Capacitance (Chapter 26)

How much charge can a conductor hold?

- Definition
- Calculating Capacitance
- Dielectric Constant

Serway and Jewett sections 26.1, 26.2, 26.5

Practice: Chapter 26,
Objective Questions 3, 13, 14
Conceptual Questions 2, 3, 6
Problems 3, 5, 6, 9, 45

Definition: Capacitance C between two conductors,

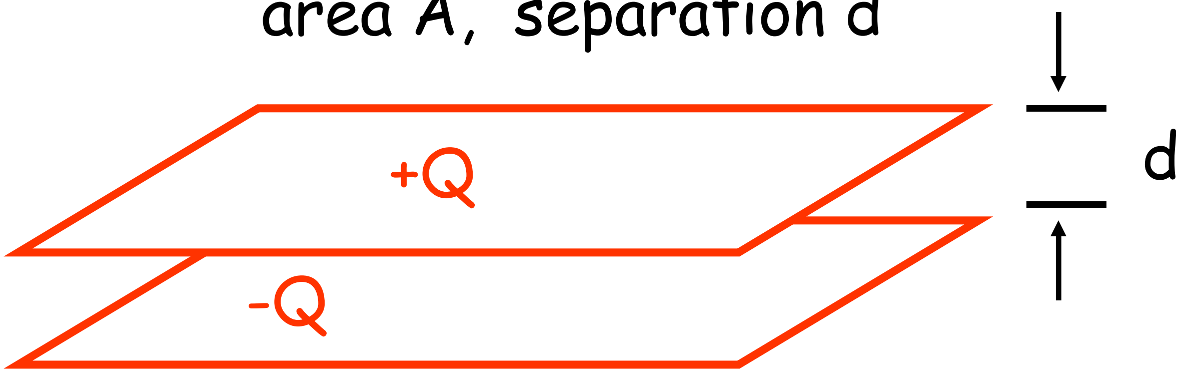
$$C \equiv \frac{Q}{V}$$

V = potential difference created when charge +Q is on one conductor, and –Q is on the other.

Unit: $\frac{\text{coulomb}}{\text{volt}} = 1$ farad (F)

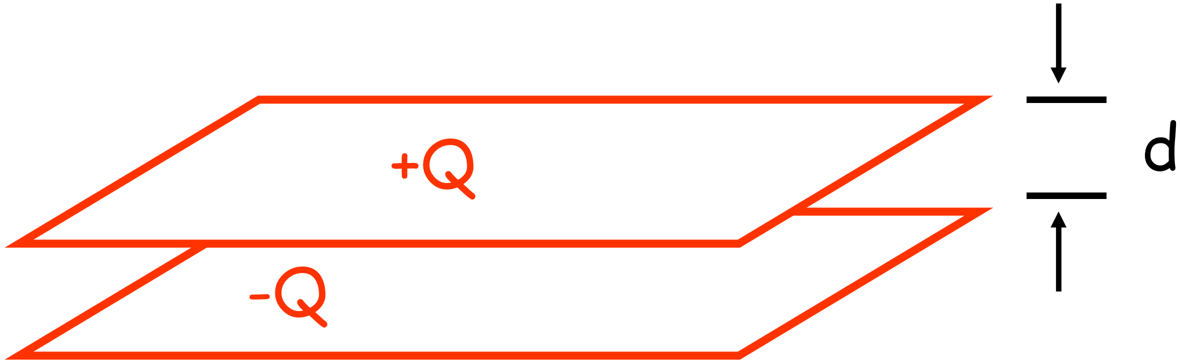
Example 1: Parallel Plates

area A, separation d



Surface charge densities: $\pm\sigma \equiv \pm\frac{Q}{A}$

Quiz:

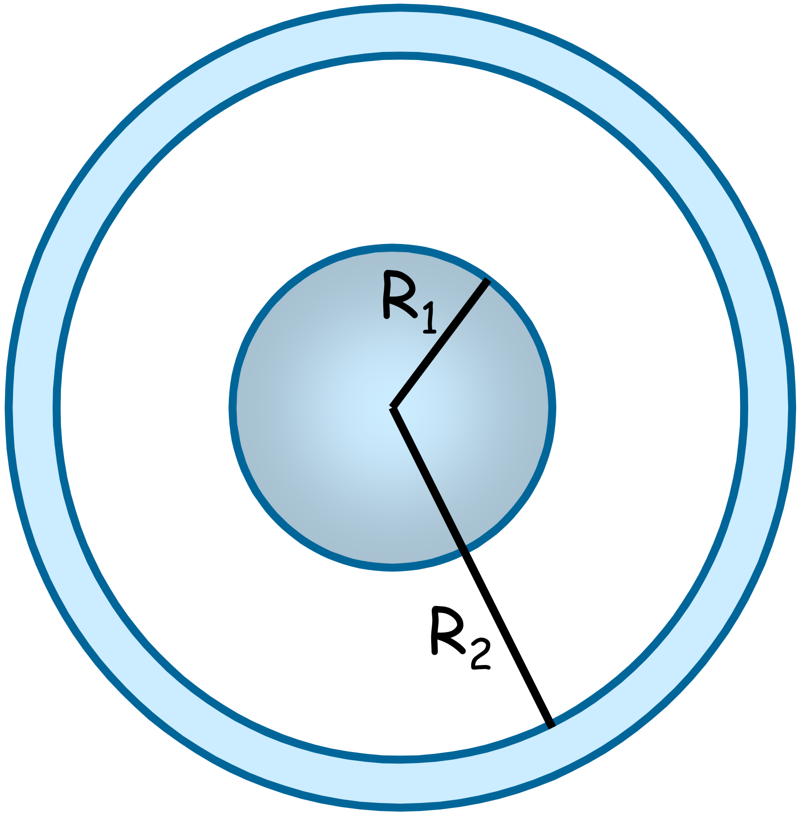


As the plates are moved apart, which of the following will increase?

A) Electric field between the plates
B) Electric potential difference between the plates
C) Capacitance between the plates
D) two of the above
E) all of the above

Example 2: Concentric Spheres

(do for homework!)



Derive:

$$C = 4\pi\varepsilon_o\left[\frac{1}{R_1} - \frac{1}{R_2}\right]^{-1}$$

Examples:

Find capacitances of…

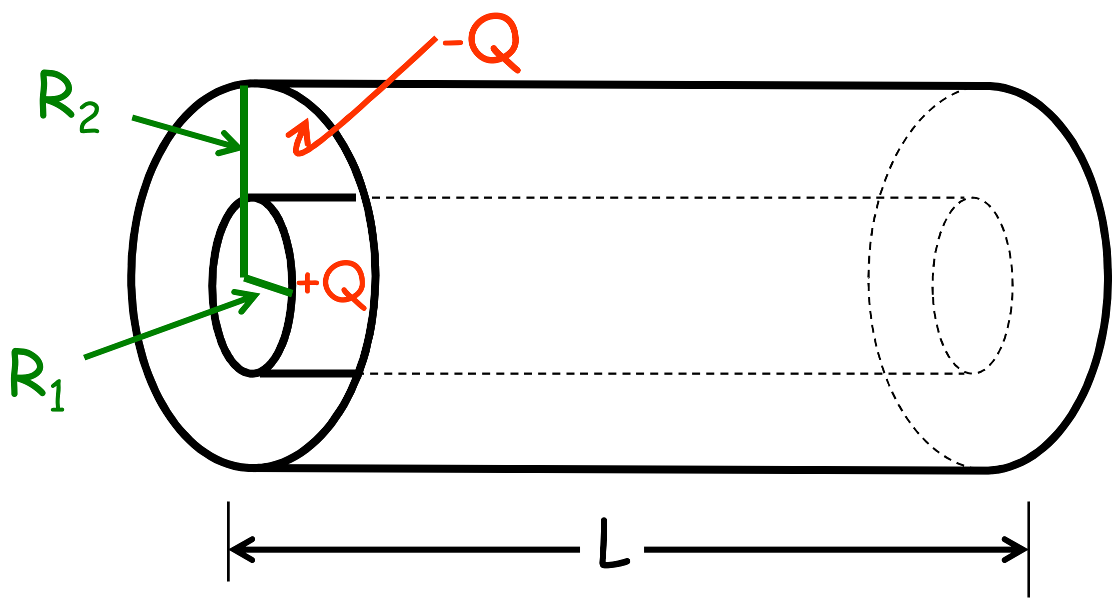
1) 2 sheets of foil, 25 cm x 40 cm, 1 mm apart (in air.)
2) The earth (as an isolated sphere): R_E = 6370 km
3) The earth and the ionosphere (height: 100 km) as a pair of concentric spheres.

answers: 1) 885 pF 2) 700 μF 3) 46000 μF

Exercise:

If the earth has a (downward) field of 100 N/C, what charge does it hold? (do for cases 2 and 3.)

Example 3: Concentric Cylinders



For L >> R_2, show:

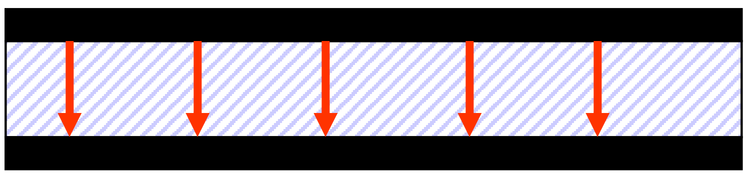
$$\frac{C}{L} = \frac{2\pi\varepsilon_o}{\ln\left(R_2/R_1\right)}$$

Dielectric Constant

$\vec{E}$ is smaller in many materials than it would be in a vacuum, for the same arrangement of charges.

Eg. <u>Parallel plates</u>:

$+\sigma$



$-\sigma$

$$E = \frac{\sigma}{\kappa \varepsilon_o}$$

κ ("kappa") = "dielectric constant"

= (a <u>pure number</u> ≥ 1)

So, eg...

$$C = \frac{\kappa \varepsilon_o A}{d}$$

(parallel plates)

Material	κ
Vacuum	1 (definition)
air	1.0006
glass	~4 - 6
polystyrene	2.6
water	80

For any geometry,

$$C = \kappa \cdot C_{vacuum}$$

i.e. Replace "ε_0" with " $\kappa\varepsilon_0$ " in the formulae.

Why use dielectrics in capacitors?

1) the capacitance is increased by a factor κ

2) the dielectric material gives mechanical strength (holds the conductors apart)

3) the plate separation can be smaller (which also increases capacitance)

4) the "dielectric strength", or maximum electric field before conduction starts, can be higher than for air, allowing higher voltage ratings

Quiz

Two charged plates with charges +Q, -Q are suspended a distance *d* apart in a beaker of oil (dielectric constant = 2). The electric field in the region between the plates is

a) the same as if they were in air
b) larger
c) smaller

What if we compared plates in oil and air with the same potential difference, rather than with the same charge?

Quiz

The two plates are connected to a battery which maintains a fixed ***potential difference*** between them. Now when the gap is filled with oil, the electric field in the region between the plates is

a) the same as if they were in air
b) larger
c) smaller

Summary of Capacitance

$$C = \frac{Q}{V}$$

+Q on one conductor, -Q on the other conductor

Potential difference

Deriving Formulae: (for plates, spheres, cylinders)

1) Assume charges +Q, -Q
2) Calculate E from Gauss's Law
3) Integrate along a field line: $\Delta V = -\int \vec{E} \bullet d\vec{s}$
4) $C = \frac{Q}{|\Delta V|}$