Adiabatic transfer of light in a double cavity and the optical Landau-Zener problem

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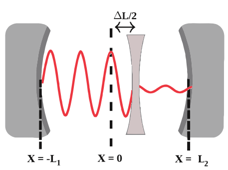
Pellizari [Phys. Rev. Lett. **79**, 5242 (1997)] has previously proposed transferring photons between two optical cavities by using a STIRAP scheme to control the coupling of atoms in the two cavities to the cavity fields. Here we analyze a variant of his idea and study the evolution of an electromagnetic field inside a double cavity when the difference in length between the two cavities is changed, e.g. by translating the common mirror. We find that this allows photons to be moved deterministically from one cavity to the other. We are able to obtain the conditions for adiabatic transfer by first mapping the Maxwell wave equation for the electric field onto a Schrödinger-like wave equation, and then using the Landau-Zener result for the transition probability at an avoided crossing. Our analysis reveals that this mapping only rigorously holds when the two cavities are weakly coupled (i.e. in the regime of a highly reflective common mirror), and that, generally speaking, care is required when attempting a hamiltonian description of cavity electrodynamics with time-dependent boundary conditions. Preprint available at: arXiv:1105.6071

1. Proposed setup

We consider a double cavity consisting of two perfectly reflecting mirrors, separated by a partially transmissive common central mirror which can move. The difference in length between the two cavities is $\Delta L = L_1 - L_2$



This setup is similar to optomechanics experiments at Yale [1,2], except here the mirror position is controlled, e.g. using piezo-electric motors (v~1m/s). Alternatively, the mirror is fixed but the optical length of the two cavities is changed by making them out of dielectric waveguides whose refractive indices can be controlled via an electro-optic effect [3].

2. Theoretical model

Dielectric permittivity (mirror reflectivity is parameterized by α) $\varepsilon(x) = \begin{cases} \varepsilon_0(1+\alpha\delta(x)) & -L_1 < x < L_2 \\ \infty & \text{elsewhere} \end{cases}$

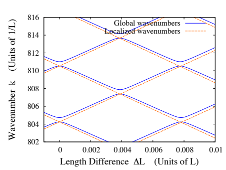
Maxwell wave equation for electric field $\dfrac{\partial^2 \mathcal{E}(x,t)}{\partial x^2} - \mu_0\varepsilon_0(1+\alpha\delta(x))\dfrac{\partial^2 \mathcal{E}(x,t)}{\partial t^2} = 0$

Solutions: $\mathcal{E}_n(x,t) = U_n(x)\exp(-i\omega_n t) \quad n = 1,2,3,\ldots$

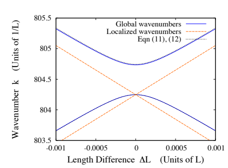
global modes: $U_n(x) = \begin{cases} A_n \sin[k_n(x+L_1)] & -L_1 \le x \le 0 \\ B_n \sin[k_n(x-L_2)] & 0 \le x \le L_2 \end{cases}$

wavenumbers: $\tan(k_n L_2) = \dfrac{\tan(k_n L_1)}{\alpha k_n \tan(k_n L_1) - 1}$

3. Anticrossing nets



Zoom in:



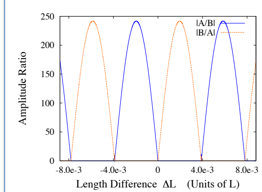
$$k_{n,e} = \frac{2\pi n}{L} + \frac{\Delta}{\hbar c} + \frac{1}{\hbar c}\sqrt{\Delta^2 + \gamma\,\Delta L^2}$$
$$k_{n,o} = \frac{2\pi n}{L} + \frac{\Delta}{\hbar c} - \frac{1}{\hbar c}\sqrt{\Delta^2 + \gamma\,\Delta L^2}$$

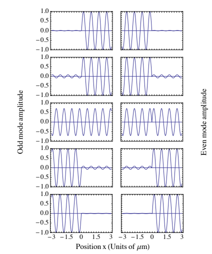
where: $\gamma = 2\Delta\,\hbar c\,\dfrac{2\pi^3 n^3 \alpha}{L^4}$

$$\Delta = \frac{\hbar c}{L}\frac{n\pi}{1 + n^2\pi^2\alpha/L}\,.$$

4. Transfer ratio

Mode functions for different mirror displacements (only need λ/4 for transfer.)





Mirror transmission~ 10^{-4}

5. Time-dependence: the Landau-Zener problem

$H_{LZ} = \begin{bmatrix} E(t) & \Delta \\ \Delta & -E(t) \end{bmatrix}$ where $E = \vartheta t/2$ are the energies of the diabatic states.

Eigenenergies: (energies of adiabatic states) $E_\pm(t) = \pm\sqrt{\vartheta^2 t^2/4 + \Delta^2}$

Rate at which diabatic energies separate: $\vartheta \approx 8\pi\hbar c n v/L^2$

Probability of transition out of original adiabatic state (t: -∞ → ∞) $P_{\mathrm{LZ}} = \exp(-2\pi\Delta^2/\hbar\vartheta)$

However, these results are for the Schrödinger eqn, which is 1st order in time.

6. From Maxwell to Schrödinger

In order to apply the Landau-Zener result to the classical electromagnetic field modes we need to reduce the 2nd order in time Maxwell wave equation to the 1st order in time Schrödinger equation.

We work in the diabatic basis: $\mathcal{E}(x,t) = A_L(t)\phi_L(x,t) + A_R(t)\phi_R(x,t)$

Put: $A_{L/R}(t) \equiv \widetilde{A}_{L/R}(t)\exp\left[-i\int_0^t \beta_{L/R}(t')\mathrm{d}t'\right]$, $\beta_{L/R}(t) = \sqrt{(E(t)/\hbar \mp \omega_{\mathrm{av}})^2 + \Delta^2/\hbar^2}$

But: $\omega_{\mathrm{av}} = 10^{15}\mathrm{s}^{-1} \gg \{\Delta/\hbar, E/\hbar\}$ ω_{av} is the optical frequency

Then: $\ddot{A}_{L/R} = (\underbrace{\ddot{\tilde{A}}_{L/R}}_{\text{neglect}} - 2i\beta_{L/R}\dot{\tilde{A}}_{L/R} - \underbrace{i\dot{\beta}_{L/R}\tilde{A}_{L/R}}_{\text{neglect}} - \beta_{L/R}^2\tilde{A}_{L/R})\exp\left[-i\int_0^t \beta_{L/R}(t')\mathrm{d}t'\right]$

7. Comparison of Maxwell and Schrödinger dynamics

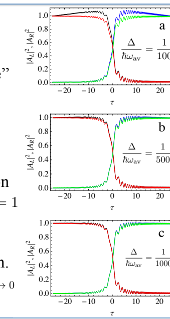
Above procedure leads to "Schrödinger-like" equation for classical amplitudes:

$$i\frac{\mathrm{d}}{\mathrm{d}\tau}\begin{pmatrix} \tilde{A}_L \\ \tilde{A}_R \end{pmatrix} = \begin{pmatrix} 0 & \mathrm{e}^{-i\tilde{\vartheta}\tau^2/2} \\ \mathrm{e}^{i\tilde{\vartheta}\tau^2/2} & 0 \end{pmatrix}\begin{pmatrix} \tilde{A}_L \\ \tilde{A}_R \end{pmatrix}$$

$\tau \equiv \Delta t/\hbar \qquad \tilde{\vartheta} \equiv \hbar\vartheta/\Delta^2$

Panels on right compare dynamics of this equation with that of full Maxwell equation. In each panel $\tilde{\vartheta} = 1$

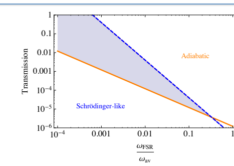
The "Schrödinger-like" eqn only depends on $\tilde{\theta}$, not $\Delta/\hbar\omega_{\mathrm{av}}$, but the Maxwell equation depends on both. We therefore see that the reduction holds when $\frac{\Delta}{\hbar\omega_{\mathrm{av}}} \to 0$



8. Regimes of validity

The Schrödinger-like equation is valid when $\dfrac{\hbar\omega_{\mathrm{av}}}{\Delta} \approx \dfrac{2\pi}{\sqrt{T}}\dfrac{\omega_{\mathrm{av}}}{\omega_{\mathrm{FSR}}} \gg 1$

where $\omega_{\mathrm{FSR}} \equiv 2\pi c/L$ is the free spectral range, and T is the transmission.



This condition is plotted as the dashed line on the plot. The adiabatic condition is plotted as the solid line, and according to the Landau-Zener theory is given by $\dfrac{2\pi\Delta^2}{\hbar\vartheta} \approx \dfrac{T}{2}\dfrac{\omega_{\mathrm{FSR}}}{\delta\omega_{\mathrm{dop}}} = \dfrac{T}{4}\dfrac{\omega_{\mathrm{FSR}}}{\omega_{\mathrm{av}}}\dfrac{c}{v} \gg 1$

Doppler shift:
$\delta\omega_{\mathrm{dop}} \equiv 2kv \approx 2\omega_{\mathrm{av}}v/c$

Both conditions are valid in the shaded region of the plot.

9. Conclusions

- By displacing the common mirror in a double cavity the electromagnetic field modes can be swapped from one side to the other. This system may be considered as a basic element in a quantum network [4].
- Because the mode wavenumbers form a network of avoided crossings, the possibility exists to make the transfer adiabatic.
- By making a slowly varying envelope approximation the Maxwell wave equation (2nd order in time) can be reduced to a Schrödinger-like equation (1st order in time) for the classical electromagnetic field amplitudes.
- The mathematical apparatus of the Landau-Zener theory can then be applied to the Schrödinger-like equation in order to obtain the condition for adiabatic transfer of light.

References: [1] J.D. Thompson, B.M. Zwickl, A.M. Jayich, F. Marquardt, S.M. Girvin, and J.G.E. Harris, Nature (London) **452**, 72 (2008), [2] G. Heinrich, J. E. Harris and F. Marquardt, Phys. Rev. A **81** 011801 (2010) ,
[3] S. F. Preble, Q. Xu, and M. Lipson, Nat. Photon. **1**, 293 (2007), [4] H. J. Kimble, Nature **453**, 1023 (2008).