Magnetic Fields (III)

Text sections 29.2, 29.3, 29.6

- Motion of charged particles in magnetic fields
- Examples: cyclotron, mass spectrometer, Hall effect

For practice:
Chapter 29, problems 13, 21, 25, 27, 30, 51

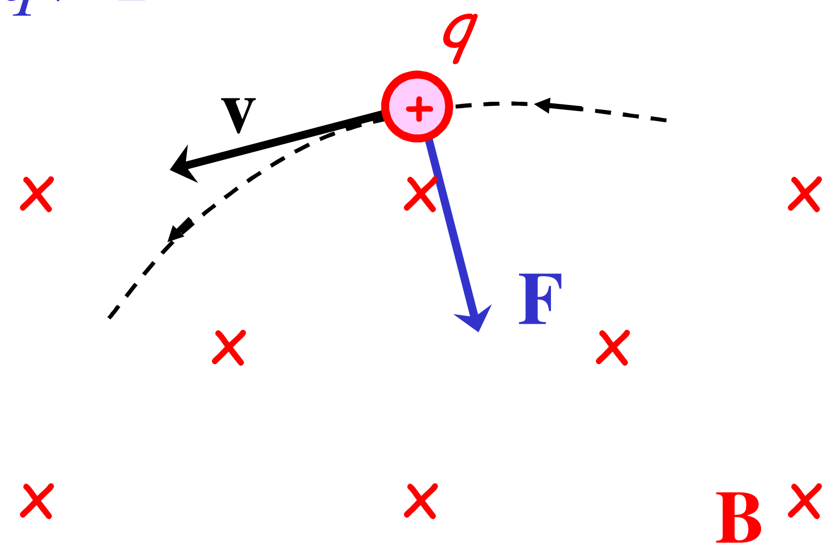
Charged Particles in Electric and Magnetic Fields

$$\vec{F} = q\vec{v} \times \vec{B}$$ in a magnetic field

or in general,

$$\vec{F} = q\vec{v} \times \vec{B} + q\vec{E}$$ ("Lorentz Force")

Magnetic Fields: $\mathbf{F} = q\mathbf{v}\times\mathbf{B}$



The force is perpendicular to the motion, so:

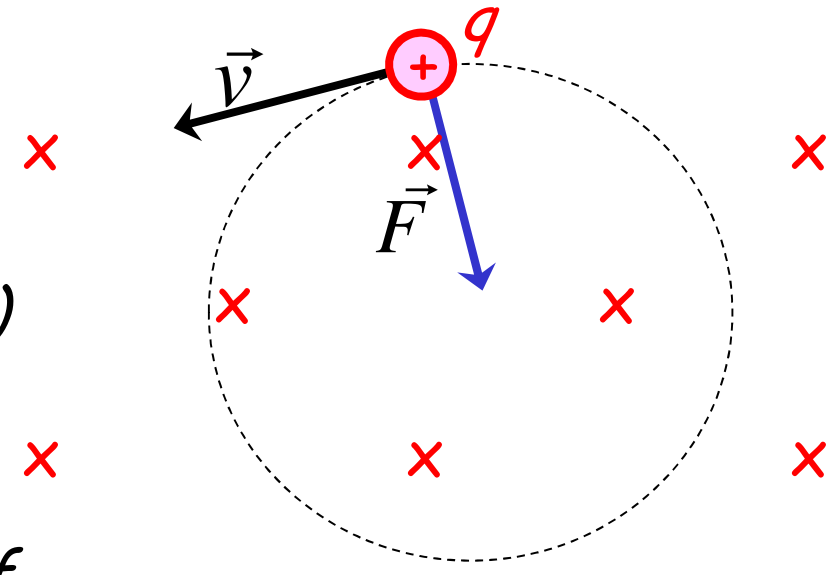
- *no work is done*
- *kinetic energy is constant*
- *speed is constant*

*Only the **direction** of the motion changes due to the magnetic force.*

1) Uniform **B**, **v** perpendicular to **B**

Radial acceleration $(\perp \mathbf{v})$ *is*

$a_r = F/m = qvB/m$ *(constant)*

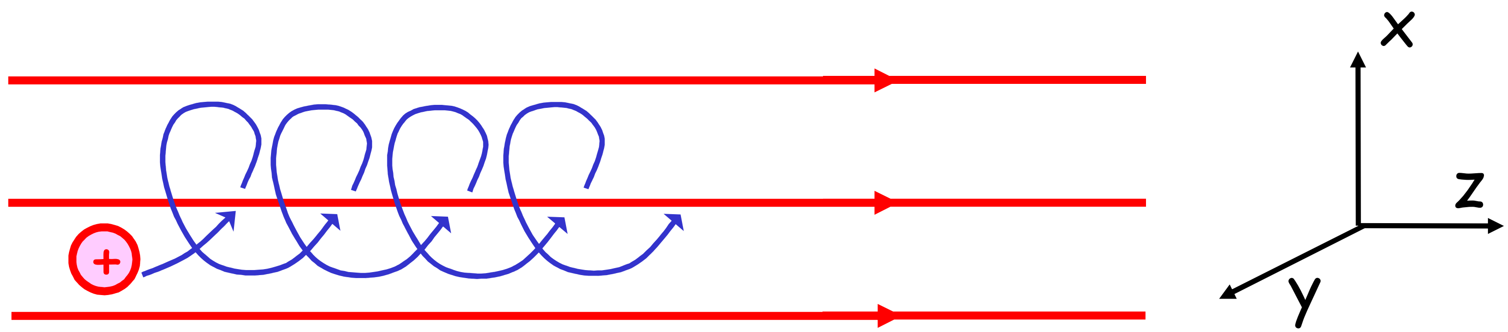


But $a_r = v^2/r$, *so the radius of curvature* r *is constant.*

$\Rightarrow$ Motion is a **circle**.

Uniform B

2) $\vec{v}$ not $\perp \vec{B}$:



-path is a circular helix along a field line

-the component $v_{\parallel}$ (parallel to $\mathbf{B}$) is constant

On a large scale, the particles *follow the field lines.*

Velocity ⊥ Field

Start with Newton's 2nd Law:

$$\vec{F} = m\vec{a}$$

Then replace: $\left|\vec{F}\right| = qvB, \;\; a = \dfrac{v^2}{r}$ *(for motion ⊥* **B***)*

→

$$q\,v\,B = m\frac{v^2}{r}$$

(Newton's 2nd Law for a particle in a uniform magnetic field ⊥ **v***.)*

From this, calculate radius, speed, etc...

Note <u>angular velocity</u>, $\omega = v / r$

$$q\, v\, B = m \frac{v^2}{r}$$

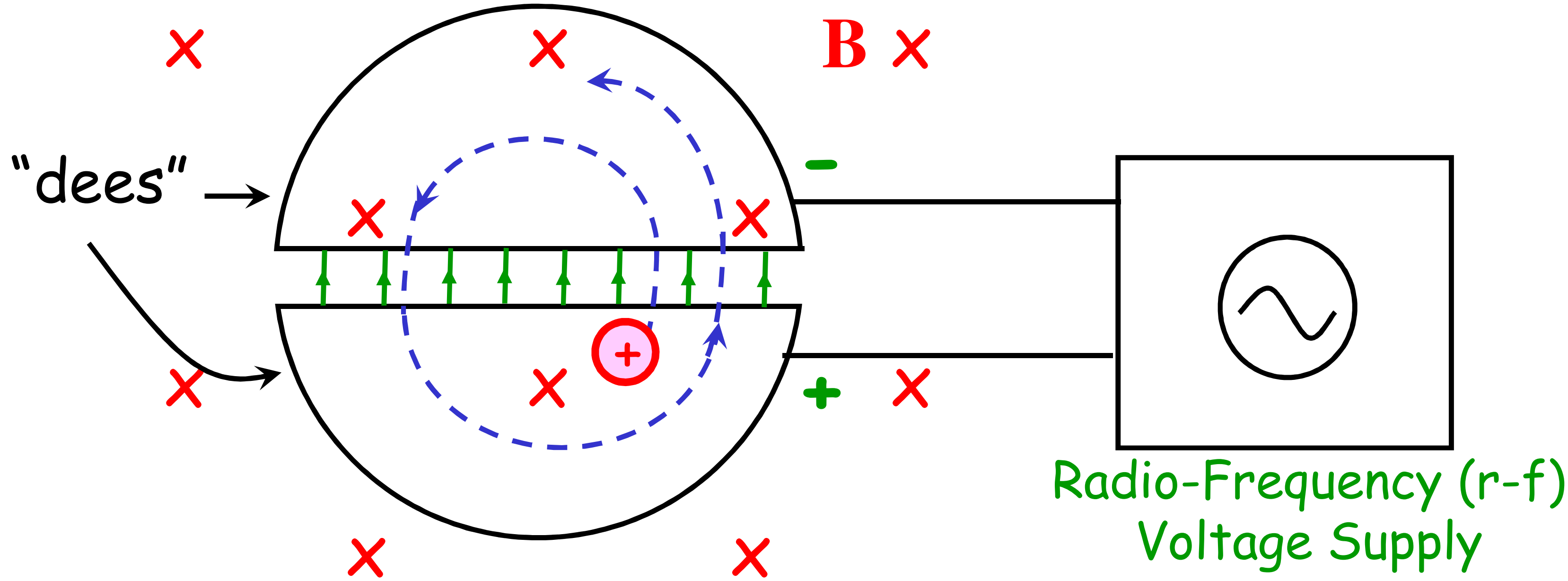
$$\Rightarrow \boxed{\omega = \frac{qB}{m}}$$ [radians/s]

frequency, $f = \dfrac{\omega}{2\pi} = \dfrac{1}{\text{period}}$ etc...

$\dfrac{\omega}{2\pi}$: "cyclotron frequency"

<u>is independent of speed</u>.

Example: The Cyclotron



*The **electric field** across the gap is* reversed *each time the proton arrives, so that its speed continually increases. Because the time for each half-circle is the same for* **any** *proton speed, the r-f supply can just be set to a constant frequency.*

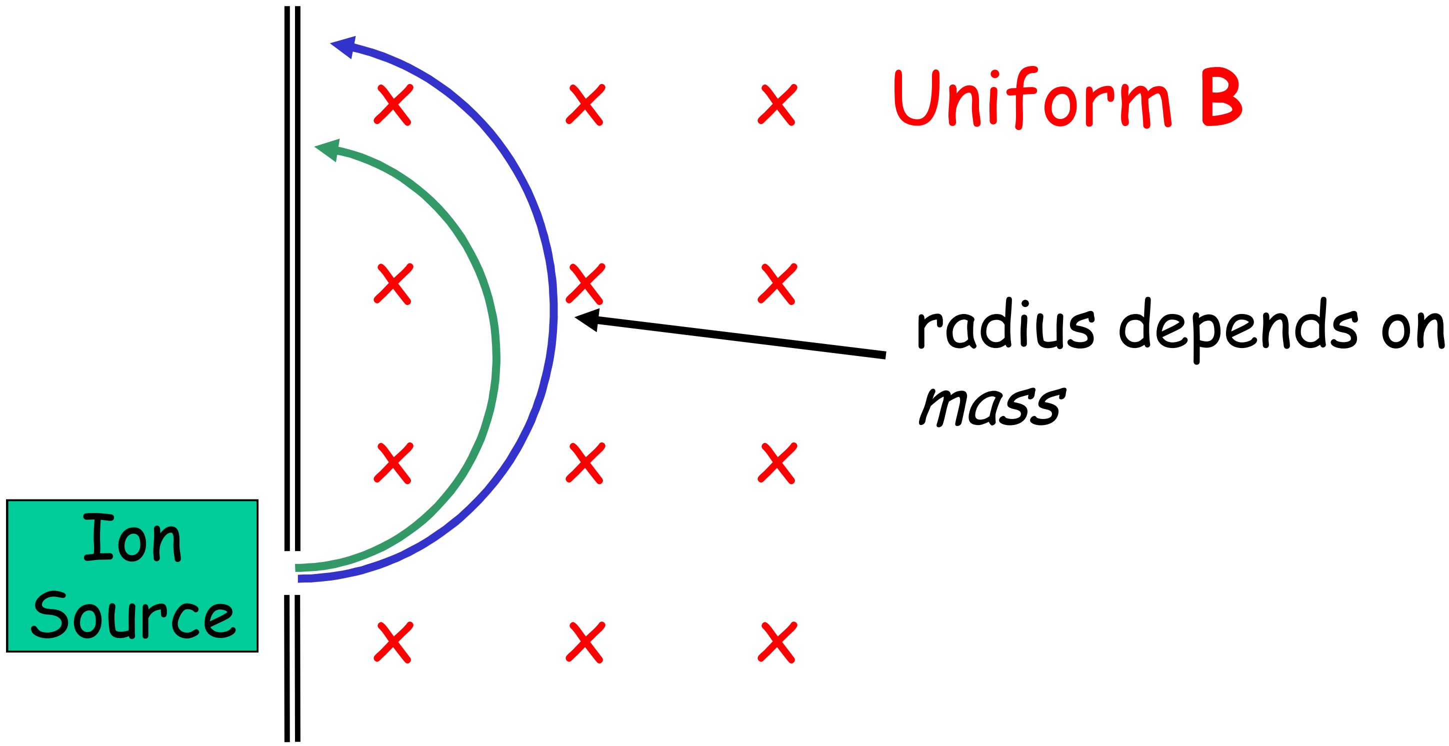
Quiz

A cyclotron can accelerate protons to a maximum kinetic energy of 1 MeV. You want to design a new, improved model with a higher maximum proton energy. Which of the following changes would help?

A) Double the magnetic field
B) Double the diameter of the dees
C) Double the voltage of the r-f supply
D) Any of the above
E) Two of the above

By what factor *would the kinetic energy increase?*

Another example: Mass Spectrometer



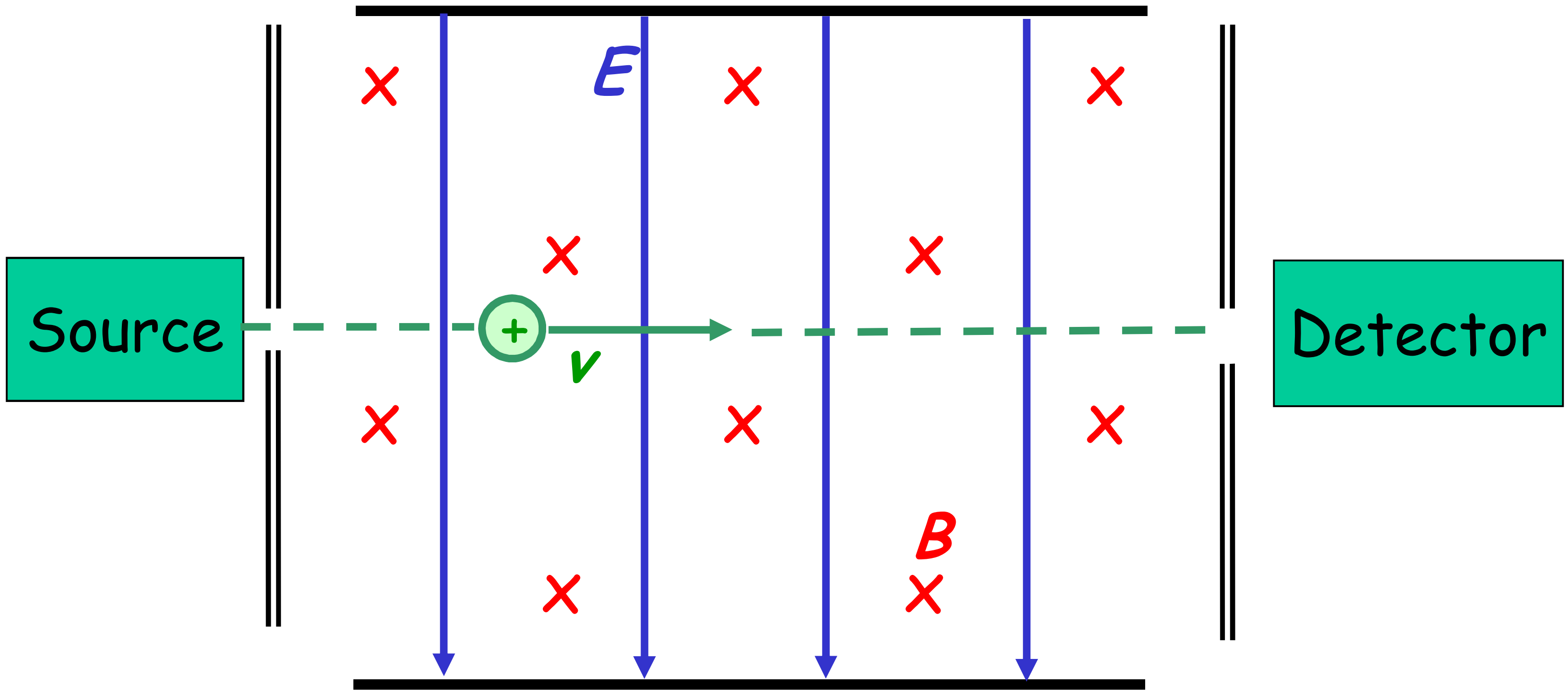
Homework Exercise:

E.g., ^{12}C and ^{13}C; same charge.
Find the ratio of diameters if the ions have

a) the same *speeds*

b) the same *kinetic energies*

Example: the "Velocity Selector"

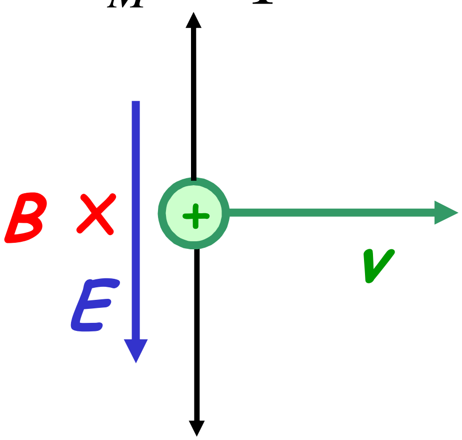


$\vec{E}$, $\vec{B}$, $\vec{v}$ all $\perp$

Find: Conditions so that the path is straight.

Forces

$$F_M = qvB$$



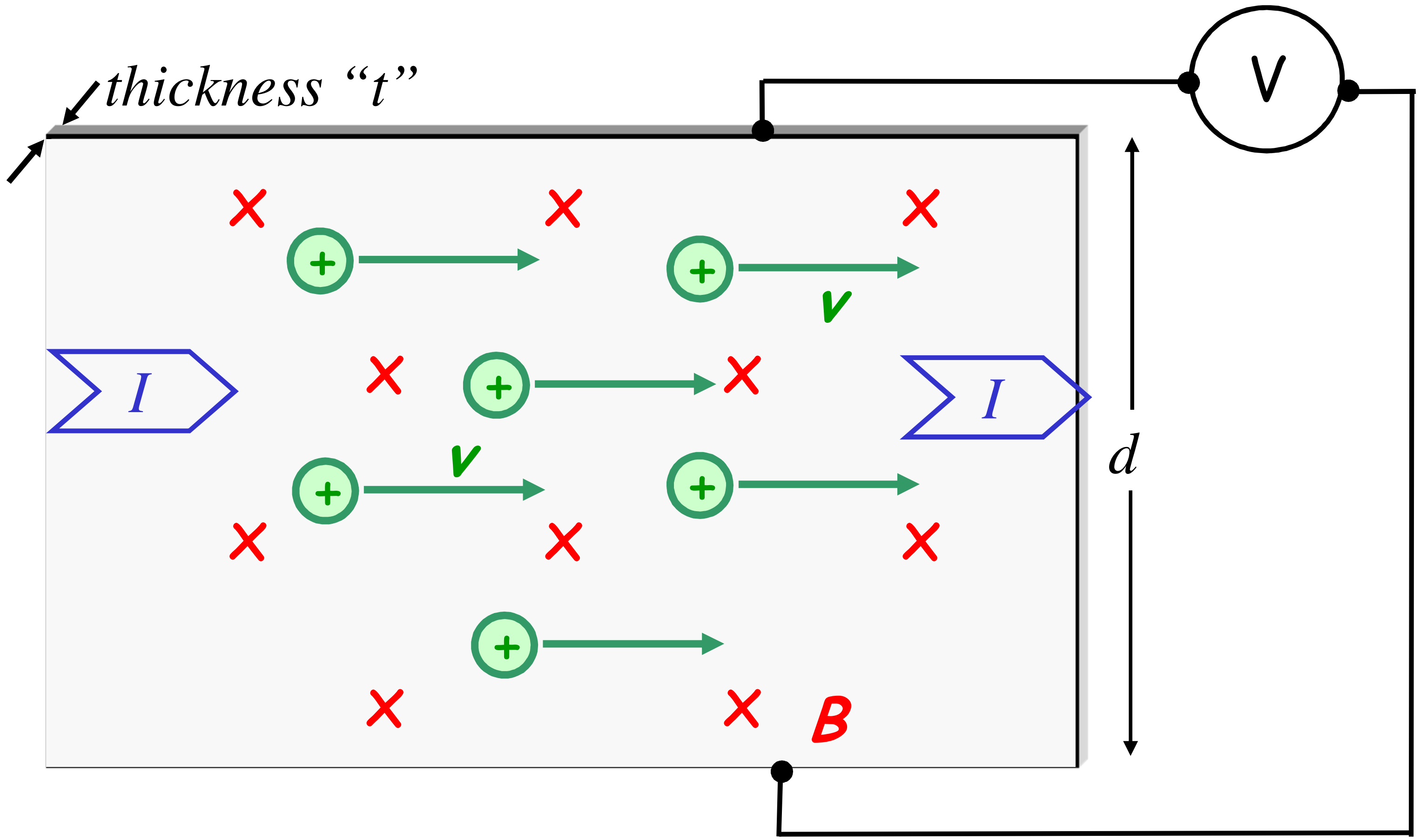
$$F_E = qE$$

Need $\Sigma\vec{F} = 0$ for straight path

$$\Rightarrow \quad qvB = qE$$

$$\Rightarrow \quad \boxed{v = \frac{E}{B}}$$ for straight path

Hall Effect

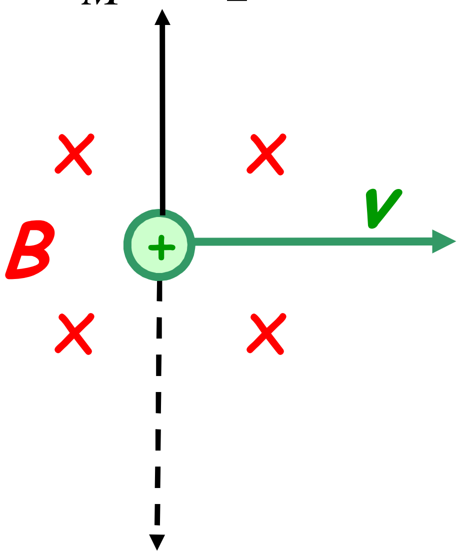


Thin conductor in uniform **B**; current I.

Measure "Hall Voltage" V_H perpendicular to current.

Forces

$$F_M = qvB$$



$$F_E = qE$$

$\vec{v}$ must be parallel to current.

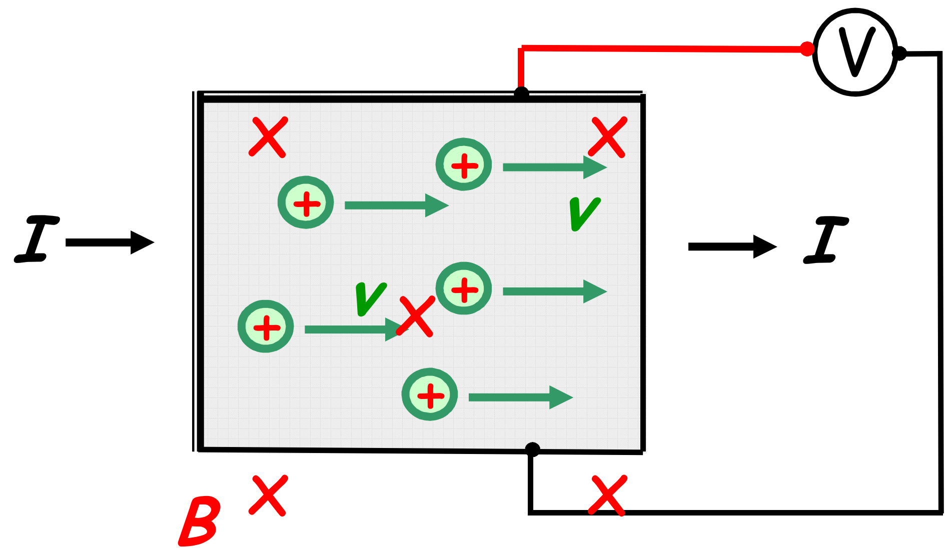
$$\Rightarrow \left|\vec{F}_M\right| = \left|\vec{F}_E\right| \Rightarrow E = vB$$

BUT: $\left|\vec{E}\right| = \dfrac{V_H}{d}$

$$\Rightarrow V_H = vBd$$

Quiz

Assuming the charge carriers are positive (as shown), the reading on the voltmeter (potential of the top edge) will be:

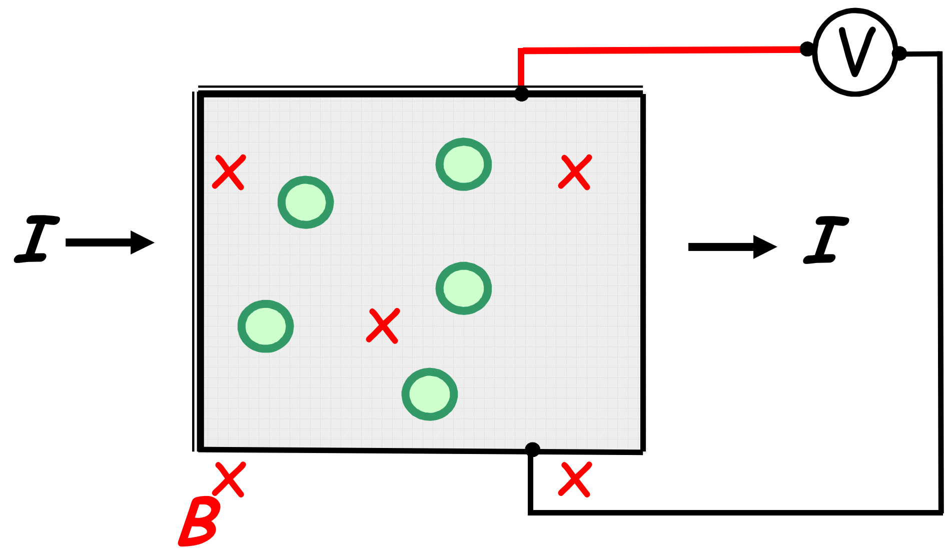


A) zero B) positive C) negative

If the charge carriers are negative, will the meter reading be the same (for the same current) ?

Quiz

*If the charge carriers are **negative**, the meter reading, for the same current I, will be:*



A) zero B) positive C) negative

Current: $I = nqvA$ (n = number/volume)

$$\Rightarrow v = \frac{I}{nqtd} \qquad (A = t \times d)$$

$$\Rightarrow V_H = \left(\frac{1}{nq}\right)\frac{IB}{t}$$

$\frac{1}{nq}$ = "Hall Coefficient"