Drude Model; and Electrical Work and Power

Text sections 27.4, 27.6

- Drude Model
- Electrical work and power

Practice: Chapter 27,
Objective Questions 11, 12
Conceptual Questions 2, 4
Problems 22, 23, 59, 60, 61, 78

Resistance

Defined by $V = IR$ ("Ohm's Law")

Unit: $1\ \text{ohm}\ (\Omega) = 1\ \frac{\text{volt}}{\text{amp}}$

For most materials ("ohmic conductors"):

$V \propto I$, so R is constant.

This is really Ohm's Law

RESISTIVITY, $\rho = \frac{1}{\sigma}$ (a property of the material)

$$R = \rho \frac{L}{A}$$

(Uniform wire, Length L, cross-section A)

Microscopic Physics of Ohm's Law

From before: $I = nqAv_d$

- We assumed that the drift velocity is proportional to the electric field, thus leading to Ohm's Law,

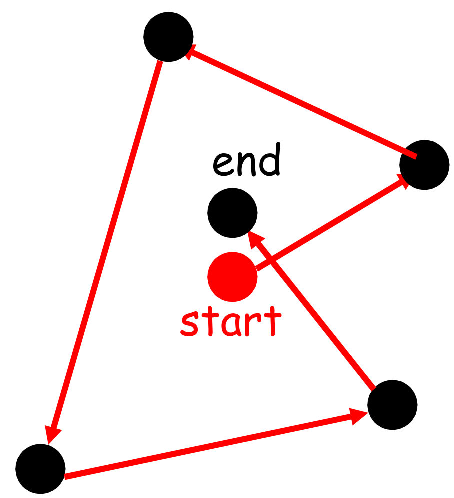
$$I \propto V$$

- $\mathbf{v}_d$ is the average velocity of an electron in a metal (net displacement per second) and is small ($\leq$ 1mm/s)
- Typical speeds are in km/s for electrons in metals

Electron Velocities With NO Field

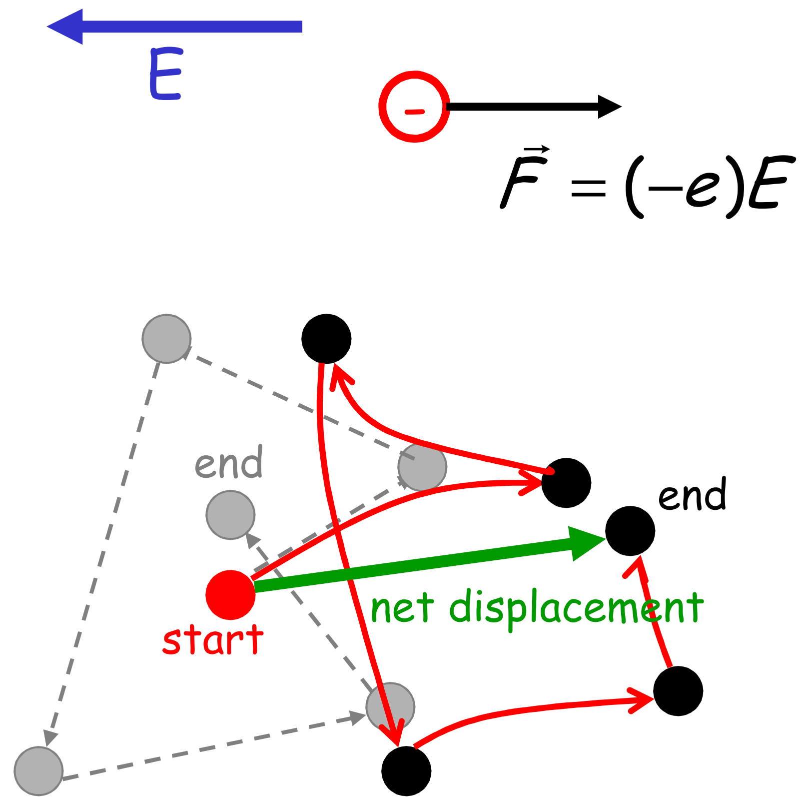
• Random velocities of electrons are large (several km/s)

With no field, the electrons scatter randomly (due to impurities and ions in the metal), so <u>on average</u> they go nowhere.



Electrons Velocities With a Field

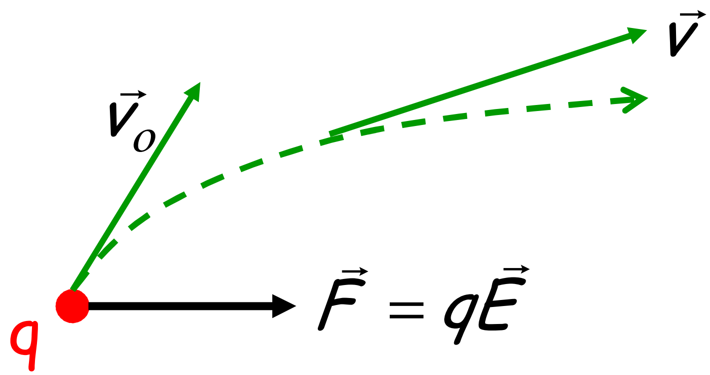
An electric field will cause the electron's path to curve slightly between collisions, but always in the same direction, leading to a slow but steady net "drift".

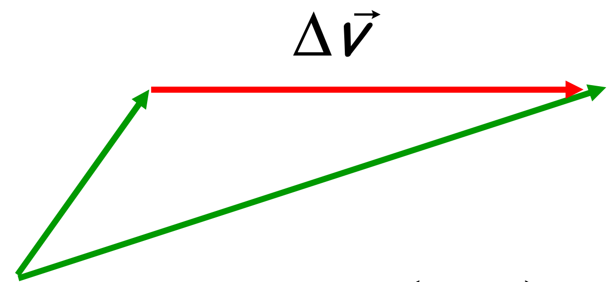


<u>Drude Model</u>

Assumptions:

1) Electrons have large random velocities (no field).
2) Electrons collide with atoms (time τ);
   τ is <u>not</u> affected by the field.
3) After a collision, velocity is random again.





$$\Delta\vec{v} = \vec{a}t = \left(\frac{q\vec{E}}{m}\right)t$$

Averages:

$$\langle \vec{v}_o \rangle = 0$$

$$\vec{v} = \vec{v}_o + \vec{a}\,t \;\; ; \;\; \langle t \rangle = \tau$$

$$\langle \vec{v} \rangle = \vec{a}\,\langle t \rangle = \frac{q\,\vec{E}\,\tau}{m}$$

So... drift velocity: $\vec{v}_d \equiv \langle \vec{v} \rangle$

$$\Rightarrow \qquad \vec{v}_d = \left(\frac{q}{m} \cdot \tau \right) \vec{E} \quad \propto \quad \vec{E}$$

and $$I = n\, q\, A\, v_d$$

$$\Rightarrow \qquad I = \left(\frac{ne^2}{m} \cdot \tau \right) AE \qquad \text{(if } |q| = e\text{)}$$

So... $$I \propto E$$

$$\vec{J} = \frac{ne^2}{m} \tau \vec{E}$$

and $$\boxed{\sigma = \frac{ne^2}{m} \tau}$$

Electrical Work and Power

Current I flows through a potential difference ΔV

Follow a charge Q : at positive end, $U_1 = QV_1$

at negative end, $U_2 = QV_2$

P.E. Decreases: $\Delta U = Q\Delta V$

➔ Electrical energy is converted to other forms of energy.

Quiz

If potential energy is steadily lost, what happens with the kinetic energy of the moving charges?

A) the average kinetic energy increases steadily

B) the average kinetic energy decreases steadily

C) none of the above

Quiz

The derivation assumed positive charges, and found that the potential energy decreases with time. If the current is actually carried by negative charges, then:

A) the potential energy will increase with time

B) the potential energy will still decrease

C) the potential energy will not change

Power:

P = work per unit time

$$\Rightarrow \quad P = \frac{Q\Delta V}{\Delta t} = I\Delta V$$

Power = current x (potential difference)

Units: 1 volt x 1 amp = 1 watt (= 1 J/s)

Eg. Resistor: $\Delta V = IR$

➔ $P = I^2 R$ *(rate of "Joule heating" in resistor)*

Examples

1) What is the resistance of a “60 watt” bulb? (for a 120-V supply)
2) Find R for a 60-W headlamp (12-V battery).
3) What power do you get from a “60-W” household bulb if you connect it to a 12-V car battery?
4) How many 200-A, 750 kV lines are needed to carry the power from a 1,500,000 kW hydro dam?

"Electromotive force" (emf)

$\mathcal{E} \equiv$ external work per unit charge

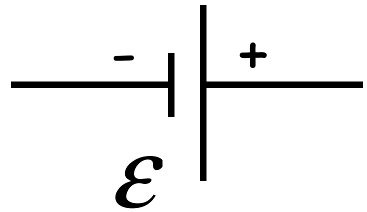
Units: J/C = volts (not actually a force)

e.g.: Battery
(chemical energy ➔ electrical energy)

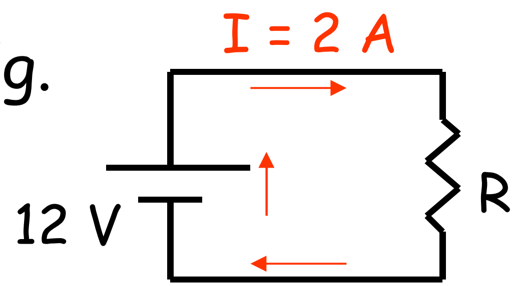
Generator
(mechanical energy ➔ electrical energy)

Solar cell, etc.

Example: battery.



Eg.



What does the energy balance look like in this circuit?

What is the resistance R?