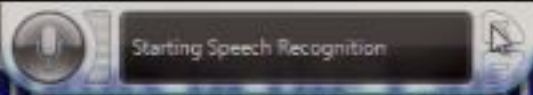


Title: Random Observations in the Landscape

Date: 2007-05-20 15:00:00

Abstract:

Random Observations in the Landscape



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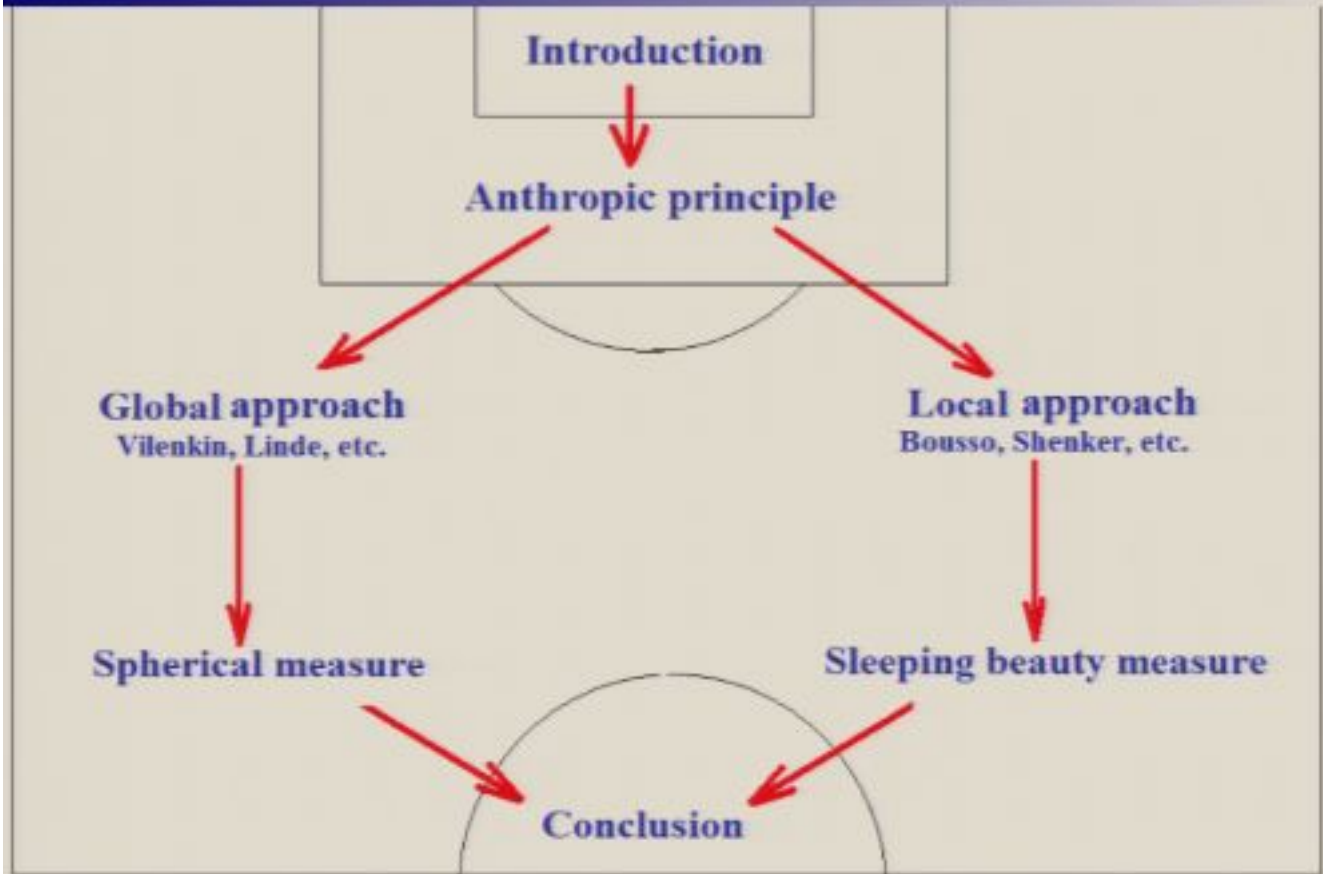
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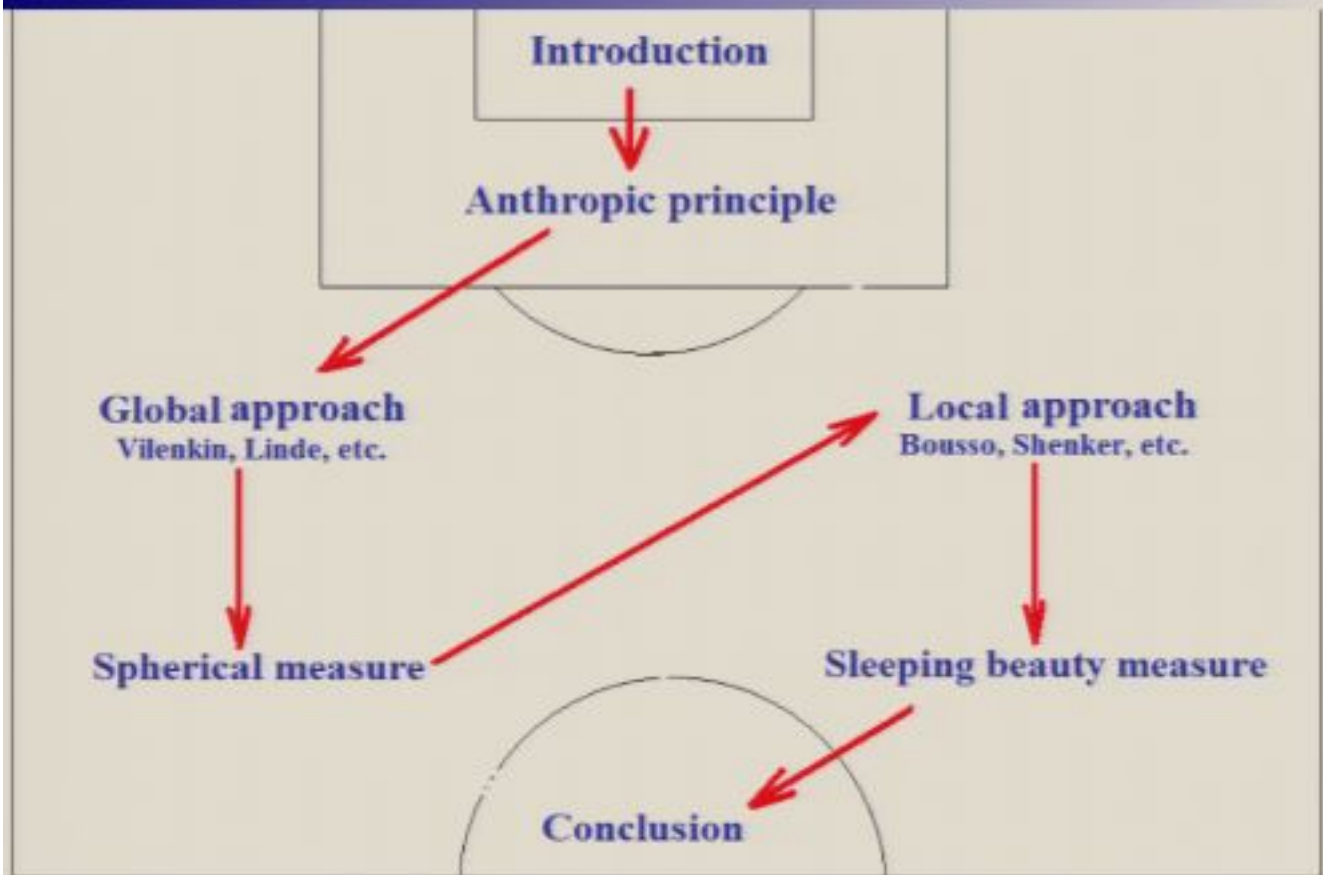
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Outline:



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Inflation:

- explains homogeneity, isotropy, flatness, etc.
[Starobinsky (1980), Guth (1981), Linde(1982), ...]
- “generically” is eternal
[Vilenkin (1983), Linde (1986), ...]
- the “measure” problem
[Linde, Linde, Mezhlumian (1996), Vanchurin, Vilenkin, Winitzki(2000), ...]

String theory:

- huge number of distinct vacua
 $N \sim 10^{500} - 10^{1000}$
- landscape picture of universe
[Bousso & Polchinski (2000), Susskind (2003), Douglas (2003), ...]

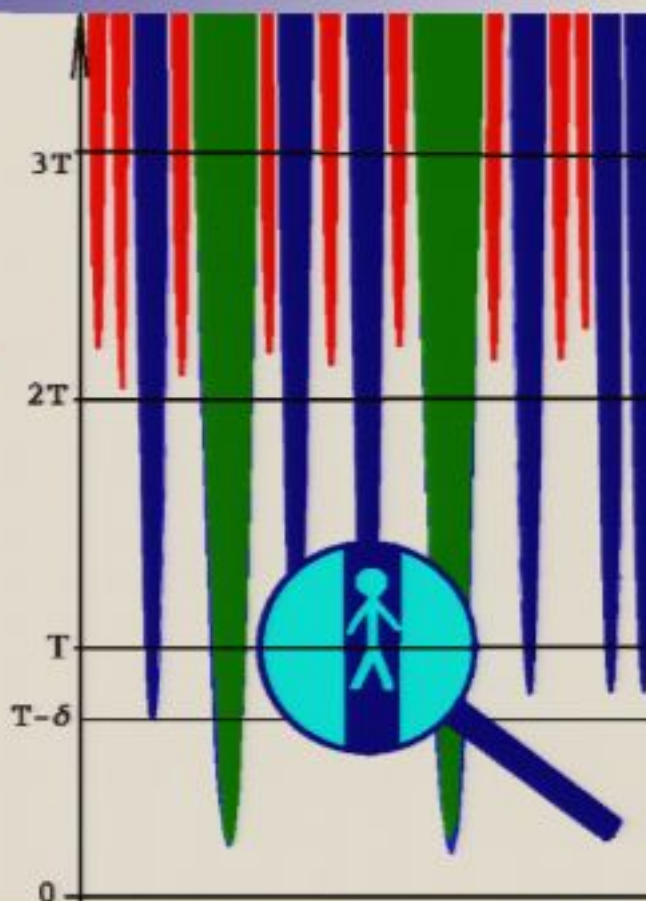
Landscape + Inflation => Eternal Inflation

Anthropic principle

A paradox of eternal inflation:

- ⇒ Semi-eternal inflation
- ⇒ We are at some finite distance T
- ⇒ Slice the space-time:
 $(0, T]$, $(T, 2T]$, $(2T, 3T]$, ...

Why are we so atypical? Why do we live so close to the origin? Why T is so small?



Anthropic principle

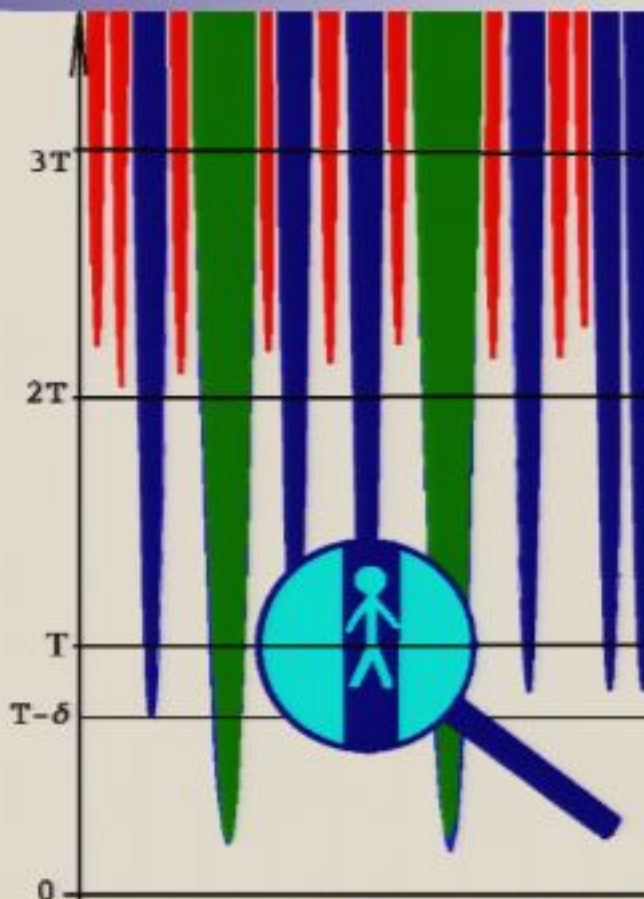
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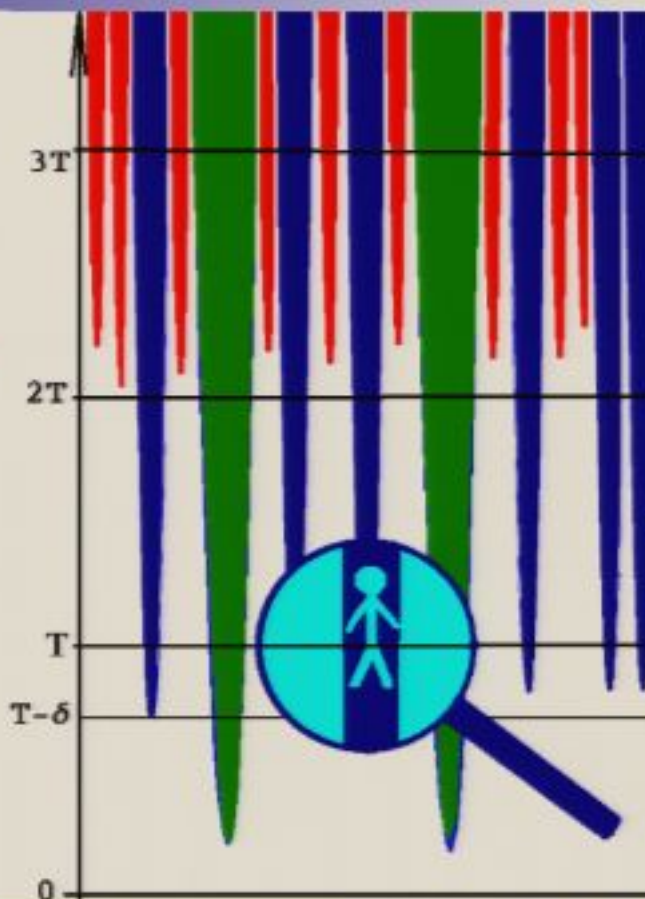
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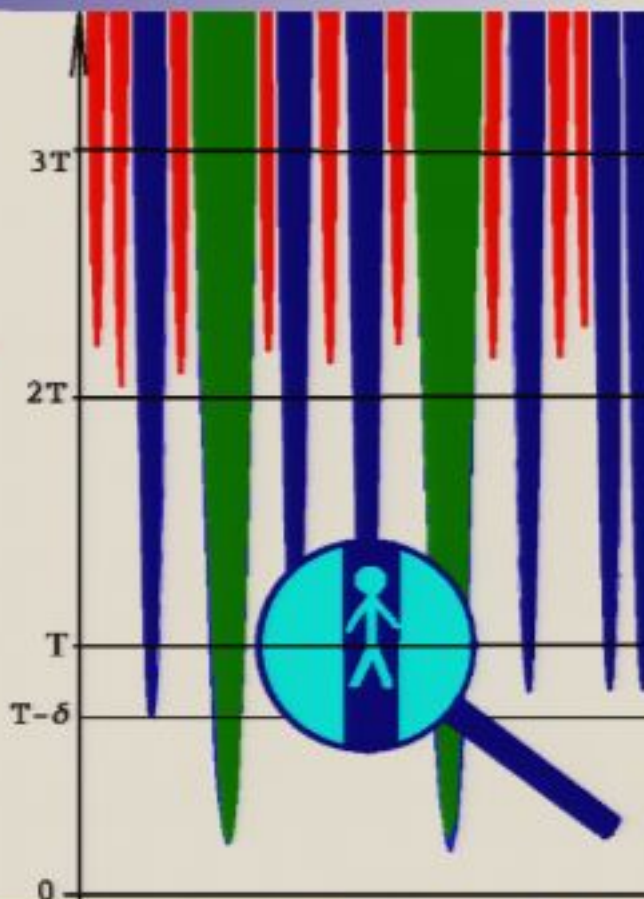
“All observers have the same problem.”

Mukhanov:

“Inflation is not semi-eternal.”

Myself:

“Typical observers do not exist.”



Anthropic principle (cont'd)

Mediocrity principle:

“We observe, what a typical (random) observer would observe”

1) Major problem: It is not possible to pick a random object from a countable set!

Consider a set of Natural numbers: $\{1, 2, 3, \dots\}$. Let $P(n)$ be the probability of choosing n .

If $P(n) = \text{const}$ for all n , then

$$\sum_{n=1}^{\infty} P(n) = \begin{cases} 0 & \text{if const} = 0 \\ \infty & \text{if const} \neq 0 \end{cases}$$

On the other hand $\sum_{n=1}^{\infty} P(n)$ must be normalized to 1. A random observer is ill-defined.

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2) Minor problem: Not interested in observers, but in observations, which is not always the same.

Possible “solutions”:

1) Define a generalized random observer (or observation), as a random observer (or observation) out of the first n observers (or observations), from an *unbiased* series of observers, for large enough n .

2) Define a generalized anthropic principle:

“We observe a (generalized) random observation.”

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Two approaches to define a generalized observation:

1) Global approach: Choose a single realization of initial conditions [Vilenkin, Linde, ...]

2) Local approach: Consider many worldlines (one for each realization of IC) [Bousso, ...]

Global approach

Consider three stochastic processes that generate:

- 1) **Geometry:** $G^{3,1}$ (eternally inflating space-time)
- 2) **Content:** $C : G^{3,1} \rightarrow \mathbb{R}$ (varying fundamental constants (e.g. Λ)
- 3) **Observations:** $O : \mathbb{N} \rightarrow G^{3,1} \times G^{3,1}$ (maybe correlated with G and C)

Questions: What is a (generalized) random observation?

Geometry:



Geometry+Content:



Geometry+Content+Observers:



Global approach (cont'd)

Euclidean space: Consider 2D painted in black and white:

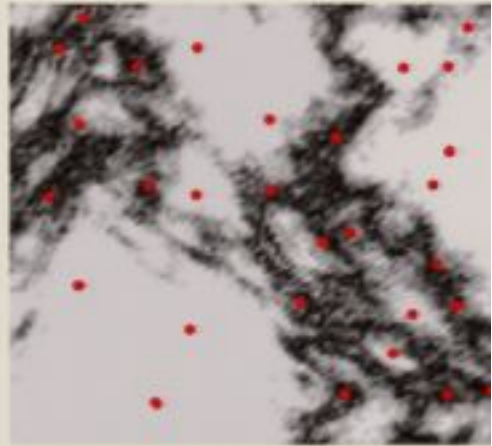
$$C: \mathbb{R}^2 \rightarrow \{0, 1\}$$

Define an infinite set of isolated points (red dots):

$$O: \mathbb{N} \rightarrow \mathbb{R}^2$$

What is the probability of a randomly chosen point to be white?

- Not known.
- Not known, even if C maps everything to $\{0\}$



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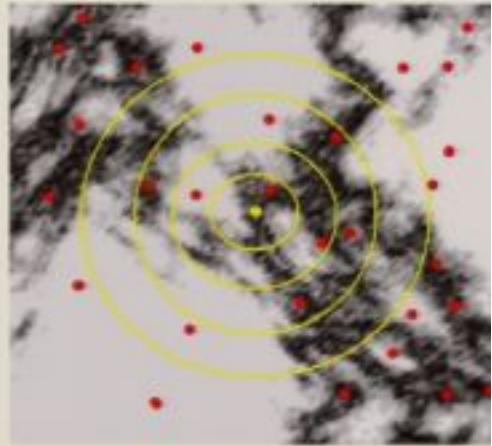
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If the limit exists, then one should also prove that it is unique.



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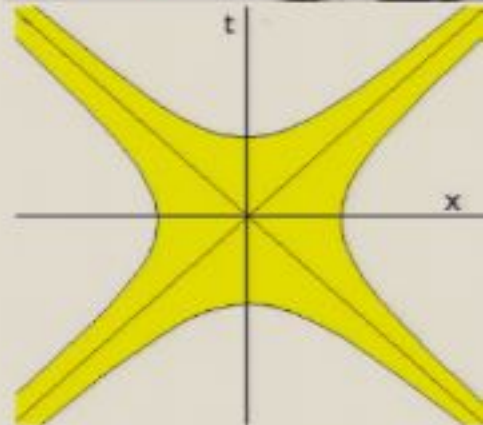
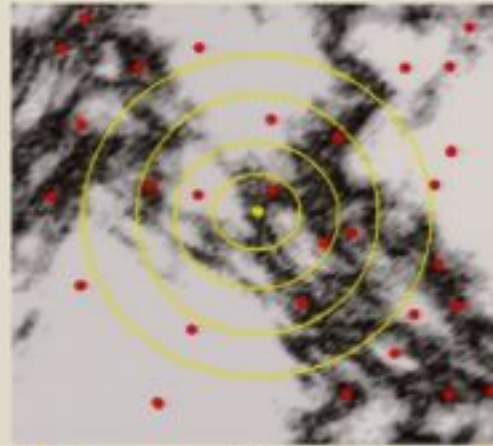
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Volume on finite proper distance is infinite
Spherical ordering of observers is ill-defined



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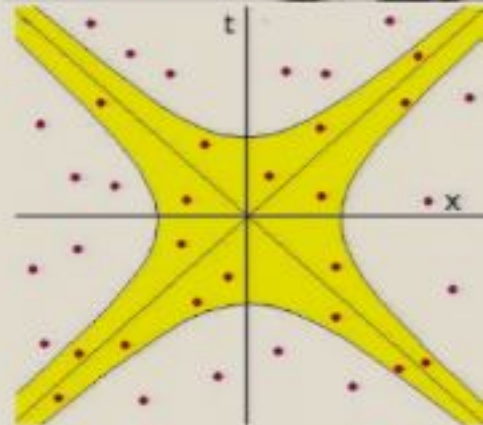
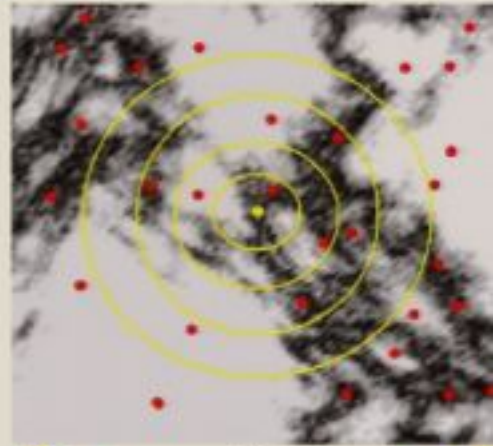
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Eternal inflation:

- 1+1D landscape models with at least one AdS vacua
- generic time-like geodesic has a finite proper length
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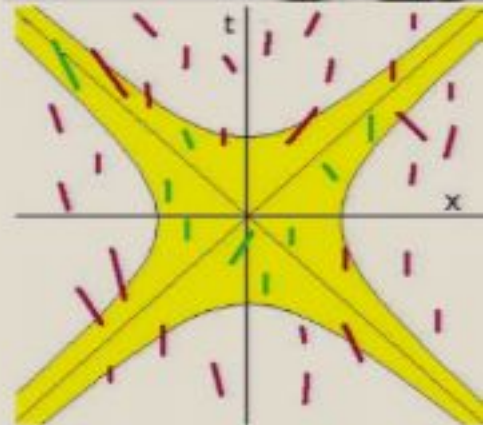
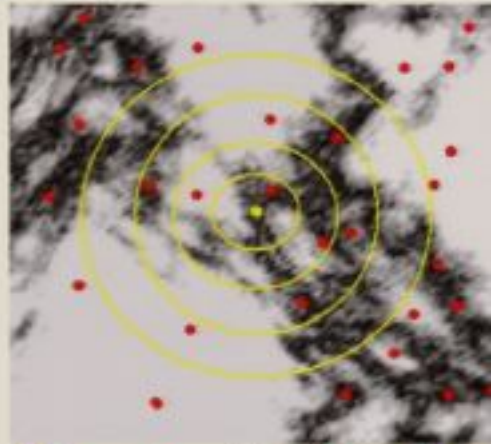
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In 3+1D the spherical ordering of observations could be well defined if we require a finite time Δ for an observation!



Spherical measure

In a pure de-Sitter:

$$ds^2 = -dt^2 + e^{2Ht}(dr^2 + r^2 d\Omega^2)$$

Tunneling rate per unit time t is given by:

$$\kappa_{ij} = \frac{4\pi}{3} H_j^3 I_{ij}$$

The bubbles nucleation rate is

$$\Gamma_{ij} = A_{ij} e^{-I_{ij} - S_j}$$

where

$$S_j = \pi H_j^{-2}$$

is the Gibbons-Hawking entropy, I_{ij} is the instanton action and A_{ij} is a prefactor.

Matrix of probability currents:

$$M_{ij} = \kappa_{ij} - \delta_{ij} \sum_{r=1}^N \kappa_{rj}$$

The magnitude of κ_{ij} is the same for all geodesic observers, but the tunneling rate per unit proper time τ varies [Garriga, Guth, Vilenkin (2006)]:

$$M_{ij}(v) = (\kappa_{ij} - \delta_{ij} \sum_{r=1}^N \kappa_{rj}) (1 - v^2)^{-\frac{1}{2}}$$

Spherical measure (cont'd)

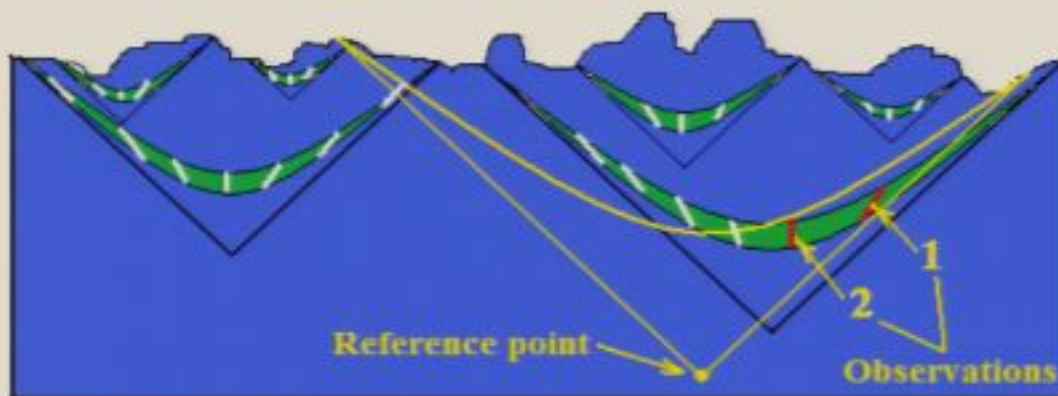
Is the spherical measure well defined?

- 1) in 1+1D the 2-volume is finite, thus the procedure is well defined
- 2) in 3+1D and fractal dimension of eternal set less than 2 the 4-volume is also finite
- 3) in 3+1D and fractal dimension of eternal set greater or equal to 2:
 - a) the spherical ordering of observers is ill-defined
 - b) the spherical ordering of observations

Write down the evolution equation: $\frac{d\vec{p}^{\text{vol}}}{dt} = (M(v) + 3H)\vec{p}^{\text{vol}}$

For large enough velocity, the largest eigenvalue is negative: $\vec{p}^{\text{vol}}(t) = \vec{s} e^{-\lambda t}$

Inflation is not eternal for highly boosted observers!



Spherical measure (cont'd)

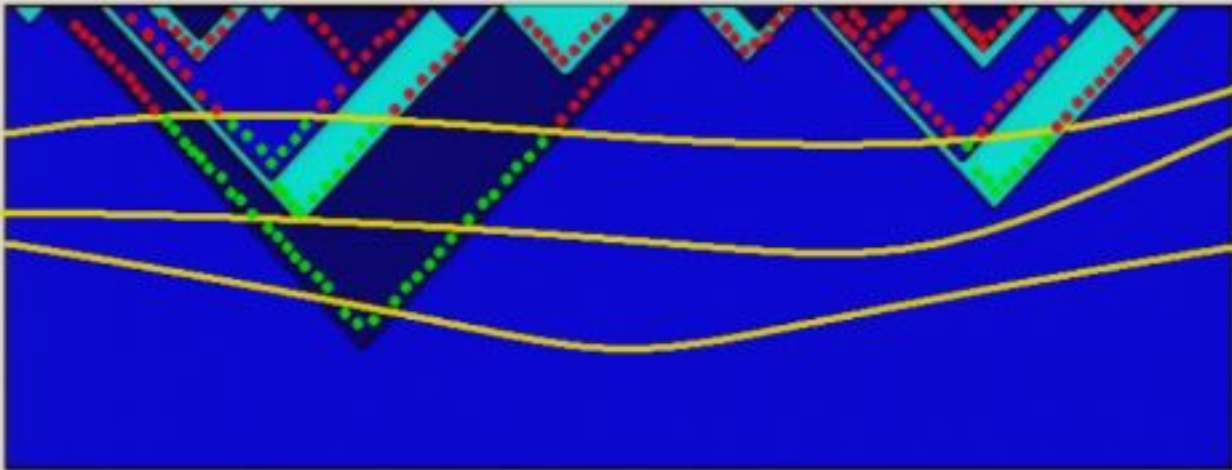
The distribution of observations must be counted in three steps (volume => bubbles => observations)

1) Volume distribution:
$$\frac{d\bar{p}^{\text{vol}}}{dt} = M\bar{p}^{\text{vol}} + 3H\bar{p}^{\text{vol}} \Rightarrow \bar{p}^{\text{vol}}(t) = \bar{s} e^{3Ht}$$

Total count of Boltzmann observations:
$$p_i^{\text{obs}}(t) = \frac{1}{3(H)} b_i s_i e^{3Ht}$$

(b_i is the probability for Boltzmann civilizations to form per unit time per unit volume)

2) Frequency of bubbles:
$$\bar{p}^{\text{freq}} = \kappa \bar{p}^{\text{vol}} \Rightarrow \bar{p}^{\text{freq}}(t) = \kappa \bar{s} e^{3Ht}$$



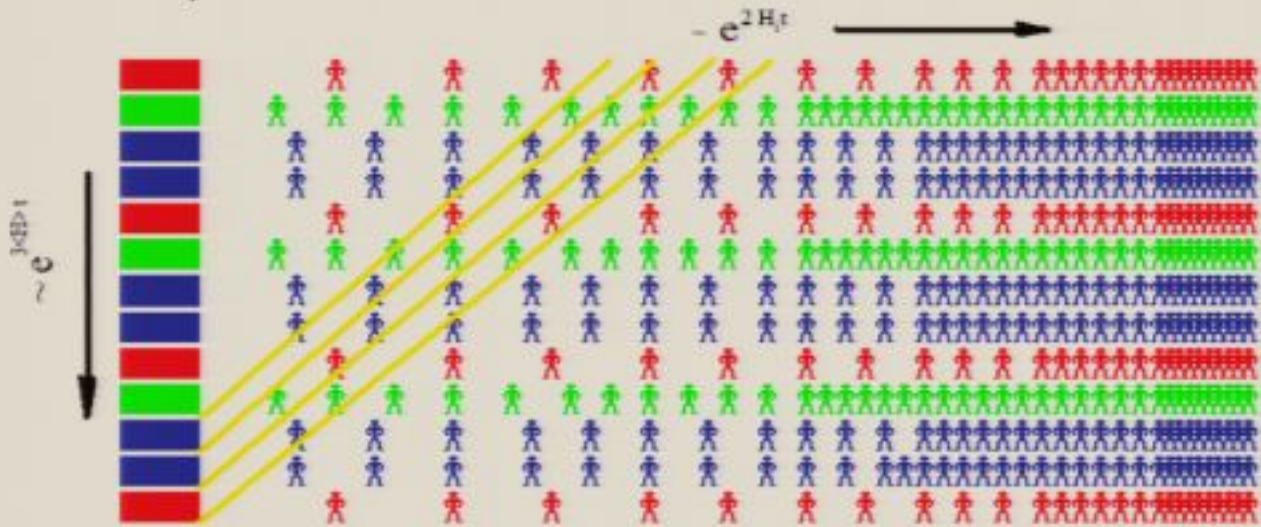
Spherical measure (cont'd)

3) Distribution of observations:

$$p_i^{obs}(t) = \int_0^t \int_0^{t_0} \sum_{r=1}^N \kappa_r s_r e^{3(H_r)t_0} \frac{4\pi l_i e^{3N_i/2H_r \tau}}{H_r^2} d\tau dt_0$$

where

- l_i is the probability for life to evolve and to live long enough to perform an observation
- t_i is the time of slow roll
- N_i is the number of e-foldings of slow-roll



Spherical measure (cont'd)

With two assumptions:

- 1) time of slow roll is negligible t_i (in general not true)
- 2) life can only exist in vacua with small cosmological constant: $H_i \ll \langle H \rangle$ for all $i \neq 0$

The total count of observations is given by:

$$P_i^{\text{obs}}(t) = \frac{4\pi l_i e^{3N_i}}{3\langle H \rangle(3\langle H \rangle - 2H_i)} \sum_{r=1}^N \frac{K_{\sigma} S_r}{H_r^2} e^{3\langle H \rangle t}$$

Spherical measure (cont'd)

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One can include the collisions of bubbles, which leads to modifications of $H_i \rightarrow \tilde{H}_i$

The asymptotic distribution of observations:

$$P_i^{\text{obs}} \propto \frac{l_i e^{3N_i}}{3\langle H \rangle - 2\tilde{H}_i} \sum_{r=1}^N \frac{\kappa_{i,r} S_r}{H_r^2}$$

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Anthropic constraints on the landscape:

- 1) $H_i \ll \langle H \rangle$ for all $l_i \neq 0$
- 2) $\sum_{r=1}^N \delta_{i S_r} < \sum_{r=1}^N \frac{4\pi l_i e^{3N_i}}{3\langle H \rangle - 2\tilde{H}_i} \sum_{r=1}^N \frac{\kappa_{\text{obs}} S_r}{H_r^2}$

Local approach

$T_{ij} = \frac{\kappa_g}{\sum_{r=1}^N \kappa_g}$ - relative transition rates

$W_{ij} = \delta_{ij} I_i$ - "Weinberg" matrix

What is the probability to find yourself in a given vacua?

The answer is not unique and depends on

- initial conditions
- ensemble of observers

Treat bubbles equally [Bousso (2006)]:

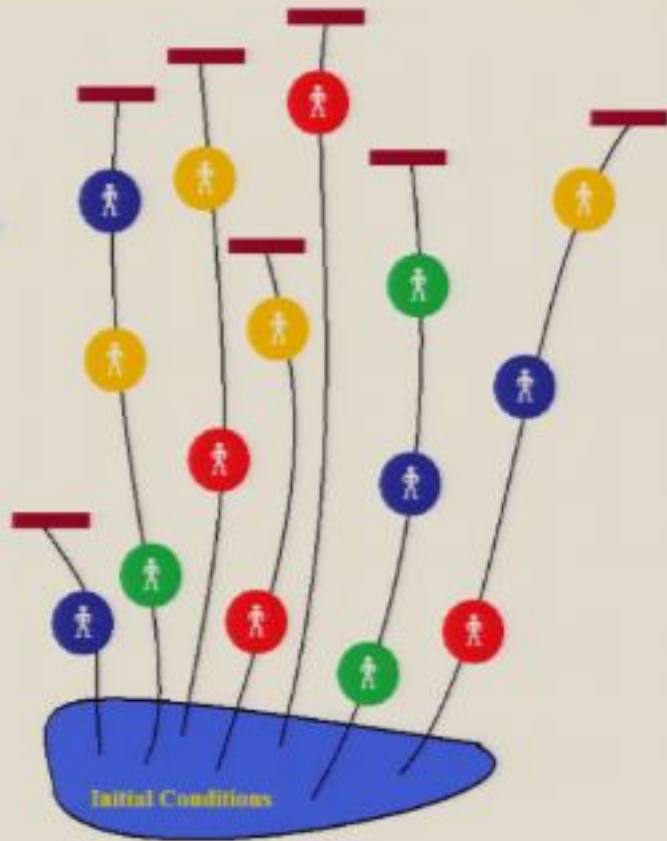
$$P_{\text{bubbles}} \propto W(I-T)^{-1} T p_0$$

Treat geodesics equally:

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Treat observers equally:

$$P_{\text{observers}} \propto N[W(I-T)^{-1} T] p_0$$



The “Sleeping Beauty” problem [Elga & Lewis]:

Beauty is put to sleep.
A fair coin is tossed.

If the coin falls heads:
She is awakened and put to sleep again.

If the coin falls tails:
She is awakened and put to sleep again.
She is administered a memory-erasing drug.
She is awakened and put to sleep again.

She knows all this!

When she awakes,
what should her credence be that the coin fell heads?

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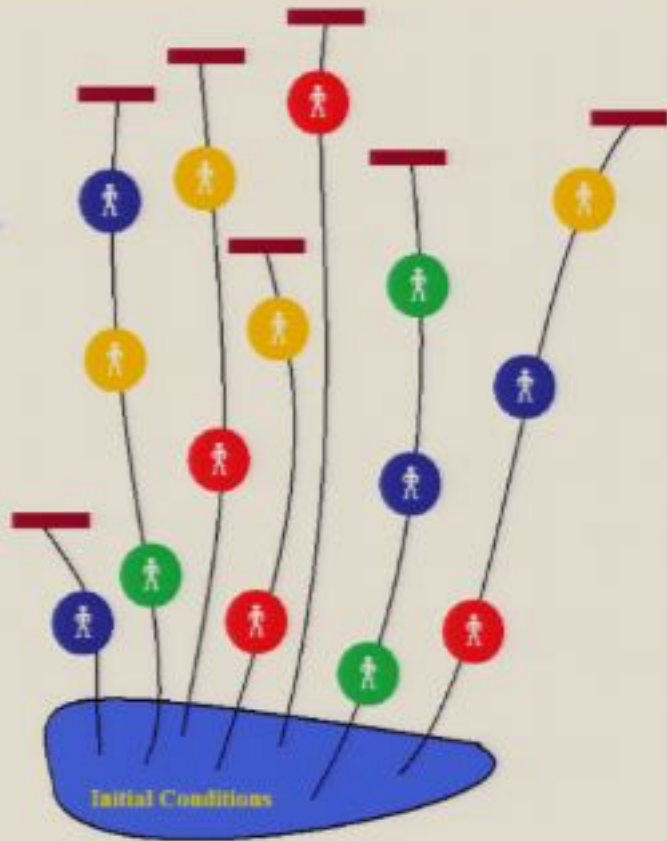
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Elga: $1/3$

Lewis: $1/2$

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Elga: $1/3 \Rightarrow$ Bousso is right $\Rightarrow \quad p \propto W(I-T)^{-1}T p_0$

Lewis: $1/2 \Rightarrow$ I am right $\Rightarrow \quad p \propto N[W(I-T)^{-1}T]p_0$

Conclusion:

Random observers do not exist, or otherwise problems and paradoxes

Generalized anthropic principle:

"We observe a (generalized) random observation"

Three-steps formalism for calculating probabilities:

1) volume distribution \Rightarrow 2) frequency of bubbles \Rightarrow 3) distribution of observations

Spherical measure:

- is well defined ordering of observations for some models
- the only ordering invariant under the Lorentz transformation
- the measure is independent of the choice of a reference point

Local approach:

- depends on the initial conditions
- ambiguous choice of the ensemble

