Systematic Corrections to the Measured Cosmological Constant due to Local Inhomogeneity

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Motivation

It appears as though the Universe is expanding at an **accelerating** rate. We deduce this in many ways:

- Type Ia supernovae are **dimmer than we would expect** in a matter dominated universe with GR (Riess *et al.*, 1998; Perlmutter *et al.*, 1999)

- The observed matter density is **too small** to close the Universe as required by CMB observations w/ H_0 priors (Bennett *et al*., 2003)

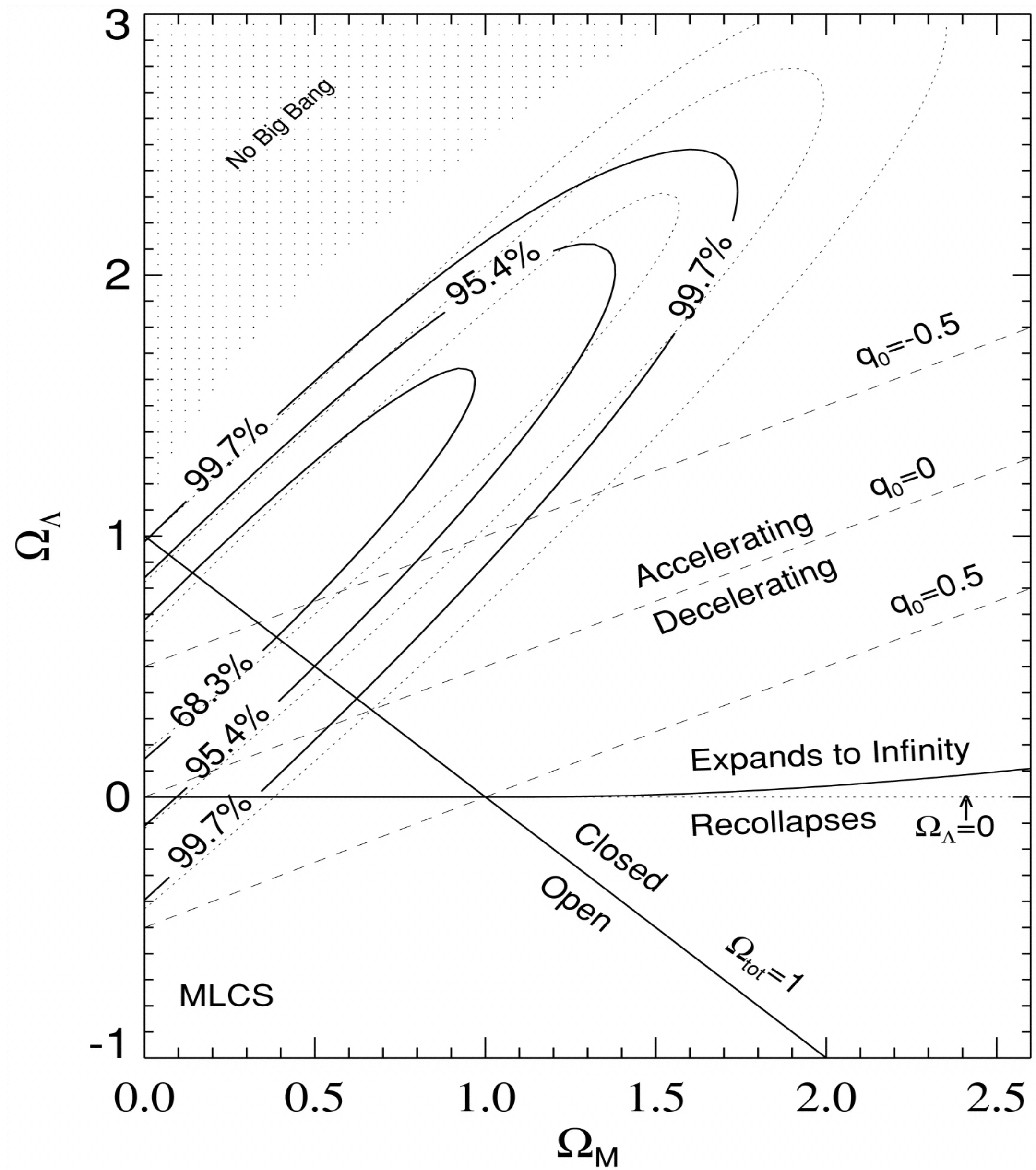
- **Other evidence** - the CMB anisotropy spectrum, baryon acoustic oscillations, structure formation studies, weak lensing maps, etc...

Fitting supernova data

Goal: given the redshifts and luminosity distances for a sample of Type Ia supernovae, learn about the Universe

Method: fit $D_L(z)$ to that of a chosen model with variable parameters

Homogeneous (FRW) model with a matter density Ω_M and a cosmological constant density Ω_Λ



Riess *et al.*, 1998

The "fitting problem"

Einstein's equations are nonlinear, and therefore **the operations for averaging and for time evolution do not commute** (Ellis, 1984; 1987). This means that...

• Although our universe in homogeneous in the mean, it will not necessarily have the same time evolution as the corresponding FRW model

• Our procedure of fitting supernova data to FRW models **introduces systematic errors, which affect the inferred properties of our Universe**

Goal: find how this affects our assessment of the acceleration of the Universe. **Could we be tricked???**

Effects of inhomogeneity

Inhomogeneity affects observations in many ways:

- Gravitational redshifts
- Doppler shifts
- Gravitational lensing
- Time delays
- Integrated Sachs-Wolfe effect, etc...

noise and **systematic effects**

Could any of these systematically affect supernova data in a way that could impact our assessment of the acceleration of the Universe?

Example: weak gravitational lensing

Holz and Wald (1998) calculated the statistical distribution for the magnification of images due to gravitational lensing with a Monte Carlo simulation to find:

• Weak lensing along the line of sight affects apparent luminosities in a **non-Gaussian** fashion

• **The dimming of a source is more likely than a brightening**

→ **a problem for small sample sizes**

• However, this is not sufficient to explain the SN data

...and this effect is negligible for large sample sizes.

Finding the size of the effects of inhomogeneity

There have been two general methods for exploring the effects of inhomogeneity:

(1) Compute **spatially averaged quantities**, as in the Buchert equations

(2) Compute **observable quantities**, such as the redshifts and luminosity distances of supernovae

→ **The dark energy problem originated with observations**, and so we will choose the second option, to find what an observer would see

Toy models

Lemaître-Tolman-Bondi (LTB) models:
inhomogeneous but still spherically symmetric, containing only dust

- **Unnatural** – we must live at the center of the Universe so as to preserve the isotropy of the CMB

- **Useful** nonetheless as toy models for the study of the fitting problem

- Can we find a model based on GR and containing only dust, where there is an apparent acceleration? **Yes...**

LTB model basics

- **The metric**: spherical symmetry (Bondi, 1947)

$$ds^2 = -dt^2 + \frac{R'^2(r,t)}{1-k(r)r^2} dr^2 + R^2(r,t)(d\theta^2 + \sin^2\theta \, d\phi^2)$$

- **The special flat case** (k=0): Einstein's equations with the stress-energy tensor of dust admit solutions of the form

$$R(r,t) \propto r\left[t - t_0(r)\right]^{2/3}$$

- **The luminosity distance:**

$$D_L(z) = (1+z)^2 R$$

(where z and R are evaluated along radially inward moving light rays)

Assessing acceleration

Consider **fitting** a measured and angle-averaged luminosity distance-redshift relation to that of a flat FRW universe. Then we can find the **effective Hubble rate**

$$H(z) = \left[\frac{d}{dz} \left(\frac{D_L}{1+z} \right) \right]^{-1}$$

and the **effective deceleration parameter**

$$q(z) = -1 + \left[\frac{1+z}{H} \right] \frac{dH}{dz}$$

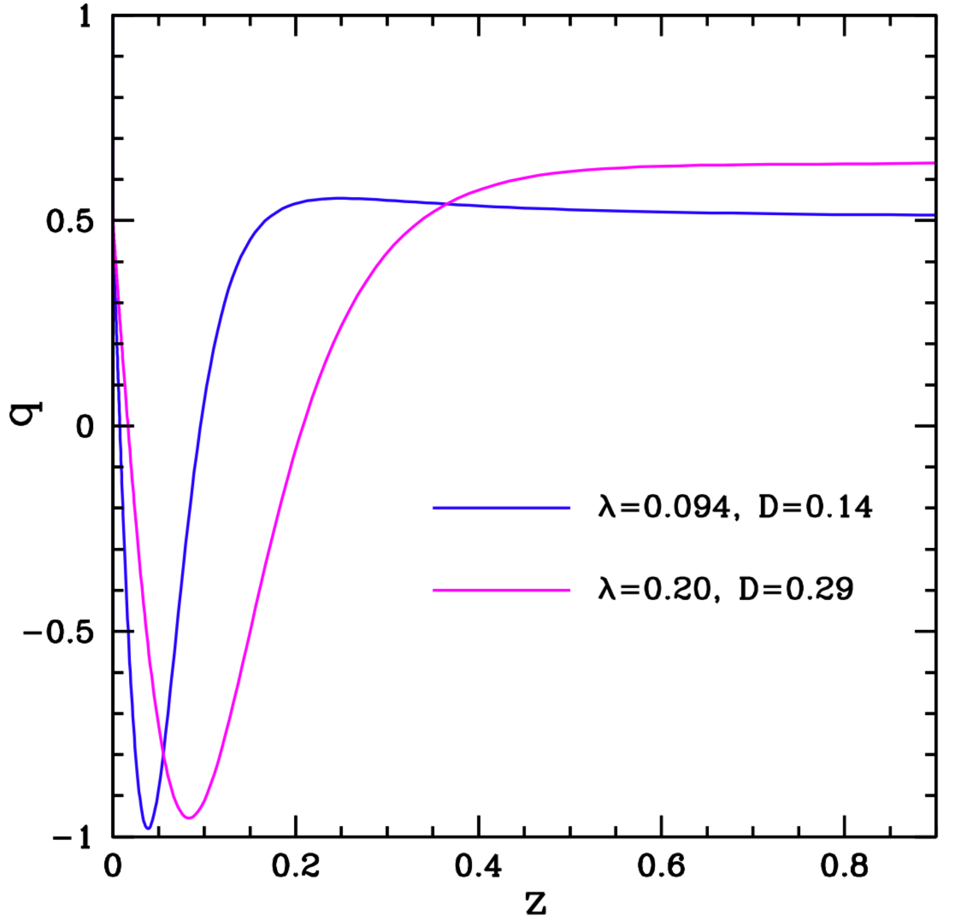
Combing these, we find

$$q(z) = 1 - \frac{D_L''(1+z)^2}{D_L'(1+z) - D_L}$$

Is apparent acceleration possible?

By appropriately choosing an LTB model, we can find ones that have a negative q(z) for nonzero redshift (AV, Flanagan, and Wasserman, 2006):

$$t_0(r) = \frac{-\lambda r^2}{r^2 + D^2}$$



Does not match the observations, **BUT** it shows that it is possible to obtain q(z)<0 **without dark energy or modified gravity**

Garfinkle (2006) claims to succeed in matching the supernova observations with an LTB model.

Perturbative calculations

- There are many reasons to believe that LTB models are **unrealistic** (but there are ways to rule them out too)

- In a more realistic perturbative scenario, assuming that density perturbations are a random process, the systematic corrections due to large scale inhomogeneity are fundamentally **nonlinear**

- **...**and so naively one would expect the (systematic) effect on the deceleration parameter and the inferred cosmological constant to be **negligible**

Small quantities

- **Density perturbations**

$$\delta = \frac{\delta \rho}{\rho} \ll 1$$

- The **perturbation wavelength**

$$\frac{perturbation\, wavelength}{horizon\, scale} \ll 1$$

- The **velocities** v_{pec}/c (peculiar velocity) and v_H/c (Hubble flow velocity)

Method

We choose the **unperturbed background model** to be

- matter dominated
- flat
- and with gravity via GR

Given this background model, we perturb it and

- **Calculate** the luminosity distance and the redshift along the past light cone of the observer, to second order in δ and to third order in v_{pec}/c and v_H/c
- **Combine** these to find $D_L(z)$
- **Fit** to a homogeneous model to find the best fit Ω_Λ

Luminosity distance perturbation

- We combine the perturbed redshift and luminosity distance expressions

- Then we **ensemble and viewing angle average**

⇒ D_L has corrections of order 10^{-5} x $D_{L(0)}$ at z ~ 0.1 (as expected)

- The leading order perturbation to $D_L(z)$ **depends on the two point velocity correlation function**:

$$\Delta D_L(z) = -\frac{1}{3}\frac{d}{dz}\left\langle \vec{n}\cdot\vec{v}(0)\,\vec{n}\cdot\vec{v}(z)\right\rangle + \ldots$$

(n is a unit vector that defines the viewing direction)

The full result before averaging

$$
\begin{aligned}
D_L(z,\theta,\phi) \approx \frac{(1+z)^2}{H_0}\Bigg\{ & z - \frac{7}{4}z^2 + \frac{19}{8}z^3 + \left(-1 + \frac{5}{2}z - \frac{33}{8}z^2\right)\left(v^r_{s(1)} + v^r_{s(2)}\right) \\
& + \left(1 - \frac{5}{2}z + \frac{29}{8}z^2\right)\left(v^r_{o(1)} + v^r_{o(2)}\right) + \left(1 - \frac{1}{2}z\right)\left(\Phi_{s(1)} + \Phi_{s(2)}\right) \\
& + \left(-1 + \frac{5}{2}z\right)\left(\Phi_{o(1)} + \Phi_{o(2)}\right) + \left(\frac{1}{2} - \frac{5}{4}z\right)\left(v^2_{o(1)} - v^2_{s(1)}\right) \\
& + \left(-\frac{7}{4} + \frac{29}{8}z\right)\left(v^r_{o(1)}\right)^2 + \left(-\frac{3}{4} + \frac{9}{8}z\right)\left(v^r_{s(1)}\right)^2 \\
& + \left(\frac{5}{2} - \frac{23}{4}z\right) v^r_{o(1)} v^r_{s(1)} + \frac{1}{2} v^r_{o(1)}\left(\Phi_{o(1)} - \Phi_{s(1)}\right) + \frac{5}{2} v^r_{s(1)}\left(\Phi_{s(1)} - \Phi_{o(1)}\right) \\
& - x^i_{(1)} v^r_{s(1),i} + \int_0^r \left(\dot{\Phi}_{(1)} + \dot{\Phi}_{(2)}\right) dr' + \left(v_{\theta(1)}k^\theta_{(1)} + v_{\phi(1)}k^\phi_{(1)}\right)_s \\
& - \left(v_{\theta(1)}k^\theta_{(1)} + v_{\phi(1)}k^\phi_{(1)}\right)_o - \left(z + v^r_{o(1)} - v^r_{s(1)}\right)\int_0^r \frac{dr'}{r'^2}\int_0^{r'} (r'')^2\,\nabla^2\Phi_{(1)}\,dr'' \\
& - z\int_0^r \frac{dr'}{r'^2}\int_0^{r'} (r'')^2\,\nabla^2\Phi_{(2)}\,dr'' \\
& + \left(v^r_{s(1)} - v^r_{o(1)}\right)\left[z\frac{d}{dz}\int_0^r \frac{dr'}{r'^2}\int_0^{r'} (r'')^2\,\nabla^2\Phi_{(1)}\,dr'' - 2\frac{d}{dz}\int_0^r \dot{\Phi}_{(1)}\,dr'\right] \\
& + \left[\Phi_{s(1)} - \Phi_{o(1)} + \left(1 + \frac{1}{2}z\right)\left(v^r_{o(1)} - v^r_{s(1)}\right)\right]\frac{d}{dz}\Phi_{s(1)} \\
& - \Bigg[2\int_0^r \dot{\Phi}_{(1)}\,dr' + \left(-1 + \frac{3}{2}z\right)\Phi_{o(1)} + \left(1 + \frac{1}{2}z\right)\Phi_{s(1)} \\
& + \left(1 - \frac{3}{2}z + \frac{13}{8}z^2\right) v^r_{o(1)} + \left(-1 + \frac{3}{2}z - \frac{17}{8}z^2\right) v^r_{s(1)}\Bigg] \\
& \times \frac{d}{dz} v^r_{s(1)}\Bigg\},
\end{aligned}
\qquad (A.3)
$$

Background

Doppler effect

Gravitational redshifts

Doppler effect

Integrated Sachs-Wolfe

Gravitational lensing

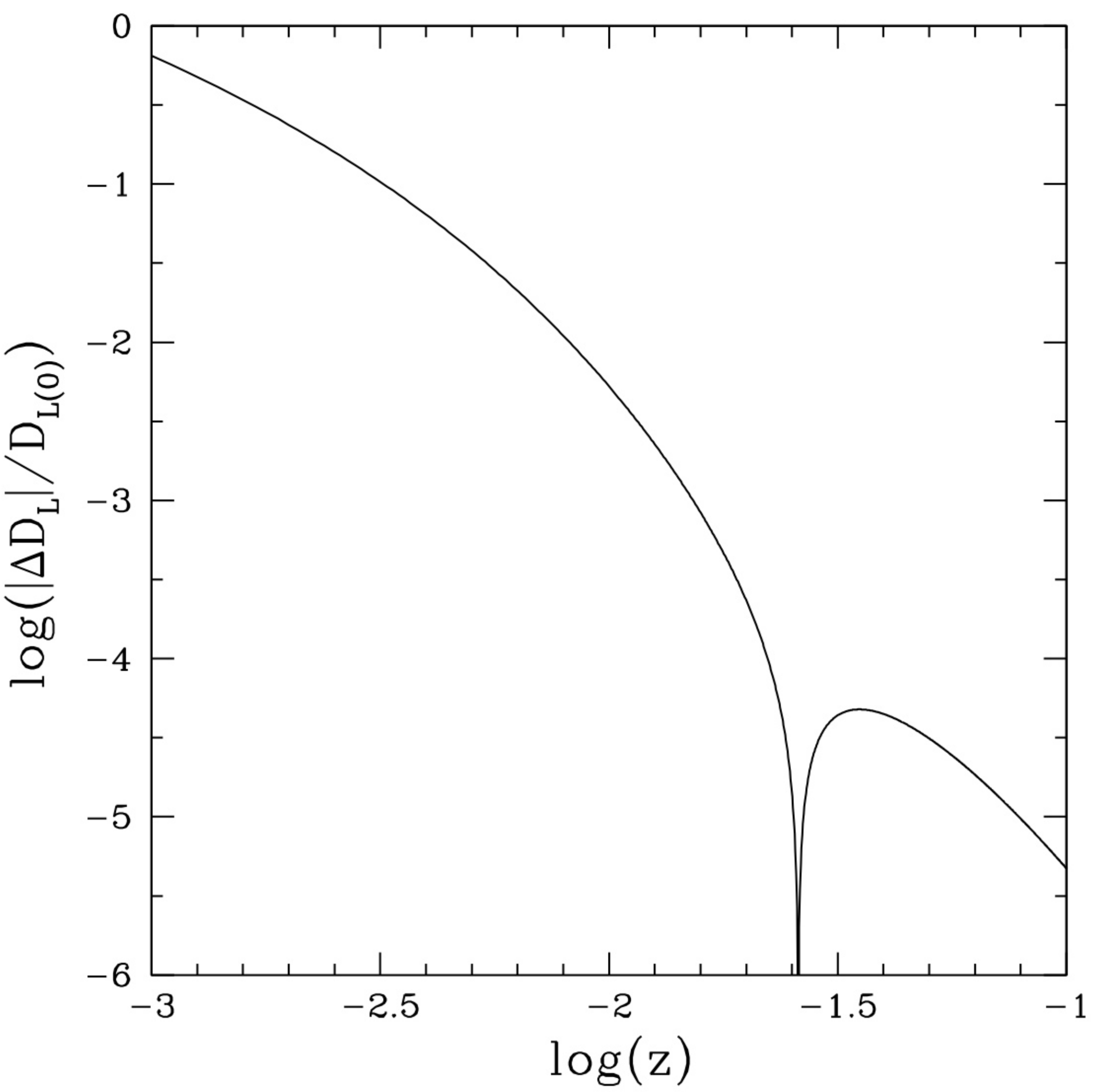
Lowest order perturbation

Problem at small redshifts

This “perturbation” is not actually a perturbation for **very small redshifts**:

i.e. for very small redshifts, **peculiar velocities can become larger than redshifts** →

...we must impose a lower redshift cutoff when we do our fits...



The best fit cosmological constant

We fit the perturbed $D_L(z)$...

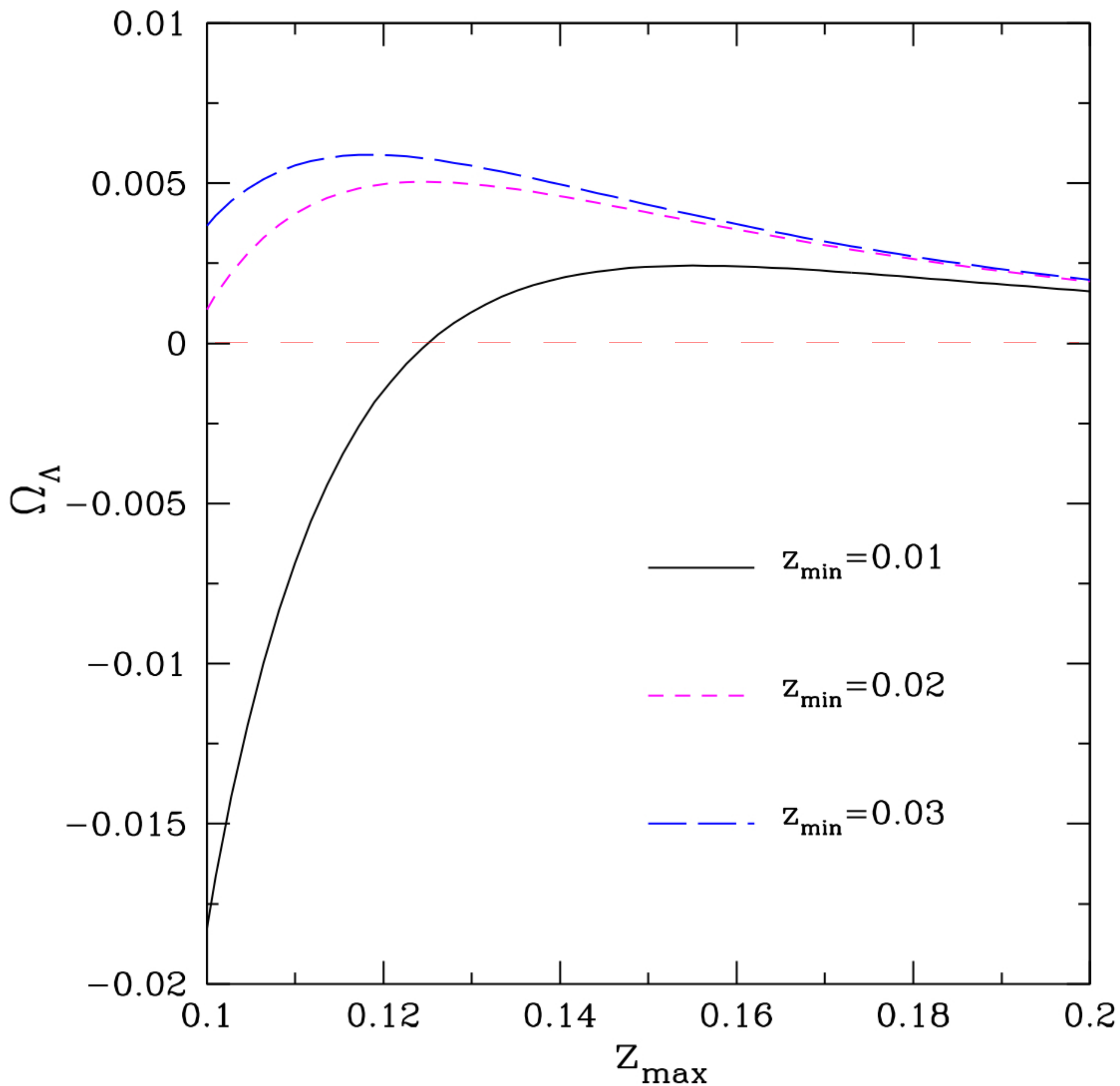
- to that of a homogeneous FRW model with Ω_M and Ω_Λ
- **using a chi squared method**
- **assuming a very large sample size** and imposing a sample redshift range from z_{min} to z_{max}

Results - best-fit models **depend on the redshift range**:

$$z_{min}=0.01,\ \ z_{max}=0.1\ \ \Rightarrow\ \ \Omega_\Lambda\approx-0.018$$

$$z_{min}=0.03,\ \ z_{max}=0.2\ \ \Rightarrow\ \ \Omega_\Lambda\approx2.0\times10^{-3}$$

Dependence on catalog limits



Actual model value

Variance

The $D_L(z)$ perturbation **linear** in the peculiar velocity:

- does not contribute to the ensemble averaged result
- **does** contribute to the **variance** in the best fit parameters

Two terms: N-dependent → Cosmic Variance →

$$\sigma_\Lambda^2 \approx \frac{100}{N}\left(\frac{z_{max}}{0.2}\right)^{-6} + 0.03\left(\frac{z_{max}}{0.2}\right)^{-8}$$

(Neill, Hudson, and Conley, 2007, computed this linear velocity error for actual supernova data)

Conclusions

- We need to explore all errors that could affect the measurement and interpretation of the luminosity distance-redshift relation.

- **The fitting effect has a nonzero impact on the determination of cosmological parameters.**

- **Our results point to the systematic effect being very small (but larger for closer supernovae)**

- **However...we find that cosmological parameter fitting with nearby supernovae can lead to a large variance**

➡ need further study in more realistic scenario