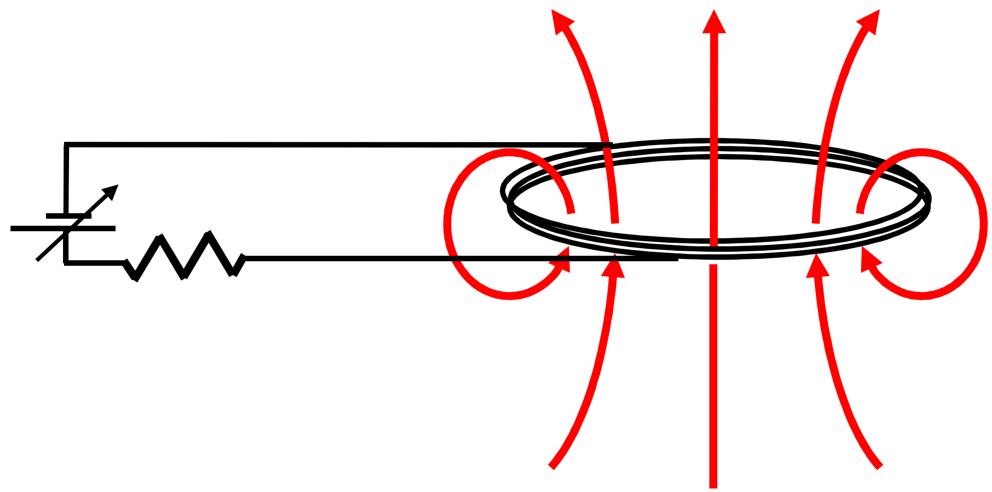
Inductance

Text sections 32.1 - 32.3

Practice: Chapter 32,
Objective Questions 2, 6
Conceptual Questions 2, 10
Problems 9, 13, 14, 39



A variable power supply is connected to a coil. The current in the coil creates a magnetic field. What happens when the power supply dial is turned down, reducing the supply current?

What happens when the current is turned up?

“Self-Inductance”

- *A circuit carrying current* $\boldsymbol{I}$ *generates its own magnetic field* $\boldsymbol{B}$*; so there will be a flux* Φ_B *through the circuit.*

  - *if* $\boldsymbol{I}$ *changes, an emf* $\mathcal{E}$ *is induced in the same circuit.*

    (“self-inductance”)

  - *Lenz’s Law:* $\mathcal{E}$ *tends to oppose the* change *in* $\boldsymbol{I}$.

(self-inductance gives “inertia” to currents)

From the Biot-Savart Law,

$$\vec{B} \propto I$$

$$\Rightarrow \Phi_B \propto I$$

$$\Rightarrow \mathcal{E} \propto -\frac{dI}{dt}$$

The **self-inductance** *L is the proportionality constant:*

$$\boxed{\varepsilon = -L\frac{dI}{dt}}$$

(definition of L)

unit: 1 henry (H) = 1 volt/(A/s) = 1 (V•s)/A

An equivalent definition of L uses the flux produced per ampere of current. For a coil with N identical turns, with flux Φ through each turn produced by current I, we can write

$$\boxed{L = \frac{N\Phi}{I}}$$

(so 1 henry
= 1 weber-turn/ampere)

Then from Faraday's Law,

$$\varepsilon = -N\frac{d\Phi}{dt} = -L\frac{dI}{dt}$$

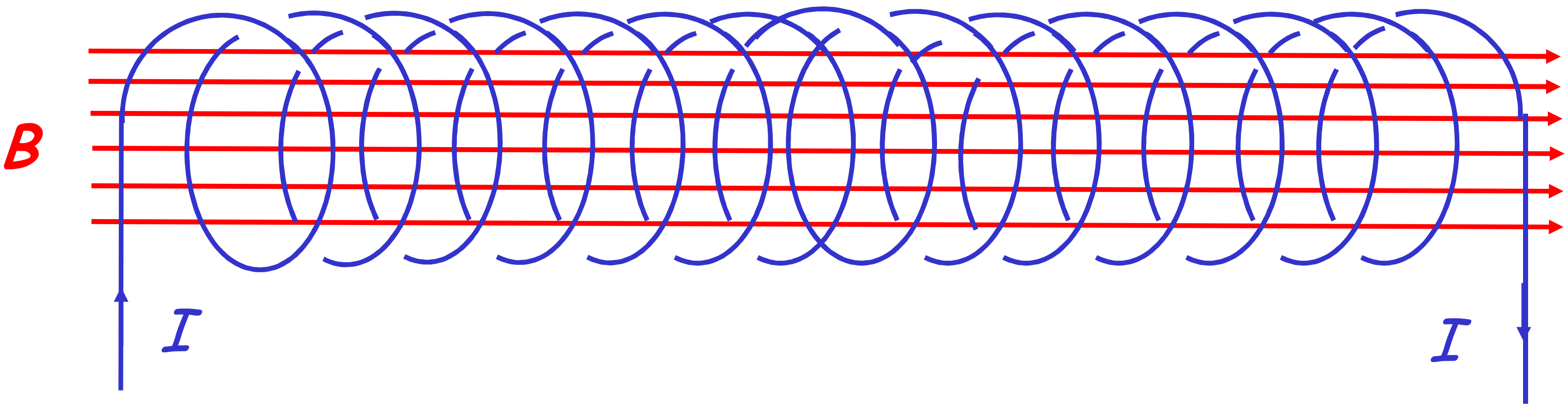
to match our previous definition of L.

Difficult Quiz

A coil with 100 turns has inductance L_1.
*A second coil, of the **same size and shape**,*
but with 200 turns, would have inductance

A) $2\ L_1$ B) $\frac{1}{2}\ L_1$ C) $4\ L_1$ D) $\frac{1}{4}\ L_1$ E) L_1

Example: Long solenoid



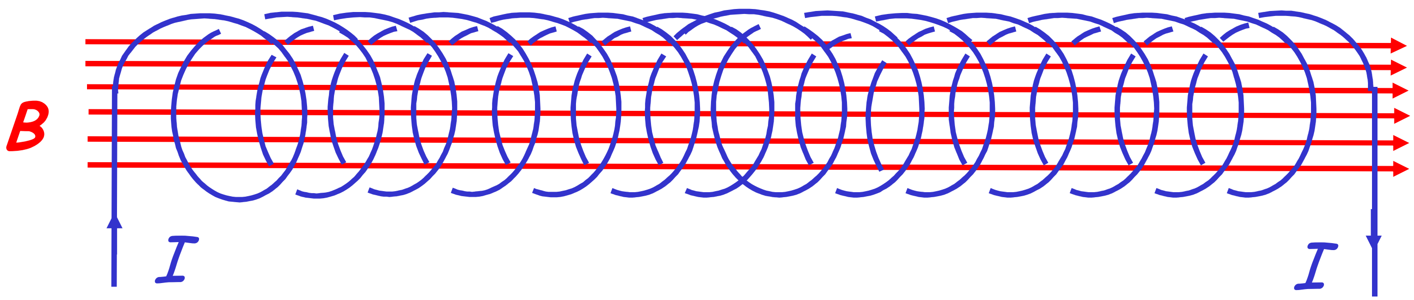
<u>Given</u>:

N = 2000 turns

r = 1 cm

ℓ = 20cm

Calculate L ("self - inductance")



Inside the coil, ***B*** *is approximately uniform:* $B = \mu_0 nI$

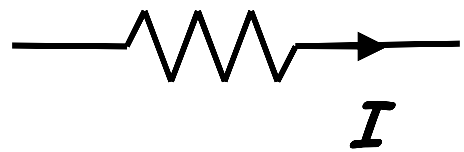
and the flux through each coil is $\Phi_B = (\pi r^2)\,\mu_0 nI$

So, if we can neglect the regions near the ends of the coil, the self-inductance of the long solenoid is

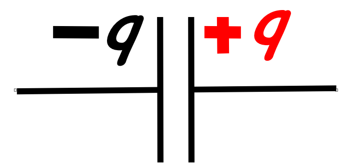
Kirchhoff's Loop Rule terms (again);

energy change/charge in going from left to right:

- resistor, $\Delta V = -IR$



- capacitor, $\Delta V = q/C$



- inductor, $\Delta V = -L\dfrac{dI}{dt}$

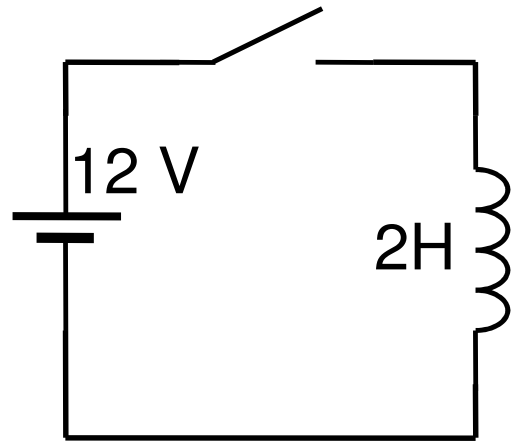
Quiz

A steady current of 5A flows through a 0.5 H inductor. The voltage across the inductor is

A) 0 B) 10V C) 2.5 V D) 0.1 V E) other

Quiz

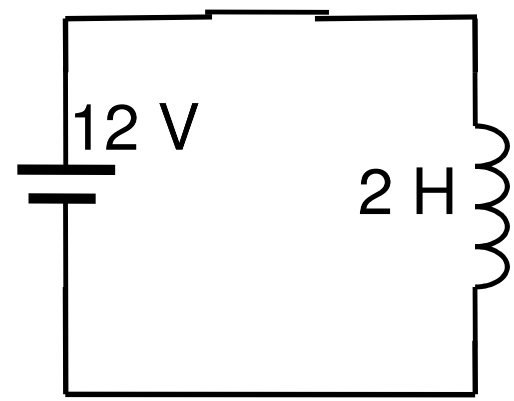
What is the current 2 milliseconds after the switch is closed?

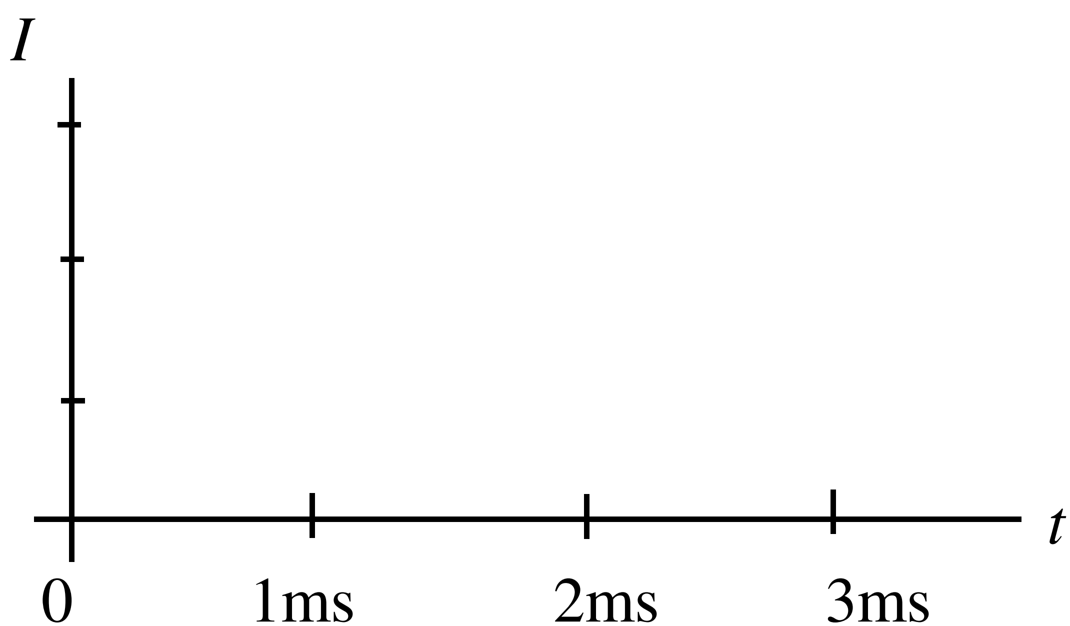


A) infinite
B) zero
C) 6A
D) 12 mA
E) 24A

Close switch
at $t = 0$

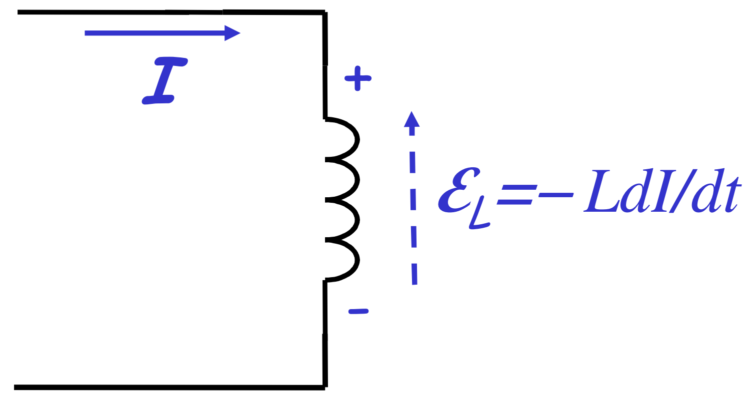
plot I vs t





Energy

How much work is done to increase the current from 0 to I_{final}?



Power supplied by inductor:

$$P_L = \mathcal{E}_L \times I = -IL\ dI/dt$$

Power supplied by external circuit:

$$P_{ext} = -P_L = +IL\ dI/dt$$

The work done by the battery in time dt is

$$dW = P_{\text{ext}} \times dt = LI\frac{dI}{dt} \cdot dt = LI \cdot dI$$

The total work done by the battery is

$$W = \int dW = L\int_0^{I_f} I\,dI = \tfrac{1}{2}LI_{\text{f}}^2$$

This work is stored as potential energy in the inductor:

$$\boxed{U_L = \tfrac{1}{2}LI^2}$$

(stored in ***B***–field of inductor)

Energy density of the field B

We can treat the energy as stored in the magnetic field set up by the current. Taking a long solenoid as an example (N turns, length ℓ, area A):

Inside the coil, $B = \mu_o \dfrac{N}{\ell} I \Rightarrow I = \dfrac{B\ell}{\mu_o N}$

The inductance is $L = \dfrac{N\Phi}{I} = \dfrac{NBA}{I}$

And the potential energy is

$$U = \tfrac{1}{2} LI^2 = \tfrac{1}{2} NBA \cdot I = \frac{B^2 \cdot A\ell}{2\mu_o}$$

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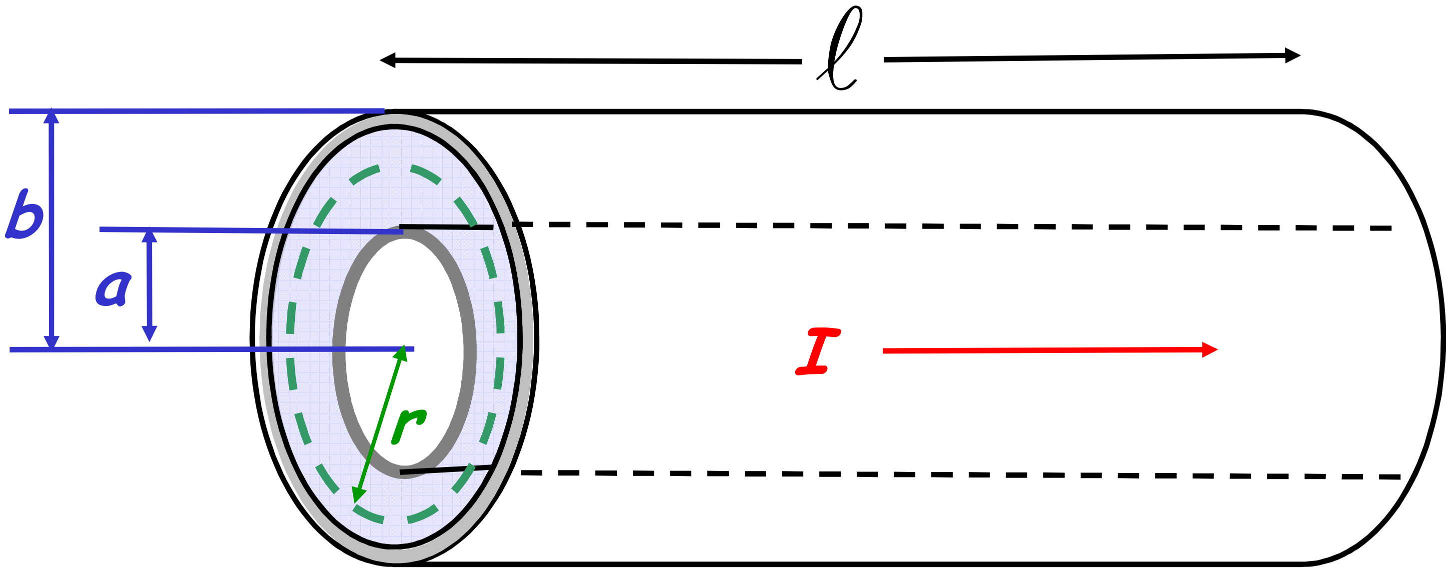
The field is uniform inside the coil, zero outside (approximately). So the field occupies a volume equal to ($A\ell$) . If the energy is stored in the field, the energy density (energy per unit volume) is

$$\boxed{u_B \equiv \frac{U}{\text{Volume}} = \frac{B^2}{2\mu_o}}$$

This is true in general for any magnetic field. For comparison, recall that the energy stored in an electric field is

$$u_E = \frac{\varepsilon_0 E^2}{2}$$

Example: Inductance of a coaxial cable



In the gap (a < r < b): $B = \dfrac{\mu_o I}{2\pi r}$

$\vdots$

Show that $L = \dfrac{\mu_o \ell}{2\pi} \ln\left(b/a\right)$