DC Circuits

Text sections 28.1 - 28.3

- Emf, internal resistance
- Series and parallel rules for resistors

Practice: Chapter 28,
Objective Questions 2, 3, 6, 10, 14, 15
Conceptual Questions 2, 3, 4
Problems 3, 4, 9, 11, 19

“DC” Circuits

“**Direct Current**”: current always flows in one direction.

(or, often: current is <u>constant in time</u>)

“Electromotive force” (emf)

$\mathcal{E} \equiv$ external work per unit charge

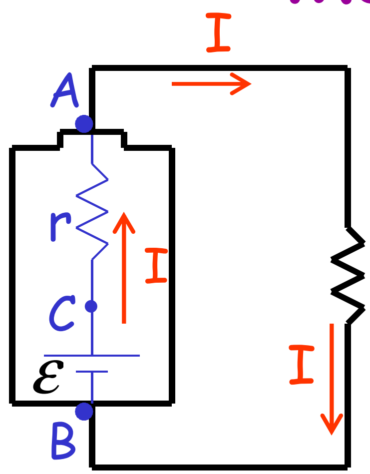
Units: J/C = volts (not actually a force)

e.g.: Battery
(chemical energy → electrical energy)

Generator
(mechanical energy → electrical energy)

Solar cell, etc.

Model of a "Real" Battery



R_L (external "load")

$\mathcal{E}$ = battery emf

r = "internal resistance"

$V_A - V_B$ = "terminal voltage" ➔ measured

$V_C - V_B = \mathcal{E}$ *(but "C" is not accessible for measurement)*

And $V_A - V_C = - I\,r$

$$\Rightarrow \quad \varepsilon - Ir = V \quad (\equiv V_A - V_B)$$

"Terminal voltage"

Quiz

For this model of a battery:
If the load resistance is decreased, then

A) the battery terminal voltage increases
B) the battery terminal voltage decreases
C) the battery terminal voltage is unchanged

Quiz

For this model of a battery:
If the load resistance is decreased, then

A) the battery internal resistance increases
B) the battery internal resistance decreases
C) the battery internal resistance is unchanged

Example

Automobile battery:

At terminals {
12.8 V (with 20 A current)
9.2 V (with 200 A current)

Find: $\mathcal{E}$ and $r_{internal}$ of battery

What is the maximum current the battery can provide?

Notes:

i) As $I \rightarrow 0$, $V_{load} \rightarrow \mathcal{E}$

ii) $I_{max} = \mathcal{E}/r$ (when $R_L = 0$, a short circuit)

Question: How would you select a load (e.g., lightbulb) to get the maximum power output, for a given battery?

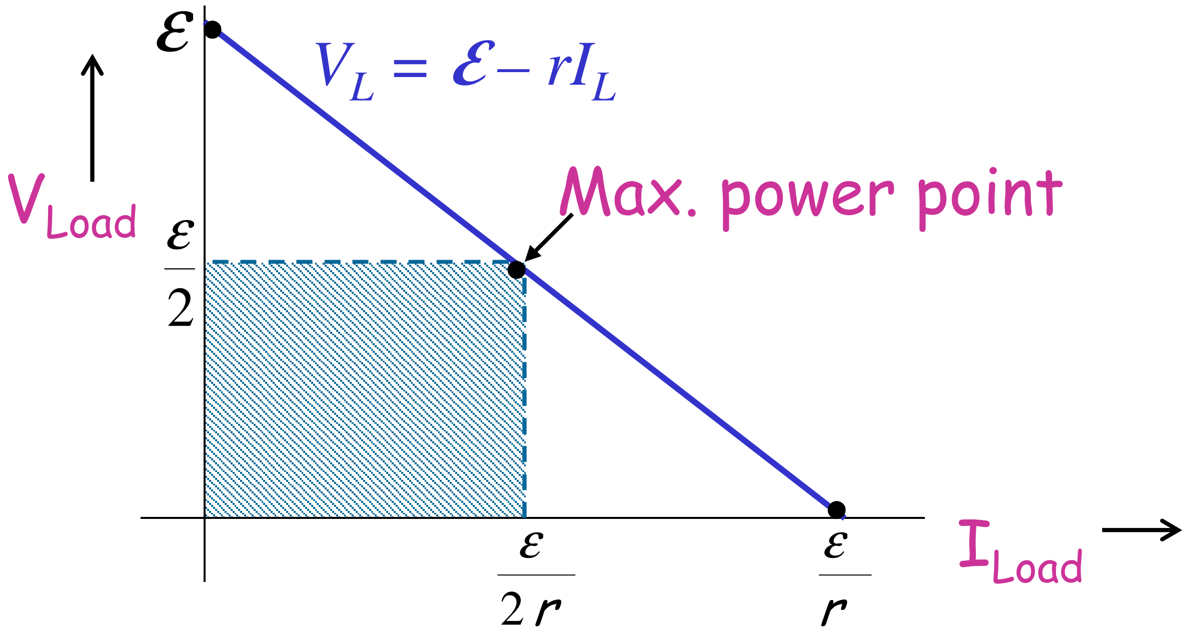
A well-known theorem: ***Maximum power transfer is achieved when the load resistance matches the source resistance.***

$$\varepsilon - Ir = V_{\text{load}} = IR_L \longrightarrow I = \frac{\varepsilon}{r + R_L}$$

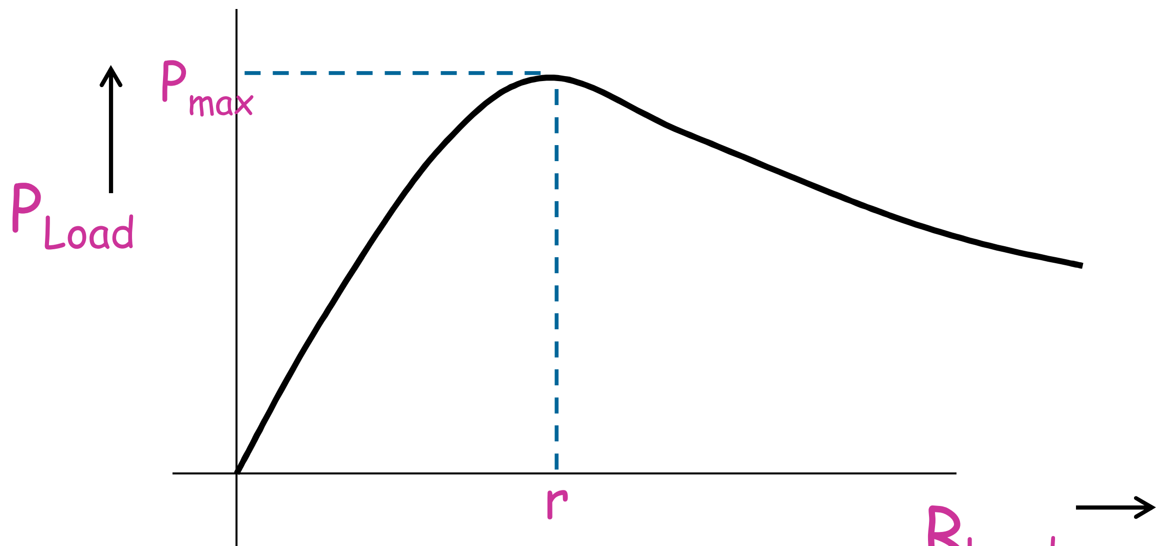
$$P_L = I^2 R_L = \varepsilon^2 \frac{R_L}{(r + R_L)^2}$$

➔ Maximum load power when $R_L = r$

Proof: Set $\frac{dP_L}{dR_L} = 0$ and solve for R_L *(exercise)*

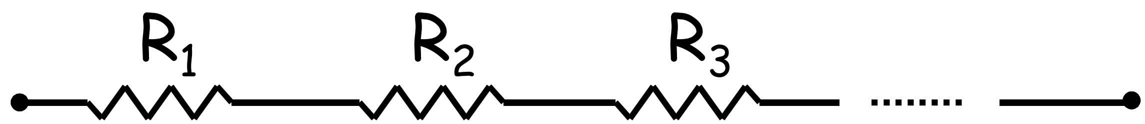


$\mathcal{E}$
$V_L = \mathcal{E} - rI_L$
Max. power point
V_{Load}
$\frac{\varepsilon}{2}$
$\frac{\varepsilon}{2r}$
$\frac{\varepsilon}{r}$
I_{Load}



P_{max}
P_{Load}
r
R_{Load}

Resistors in Series



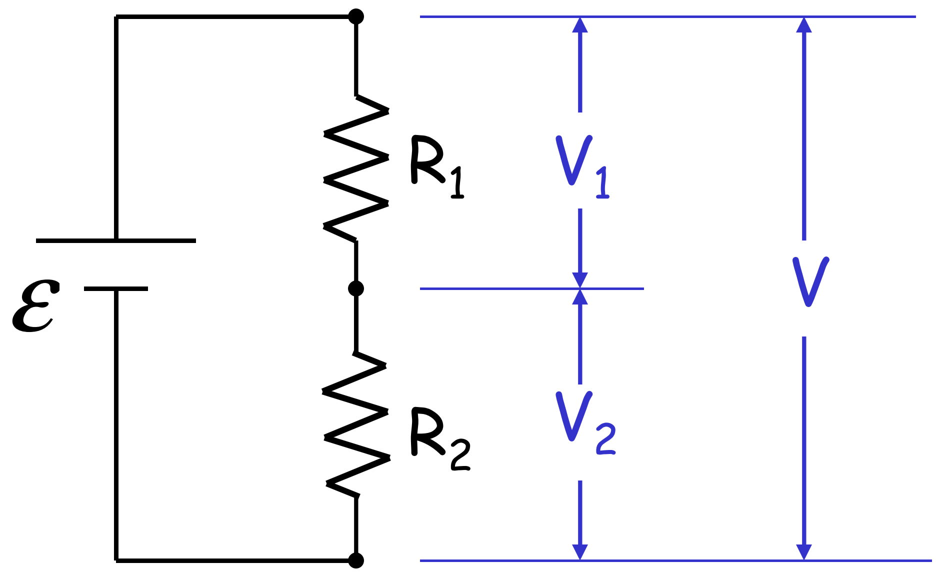
- Same <u>current</u> through all resistors
- Voltages add:

$\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3 + \ldots$

➔ $IR_{eff} = IR_1 + IR_2 + IR_3 + \ldots$

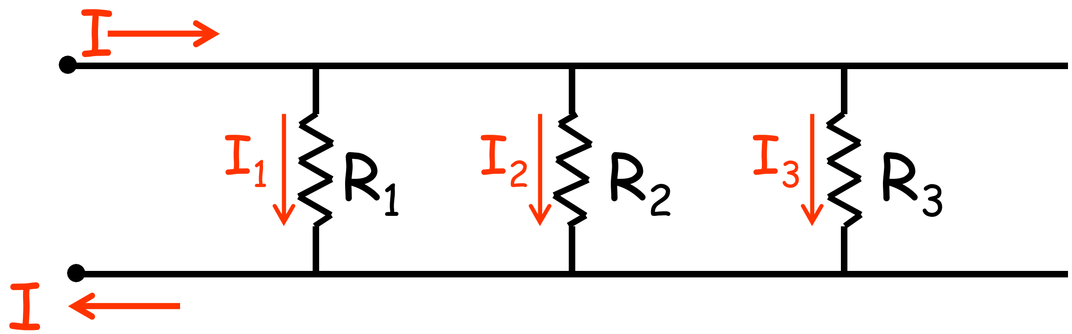
So, $$R_{eff} = R_1 + R_2 + R_3 + \ldots$$

Example: "Voltage Divider" Circuit



Show that $V_1 = \left(\dfrac{R_1}{R_1 + R_2} \right) \times V$ and $V_2 = \left(\dfrac{R_2}{R_1 + R_2} \right) \times V$

Resistors in Parallel



- same voltage across each resistor
- currents add:

$$I = I_1 + I_2 + I_3 + \ldots \qquad \Rightarrow \frac{\Delta V}{R_{eff}} = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} + \frac{\Delta V}{R_3} + \ldots$$

so,

$$\boxed{\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots}$$

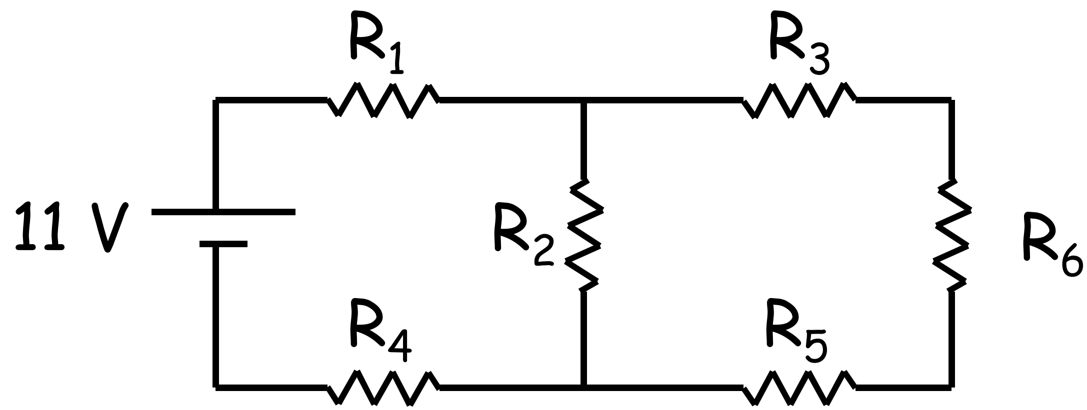
Quiz

Resistor A and resistor B are connected in parallel across 120 volts; and resistor A dissipates twice as much power as resistor B.

If instead they are connected together in series across 120 volts, then:

A) resistor A will dissipate more power than B
B) resistor B will dissipate more power than A
C) they will dissipate equal powers

Example



All resistors = 1 Ω

Find:

a) Battery current and power

b) Current in R_5, R_2

*Homework: Calculate the power dissipated in **each** resistor, and check that it adds up to the power supplied by the battery.*