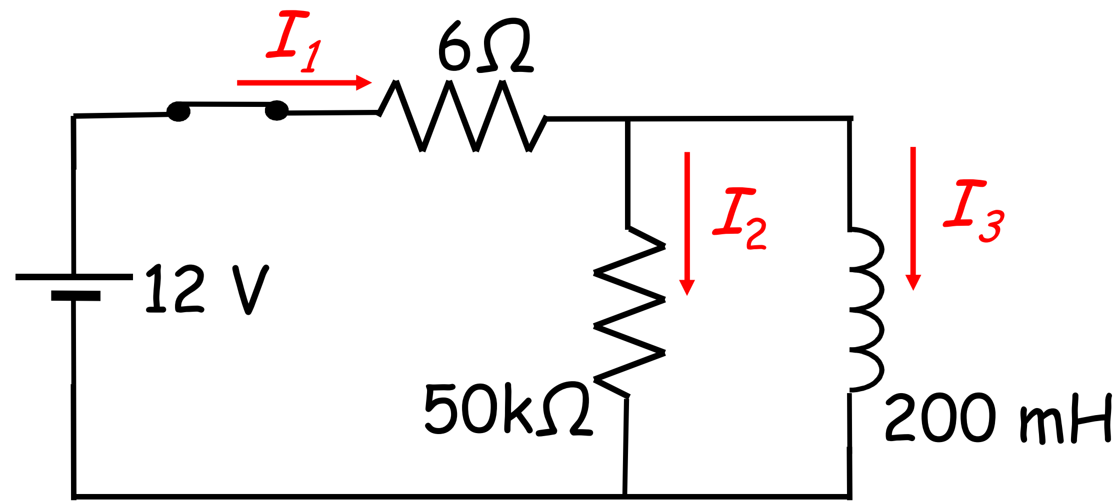


Initial: $I_i = 10A$, $U_i = \frac{1}{2}(0.2\ H)(10\ A)^2$
$= 10J$

Final: $I_f = 0$, $U_f = 0J$ (after switch is opened)

Where does the energy go?

Example:



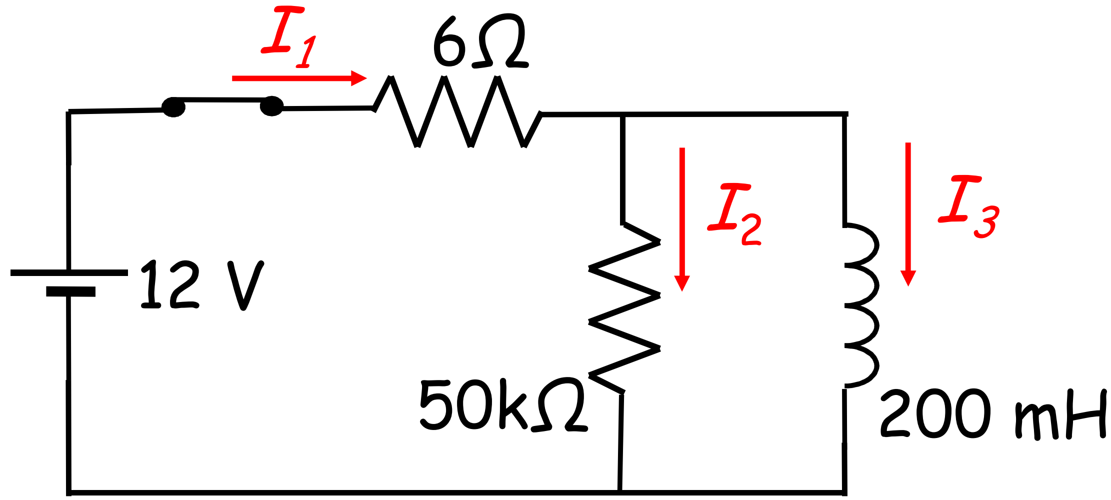
a) *The switch has been closed for a long time. Find the current through each component, and the voltage across each component.*

b) *The switch is now opened. Find the currents and voltages just afterwards.*

c) *How long does it take for the inductor current to drop to zero (after the switch is opened)?*

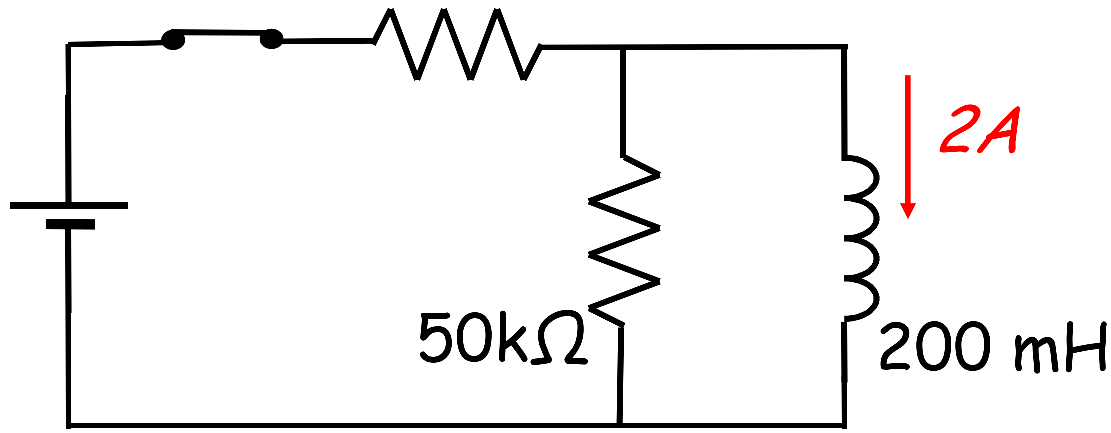
Quiz

The switch has been closed for a long time. Which currents are zero?



A) I_1
B) I_2
C) I_3
D) All of them
E) None of them

Quiz

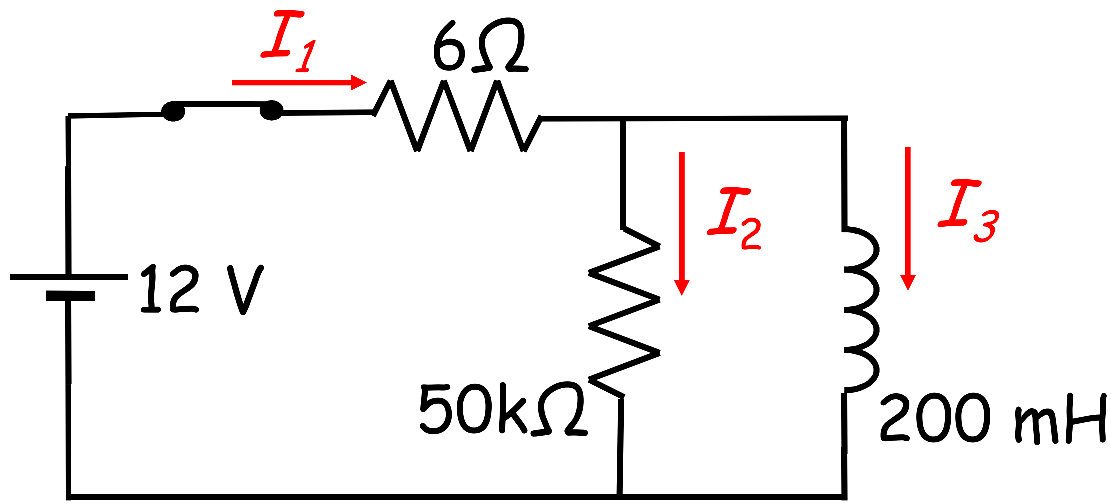


There is a 2A current through the inductor, and the switch is suddenly opened. What is the voltage across the inductor immediately after the switch is opened?

A) 12 V
B) 100kV
C) zero
D) 10V

Quiz

*Suppose the switch had been **open** for a long time. What is the current I_2 just after it is closed?*



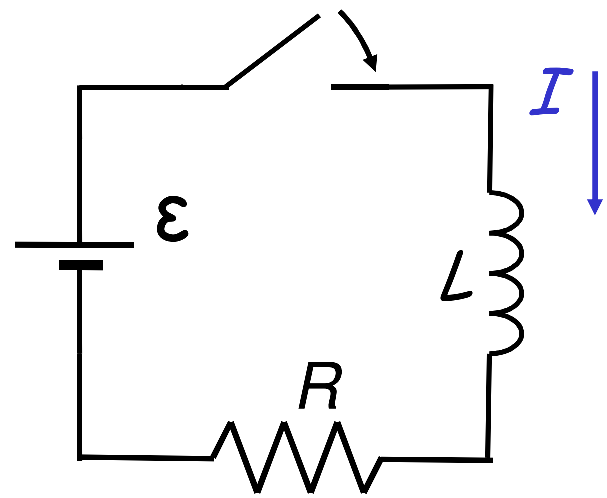
A) 2.0 A

B) 0.24 mA

C) zero

RL circuits: time dependence

The switch is closed at $t = 0$;
Find $I(t)$.



Kirchhoff's loop rule:

$$\varepsilon - L\frac{dI}{dt} - IR = 0$$

$$\Rightarrow \frac{dI}{dt} = \frac{\varepsilon - IR}{L} = \frac{R}{L}\left(\frac{\varepsilon}{R} - I\right)$$

separate the variables: $$\frac{dI}{(\varepsilon/R) - I} = \frac{R}{L}dt$$

Integrate from time zero to time t, or zero current to current $I(t)$:

$$\int_0^{I(t)} \frac{dI}{(\varepsilon / R) - I} = \int_0^t \frac{R}{L}\,dt$$

But, $\displaystyle\int \frac{dx}{a - x} = -\ln(a - x)$

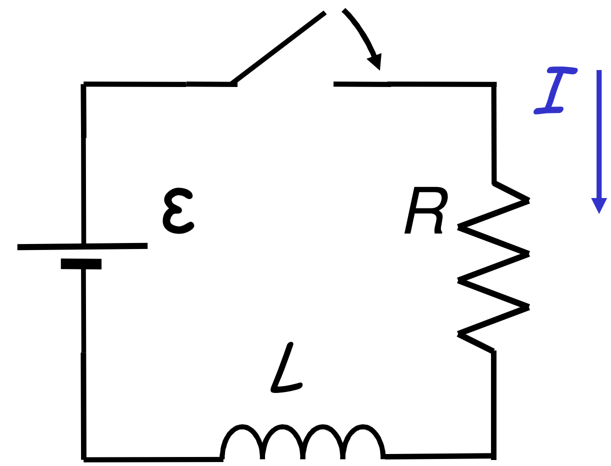
So $$-\ln\left(\frac{\varepsilon}{R} - I(t)\right) + \ln\left(\frac{\varepsilon}{R} - 0\right) = \frac{R}{L}t$$

and with a little work, $$I = \frac{\varepsilon}{R}\left(1 - e^{-(R/L)t}\right)$$

Time Constant: $$\tau = \frac{L}{R}$$

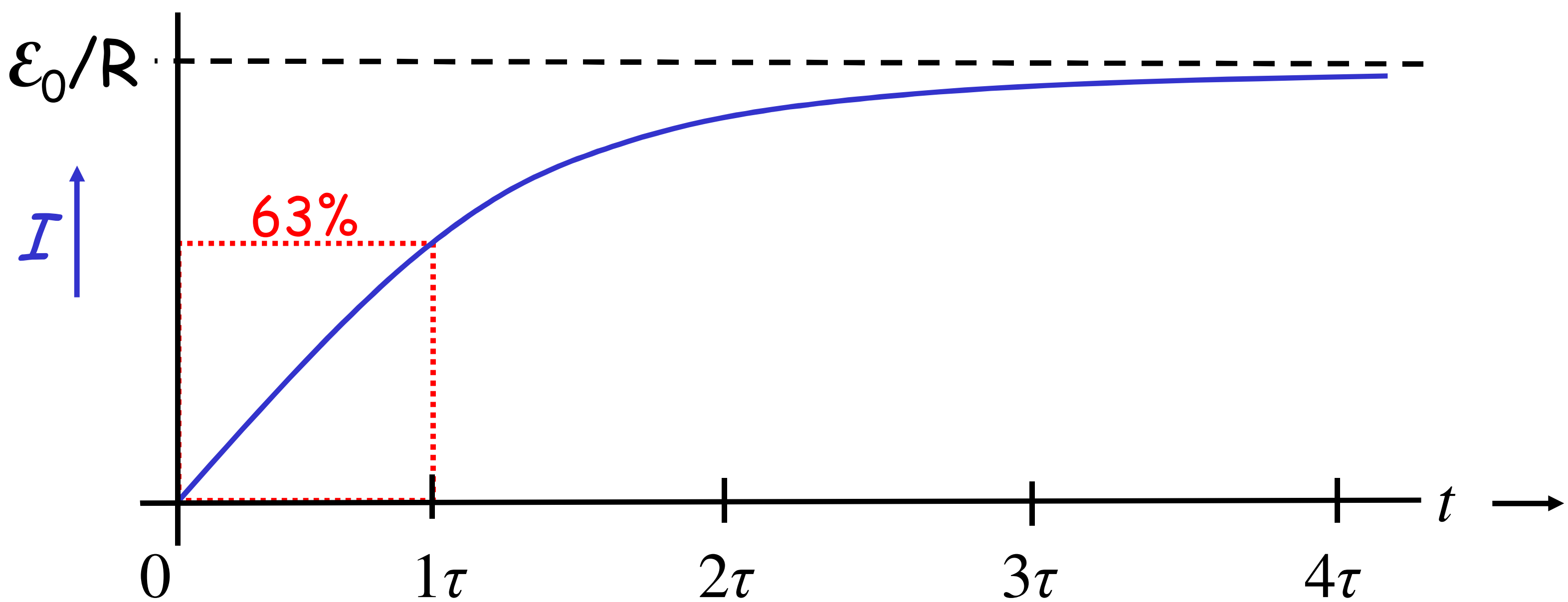
note H/Ω = seconds (exercise!)

Time Constant: $\tau = \frac{L}{R}$



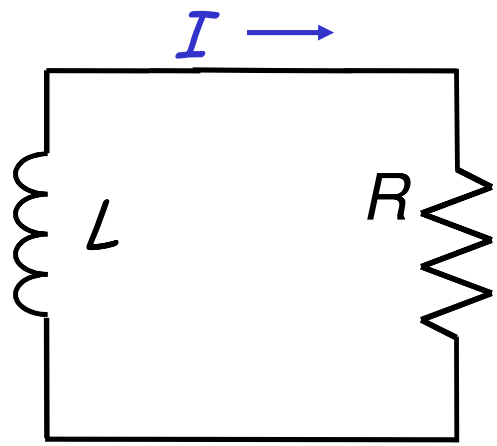
Close switch at $t = 0$:

$$I(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right)$$

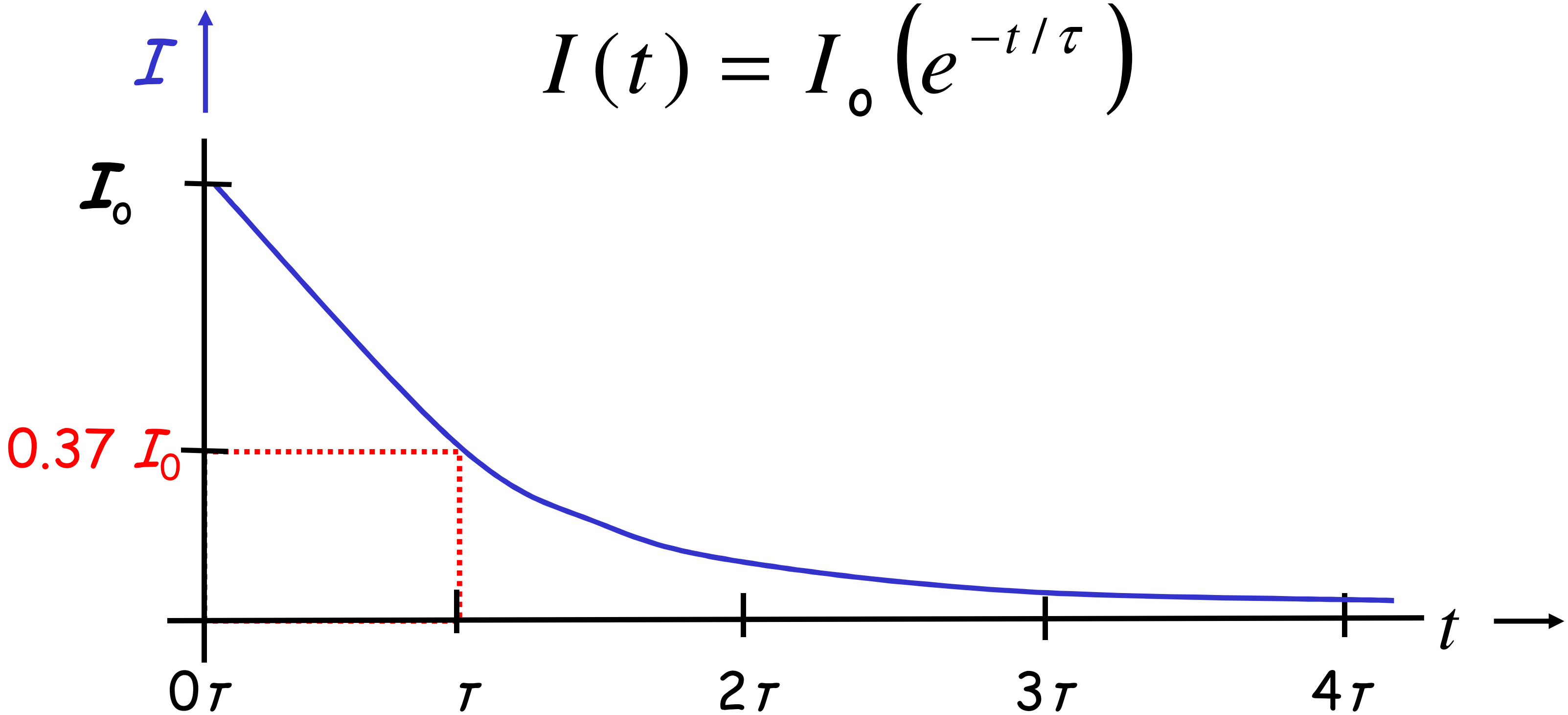


Current decreasing:

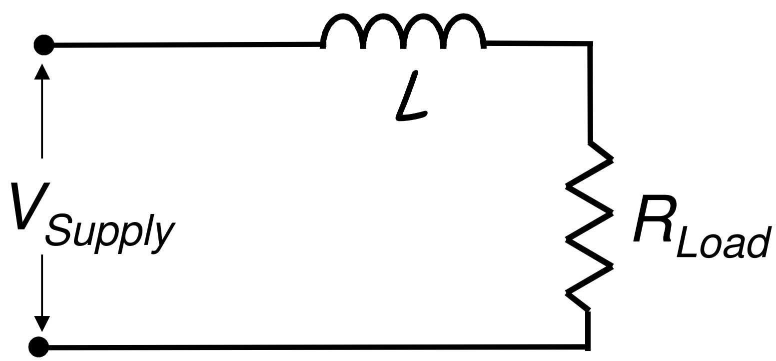
Time Constant: $\tau = \frac{L}{R}$



$$I(t) = I_o\left(e^{-t/\tau}\right)$$



Inductor application: *Filtering voltage spikes*



R_{Load} = 10 Ω

L = 0.5 H

V_{Supply} = 10 volts

What is the steady-state voltage across R_{Load}?

What is the maximum voltage across R_{Load} if the supply voltage is increased to 100 volts for 1 millisecond?