# Biot-Savart Law, Ampère's Law 

$$
\text { Text 30.2, } 30.3
$$

-Fields and forces for straight wires

- Ampère's Law

Practice: Chapter 30, Objective Questions 3, 106
Conceptual Questions 3, 4
Problems 25, 31, 36

## Parallel Long Wires:



Find: Force on segment $a b$ (magnitude \& direction)

$$
\text { (Take } \left.I_{1}=20 \mathrm{~A}, I_{2}=30 \mathrm{~A}, \mathrm{~d}=0.01 \mathrm{~m}, \mathrm{~L}=1.5 \mathrm{~m}\right)
$$

## Quiz

The two long wires shown cross at right angles. How will

A) slide to the right
B) slide to the left
C) rotate clockwise
D) rotate counterclockwise
E) none of the above

## Ampère's Law (an integral theorem)

For any closed path:

$$
\oint_{\text {path }} \mathbf{B} \cdot \mathbf{d s}=\mu_{\mathrm{o}} I_{\text {encircled }}
$$

"Circulation" of $\mathbf{B}=\mu_{\mathrm{o}} \times$ current encircled

The current calculated this way flows through the surface bordered by the path.

positive current is out of the page
through the shaded surface

## $\oint_{\text {path }} \mathbf{B} \cdot \mathbf{d} \mathbf{S}=\mu_{\mathrm{o}} I_{\mathrm{encircled}}$

The "circulation" integral means, roughly speaking,

## (average component of B parallel to the path) times (length of path).

The "path" must be a closed loop, but has no other restrictions. If we choose a field line as the "closed path", we can think of the circulation as
(length of the field line)
times (average strength of $B$ ).

## Quiz: Long Straight Wire

On which of the curves
$C_{1}, C_{2}$, and $C_{3}$ is $\oint_{\text {pat }} \mathbf{B} \cdot \mathbf{d s}$
i) smallest
ii) largest?


## Example: Long Straight Wire

Show that the field produced by a long straight wire is:

$$
B=\frac{\mu_{0} I}{2 \pi r}
$$

Steps:
a) Symmetry: Assume field lines are circles,
$|\vec{B}|$ depends only on $\mid r$.
b) Calculate $\int \bar{B} \cdot \overline{d s}$ along a circle of radius $r$
c) Calculate I through the circle
d) $\int \vec{B} \cdot \overrightarrow{d s}=\mu_{0} I$, solve for $|\vec{B}|$


## Exercise:

Wire, radius $a$, uniform current density $J=\frac{I_{\text {Total }}}{\pi a^{2}}$
Find $|\vec{B}|$ for i) $r<a$
ii) $r>a$


## Quiz

The "current encircled" by the green dashed circle is (for $r<a$ ):
A) independent of $r$
B) proportional tor
C) proportional to $r^{2}$
D) proportional to $1 / r$
E) zero

What is the answer when $r>a$ ?

## Quick Quizzes

What is the magnetic field inside and outside

1) a hollow copper pipe carrying current $I$ ?
2) a coaxial cable carrying equal currents (in opposite directions) on the inner and outer conductors?


## Coaxial Cable:


i) in the gap between the inner and outer conductors
ii) outside the outer conductor

## Quiz:



A beam of fast protons is also a current. Electric forces cause the beam to widen. The magnetic forces tend to
A) cause the beam to spread even more
B) cause the beam to spread less
C) have no effect on the spreading

What about a beam of fast electrons?

A puzzle: Force between two proton beams charge/unit length $\lambda$, speed $v$; current $I=\lambda v$

lines of positive charge: electrostatic repulsion parallel currents: magnetic force

