Current & Resistance

(Conductors **NOT** in equilibrium; **E** $\neq 0$)

Text sections 27.1, 27.2

Current and current density
Ohm's Law, resistivity, and resistance

Practice: Chapter 27,
Objective Questions 1, 2, 4
Conceptual Questions 3, 7, 8
Problems 1, 5, 10, 19

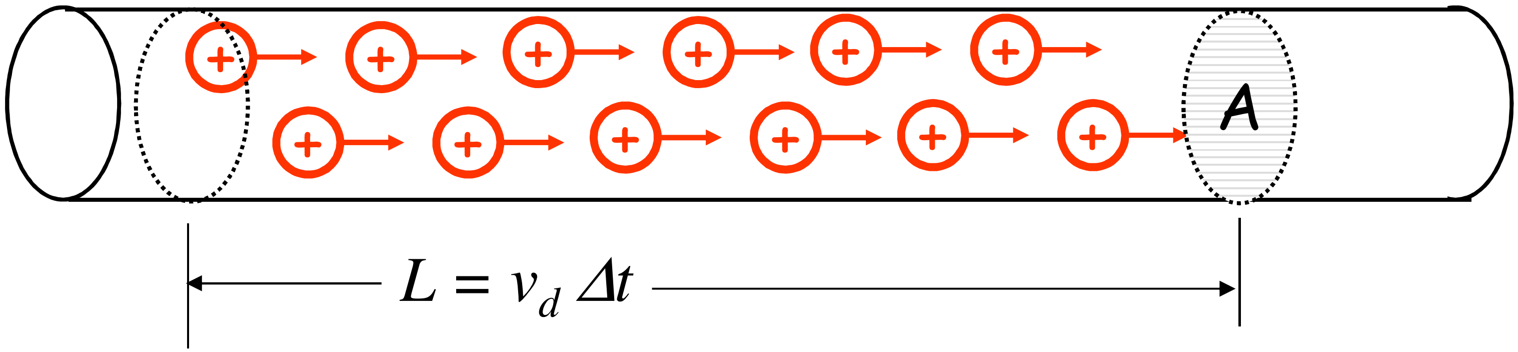
Electrical Current

CURRENT I is the charge per unit time flowing along a wire:

$$I \equiv dQ/dt$$ if charge dQ flows past in time dt

Units: 1 ampere (A) = 1 C/s

Direction: *by convention*, the direction of movement of **positive** charge



Charge passing through the shaded circle in time Δt :

Q = (number of charges/volume) x (charge on each one) x volume

$$Q = n \cdot q \cdot (AL) = nqAv_d \, \Delta t$$

Current: $I = Q/\Delta t = nqAv_d \, \Delta t / \Delta t$

So,

$$\boxed{I = nqAv_d}$$

v_d = average ("drift") velocity of each charge

Current Density (a **vector**):

$$J \equiv \frac{I}{A}$$ where A is area $\perp$ current flow

Units: Amps/m^2

(Note that the "current through a surface" is the ***flux of the current density*** *through that surface.)*

So,

$$\mathbf{J} = nq\mathbf{v}_d$$ (a *vector* equation)

In normal conductors, $\mathbf{J}$ *is caused by an* ***electric field*** *in the conductor—which is* ***not*** *in equilibrium.*

Quiz

Mobile positively-charged sodium ions in a salt solution carry an electrical current when a battery is connected. There are also some negatively-charged chloride ions in the solution. The presence of mobile chloride ions

A) causes the net current to be even larger
B) causes the net current to be smaller
C) causes the net current to be zero
D) has no effect on the net current

Example

The mobile charges in most metals are electrons, with about one or two electrons per atom being free to move. So there are about 10^{23} charges per cm^3 (or 10^{29} m^{-3}).

$$n \approx 10^{29} \text{ electrons/m}^3$$

$$nq = ne = 1.6 \times 10^{10} \text{ C/m}^3$$

Take Area = 1 mm^2, I = 1 A

$$\vdots$$

$$\Rightarrow v_{drift} = 0.06 \text{ mm/s}$$

Which way are the electrons moving?

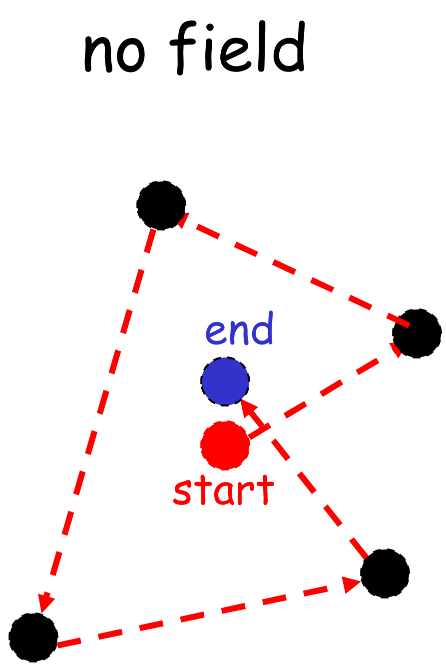
Questions

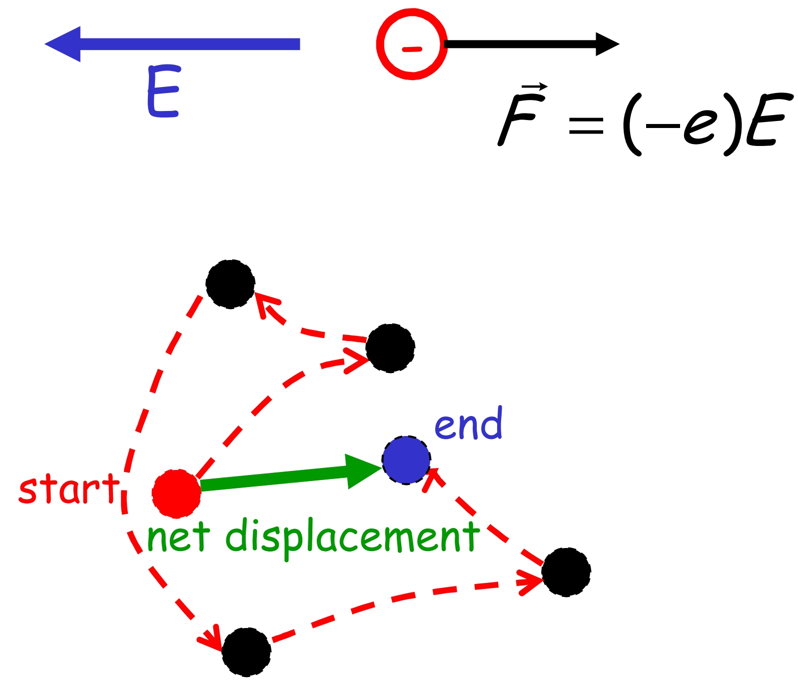
- *When you turn on a flashlight, how long does it take for the electrons from the battery to reach the bulb?*

- *How long does it take for the electrons in the bulb to start moving?*

Electron Velocities

- *Random* velocities of electrons are large (several km/s)
- *Drift velocity* is a slow, average motion parallel to $\mathbf{E}$





Ohm's Law

The charges are pushed by the electric field; the drift *velocity* is proportional to the field.

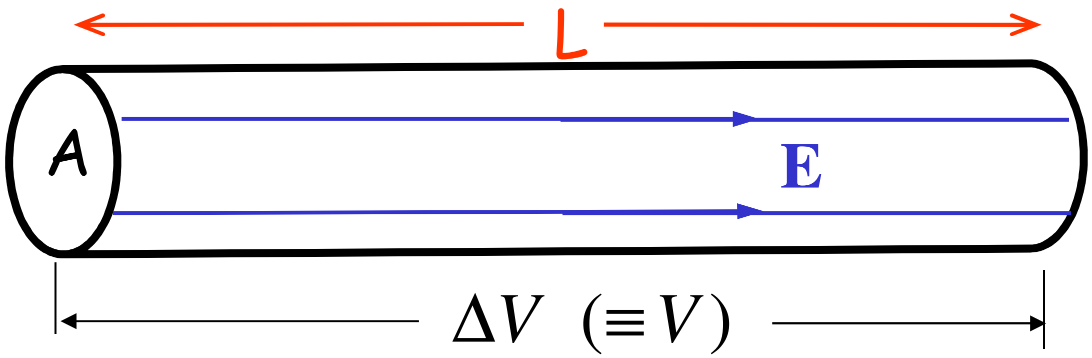
This leads to:

- Current density ∝ field: $\mathbf{J} = \sigma \mathbf{E}$
- Current ∝ potential difference: $V = RI$

R = "resistance"; unit, $1\ \text{ohm}\ (\Omega) = 1\ \dfrac{\text{volt}}{\text{amp}}$

σ = "conductivity"; units, $\dfrac{A/m^2}{V/m} = \dfrac{1}{\Omega \cdot m}$

*Because J is proportional to the field, current in a wire is proportional to the **potential difference** between the ends of the wire.*



Uniform $\mathbf{E}$, $\Rightarrow \quad V = E \times L$

$$J = \sigma E \Rightarrow \left(\frac{I}{A}\right) = \sigma \times \left(\frac{V}{L}\right)$$

$$\Rightarrow V = \underbrace{\left(\frac{L}{A\sigma}\right)}_{\text{"Resistance", } R} \times I$$

RESISTIVITY: the inverse of conductivity.

$$\rho = \frac{1}{\sigma}$$

(these depend on the type of material)

Since we found $V = \left(\frac{L}{A\sigma}\right) \times I = R \times I$,

we can write

$$R = \rho \frac{L}{A}$$

(Uniform wire, Length L, cross-section A)

Resistivities of a few materials

	ρ (20°C) ($\Omega \cdot$m)
Cu	1.7×10^{-8}
Al	2.8×10^{-8}
Nichrome	150×10^{-8}
Graphite	3500×10^{-8}
Si	~640
Quartz	$\sim 10^{18}$

Quiz

The current in a particular wire is doubled. Then:

A) the resistance decreases by half
B) the conductivity doubles
C) the electron number density doubles
D) the electron drift speed doubles
E) more than one of the above is true

Example

A copper wire, 0.2 mm in diameter and 30 m in length, has a 5 A current.

$\rho = 1.7\times10^{-8}\ \Omega\cdot$m for Cu

Find: a) resistance

b) potential difference between the ends

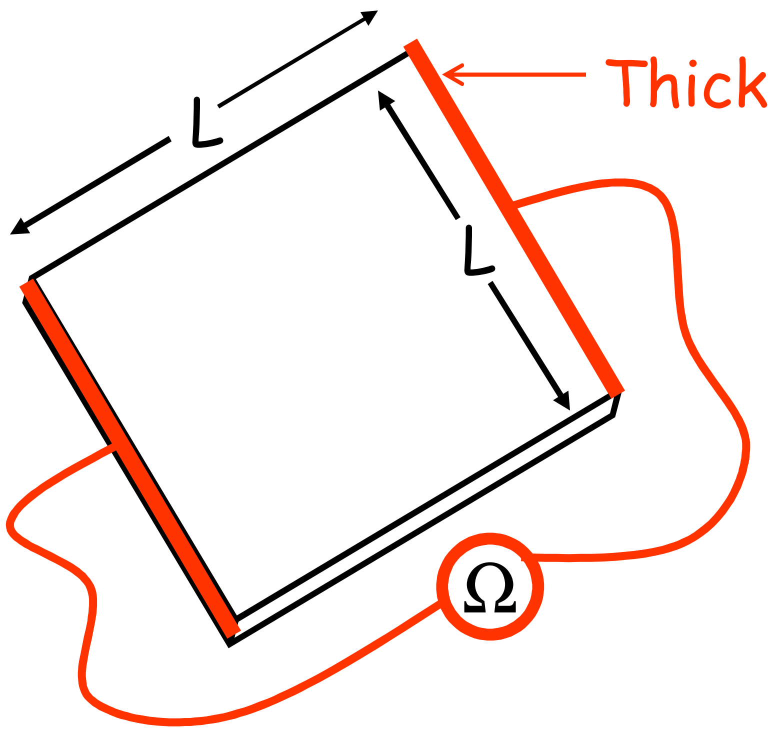
c) electric field

d) current density

Example: Aluminum Foil

$$\text{Thickness} = 0.1\,\mu\text{m} = 1.0\times 10^{-7}\ \text{m}$$

Square piece, side L



Al:

$$\rho = 2.8\times 10^{-8}\ \Omega\cdot\text{m}$$

Find: resistance between opposite edges

Summary

Current Density: $J = I/A = nqv_d$

Conductivity: σ :

$$\vec{J} = \sigma\vec{E} \quad \text{(defines } \sigma\text{)}$$

Resistivity: $\rho \equiv \frac{1}{\sigma}$

Resistance: $R = \frac{V}{I}$ and so... $R = \rho\frac{L}{A}$

Ohm's Law: $I \propto V$

($\Rightarrow$ R is constant)