Title: The DGP Braneworld

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Abstract:

The DGP braneworld

Kazuya Koyama, University of Portsmouth

KK Phys. Rev. D72, 123511,2005 [hep-th/0503191]
KK, R.Maartens JCAP0601, 016, 2006 [astro-ph/0511634]
D. Gorbunov, KK, S. Sibiryakov, Phys. Rev. D73, 044016 2006 [hep-th/0512097]
K.Izumi, KK, T.Tanaka, JHEP 0704, 053, 2007[hep-th/0610282]
KK, F.P. Silva, Phys. Rev. D75, 084040, 2007 [hep-th/0702169]
K.Izumi, KK, O.Pujolas, T.Tanaka to appear soon

DGP brane world

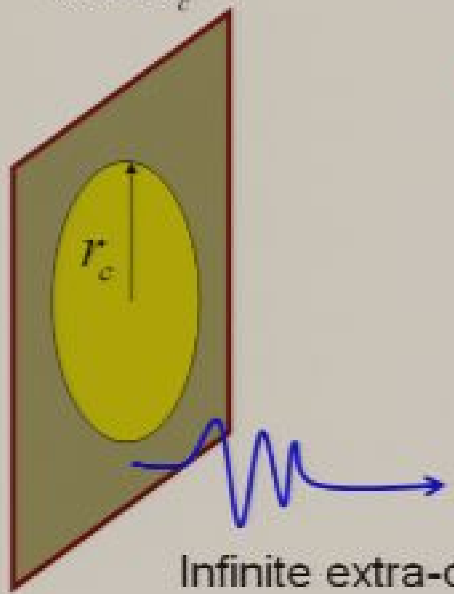
(Dvali, Gabadadze, Porrati)

$$S = \frac{1}{32\pi G r_c}\int d^5x\sqrt{-g_{(5)}}R_{(5)} + \frac{1}{16\pi G}\int d^4x\sqrt{-g}R + \int d^4x\sqrt{-g}L_m$$

- Crossover scale r_c

$r < r_c$ 4D Newtonian gravity

$r > r_c$ 5D Newtonian gravity



Cosmology in DGP model (Deffayet)

- Friedmann equation

$$\frac{H}{r_c} = H^2 - \frac{8\pi G}{3}\rho$$

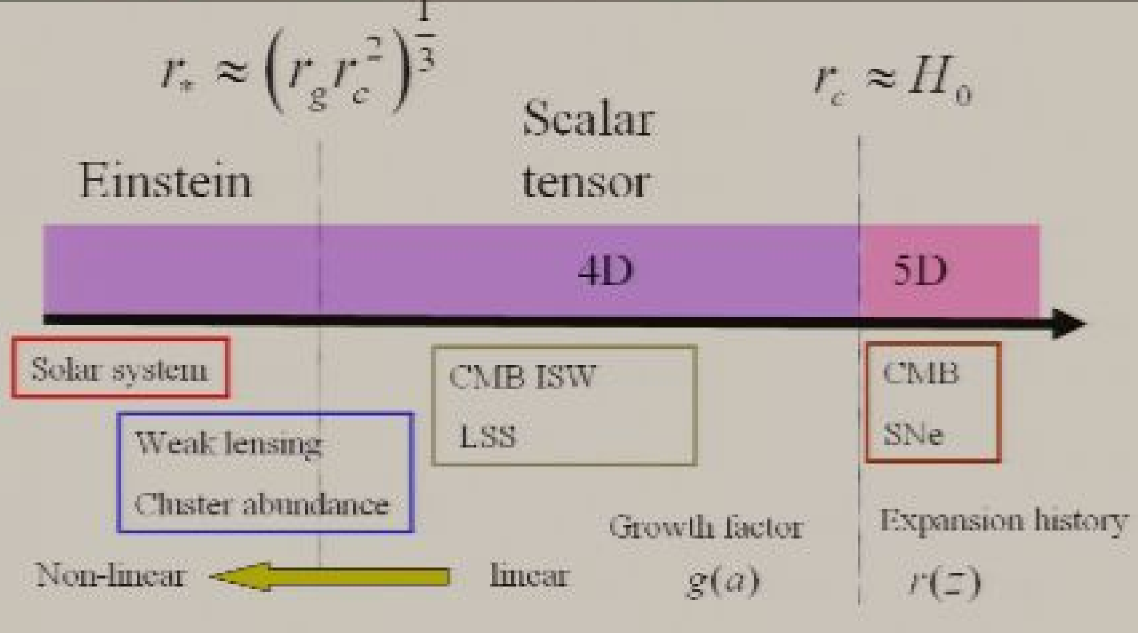
early times $Hr_c >> 1$ 4D Friedmann

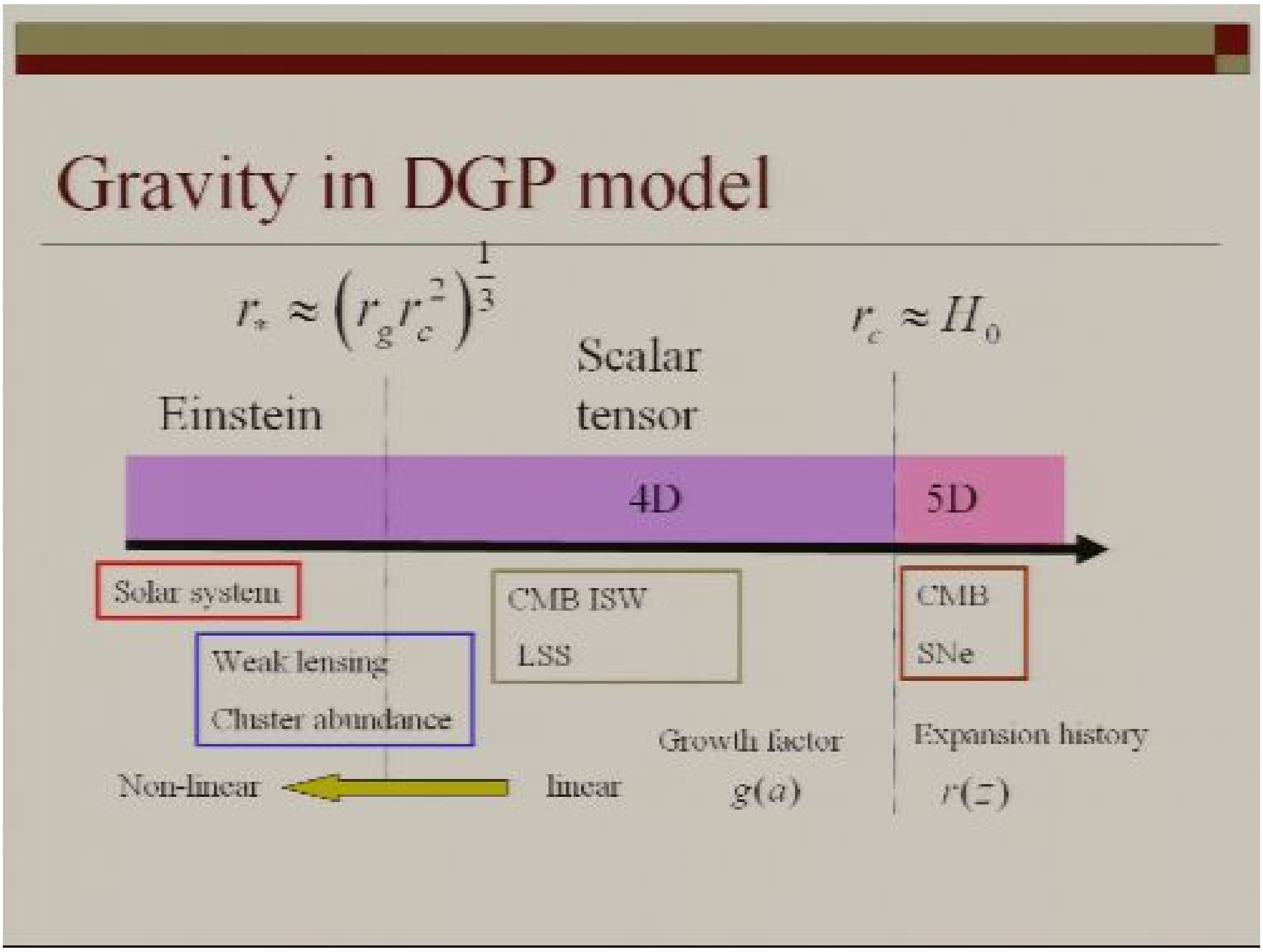
late times $\rho \to 0$ $H \to \frac{1}{r_c}$

As simple as LCDM model

(and as fine-tuned as LCDM $r_c \approx H_0$)

Gravity in DGP model

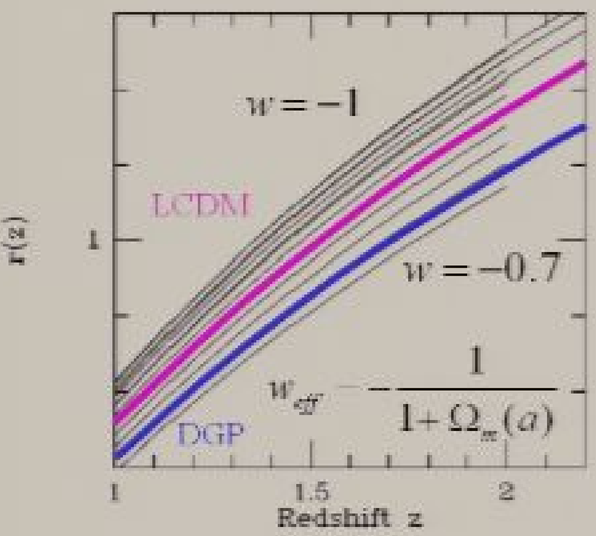


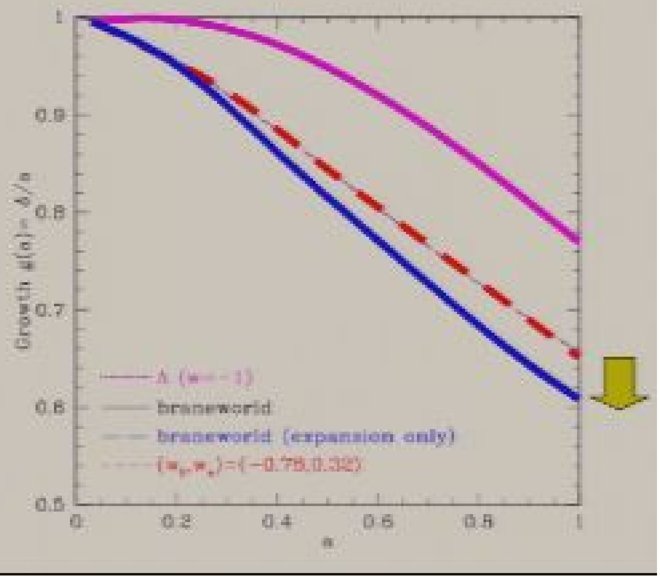

Gravity in DGP model
$r_* \approx \left(r_g r_c^2\right)^{\frac{1}{3}}$
$r_c \approx H_0$
Einstein
Scalar tensor
4D
5D
Solar system
Weak lensing
Cluster abundance
CMB ISW
LSS
CMB
SNe
Growth factor
$g(a)$
Expansion history
$r(z)$
Non-linear
linear

Large scale structure

- Expansion history vs growth rate of structure

$r(z) = \int^{z} dz\, H^{-1}(z)$ $\qquad$ $g(a) = \delta / a \,(= \text{const. for } \Omega_m = 1)$





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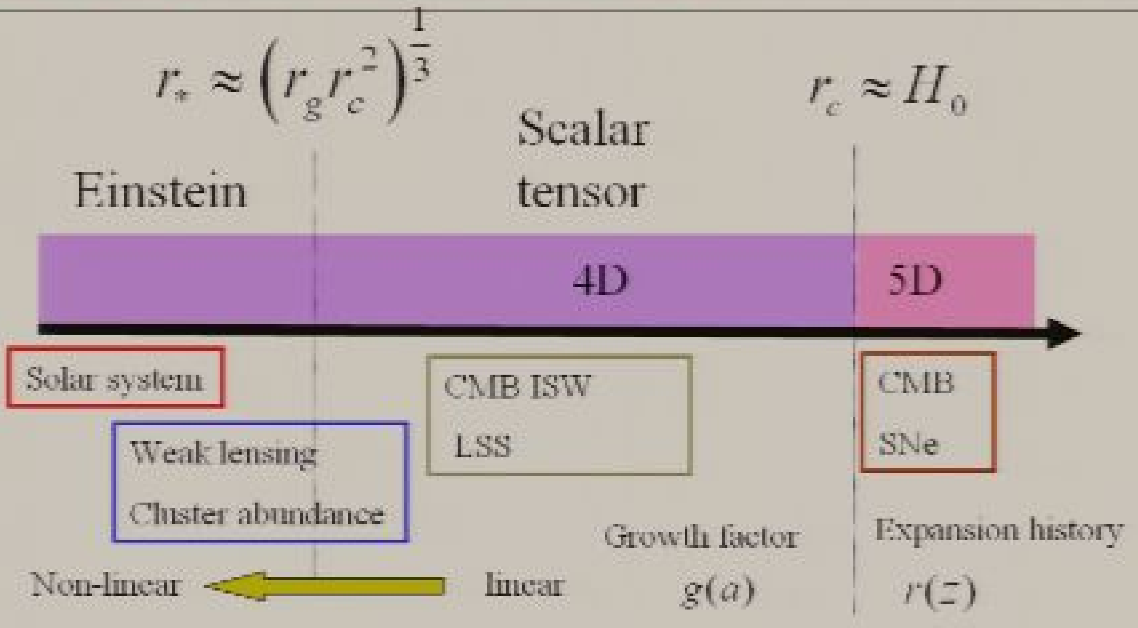

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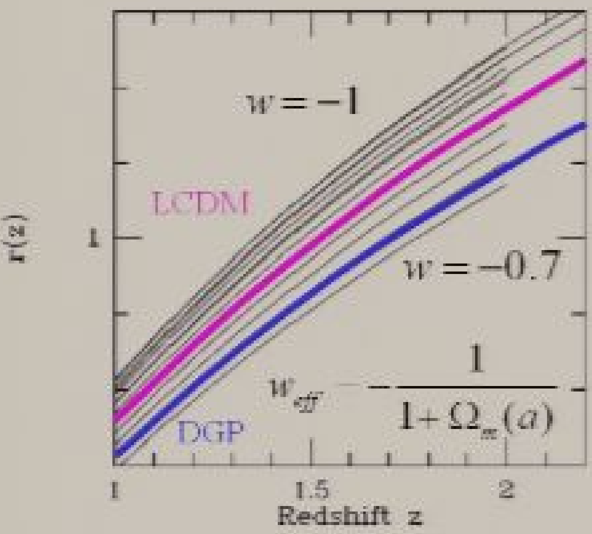
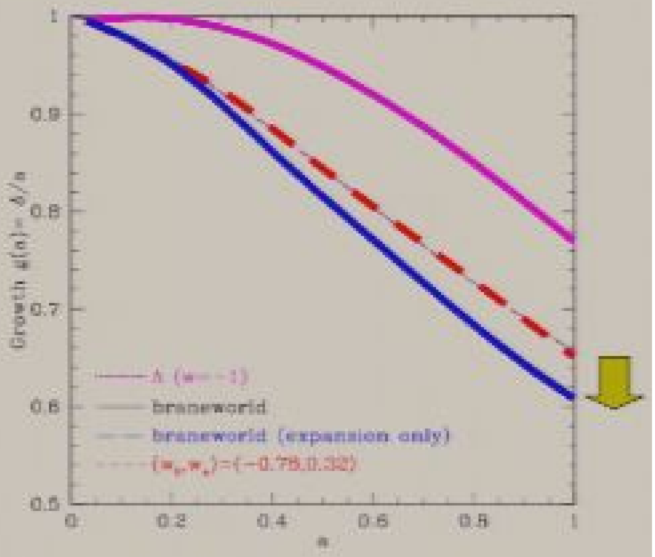




Linear theory

KK and Maartens,
JCAP [astro-ph/0511634]

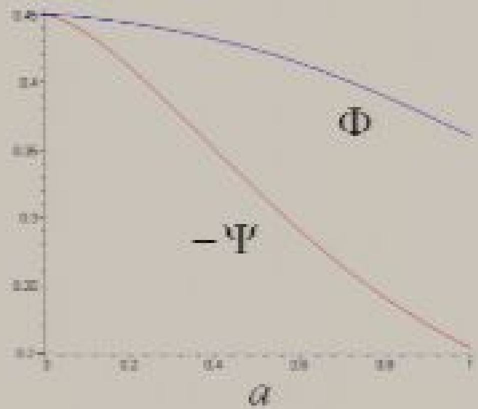
- Solutions for metric perturbations

$$ds^2 = -\left(1+2\Psi\right)dt^2 + a(t)^2\left(1+2\Phi\right)d\vec{x}^2$$

$$\frac{k^2}{a^2}\Phi = 4\pi G\left(1-\frac{1}{3\beta}\right)\rho\delta,$$

$$\frac{k^2}{a^2}\Psi = -4\pi G\left(1+\frac{1}{3\beta}\right)\rho\delta,$$

$$\beta = 1 - 2Hr_c\left(1+\frac{\dot{H}}{3H^2}\right)$$



Growth rate is determined by Ψ

(Lue and Starkman)

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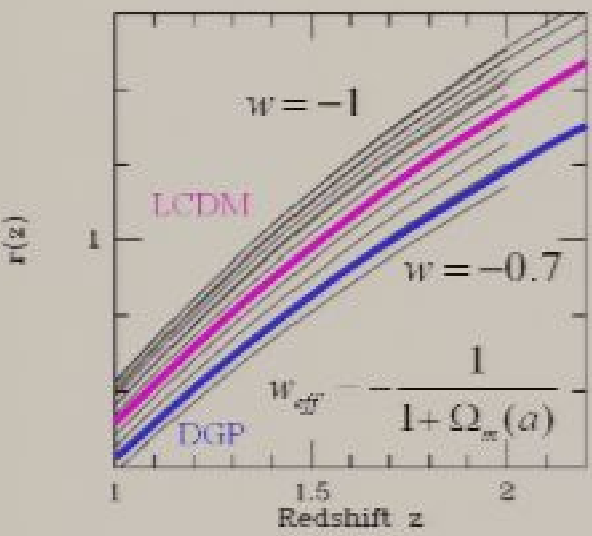

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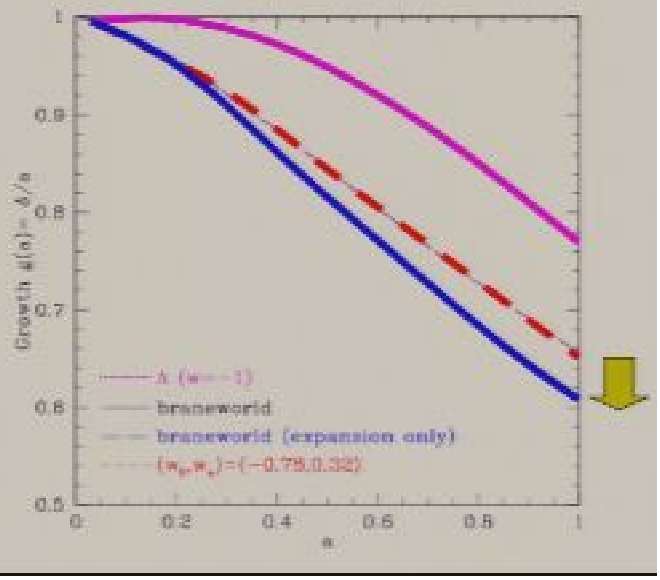

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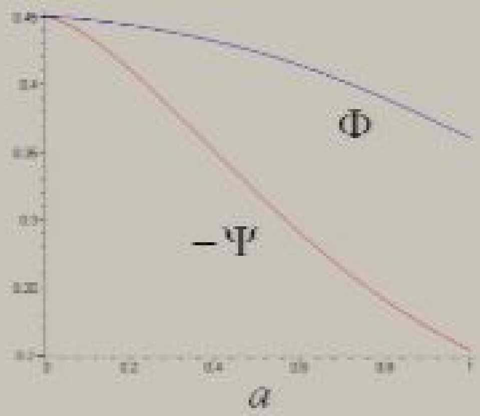


(Lue and Starkman)

Growth rate is determined by Ψ

Ghost suppresses growth of structure

- Negative BD parameter

$$\omega = \frac{3}{2}(\beta - 1) \qquad \beta = 1 - 2Hr_c\left(1 + \frac{\dot{H}}{3H^2}\right)$$

In Einstein frame, kinetic term for the scalar $-\frac{3}{2}\beta$

if $\beta < 0$ the scalar becomes a ghost

cf. de Sitter spacetime

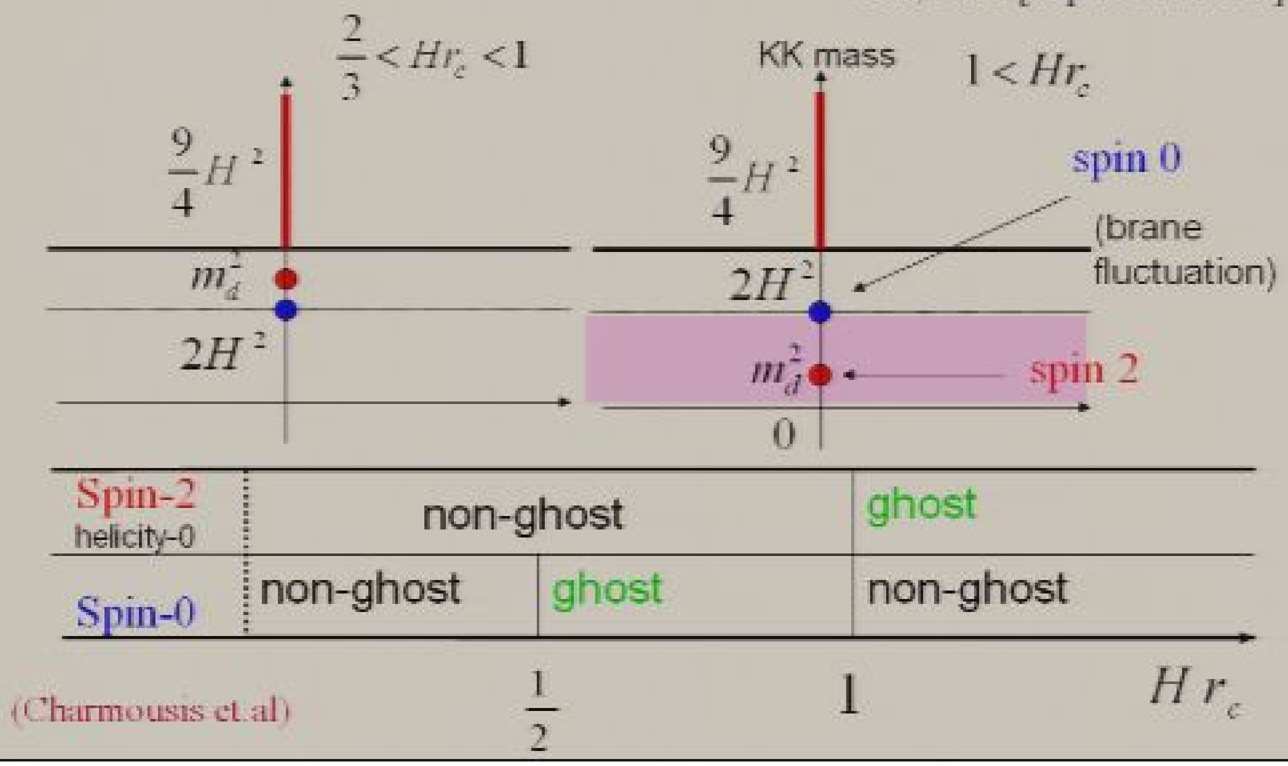
$$\beta < 0 \quad \Longleftrightarrow \quad Hr_c > \frac{1}{2}$$

$$Hr_c \geq 1, \quad (\sigma \geq 0)$$
$$Hr_c < 1, \quad (\sigma < 0)$$

(Luty et.al, Nicolis and Rattazi)

Ghost in de Sitter spacetime

KK, PRD [hep-th/0503191]



Self-accelerating universe

Gorbunov, KK and Sibiryakov PRD [hep-th/0512097]

- No ghost in massive gravity if $m^2 = 2H^2$

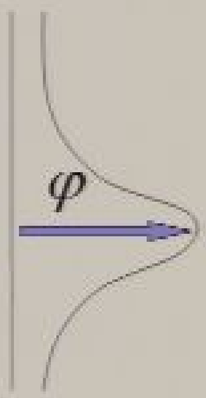
  enhanced symmetry $\chi_{\mu\nu} \to \chi_{\mu\nu} + (\nabla_\mu \nabla_\nu + H^2 \gamma_{\mu\nu}) X$

- Spin-2 and spin-0 degenerate

  $$h_{\mu\nu} = A_{\mu\nu}(x) + B_{\mu\nu}(x) \ln N(y)$$

  $$\Box A_{\mu\nu} - 4H^2 A_{\mu\nu} = H^2 B_{\mu\nu},$$

  $$B_{\mu\nu} = \frac{1}{H}\left(\nabla_\mu \nabla_\nu + H^2 \gamma_{\mu\nu}\right)\varphi$$



- In general the two cannot be diagonalized

  On small scales

  There is a ghost from the mixing

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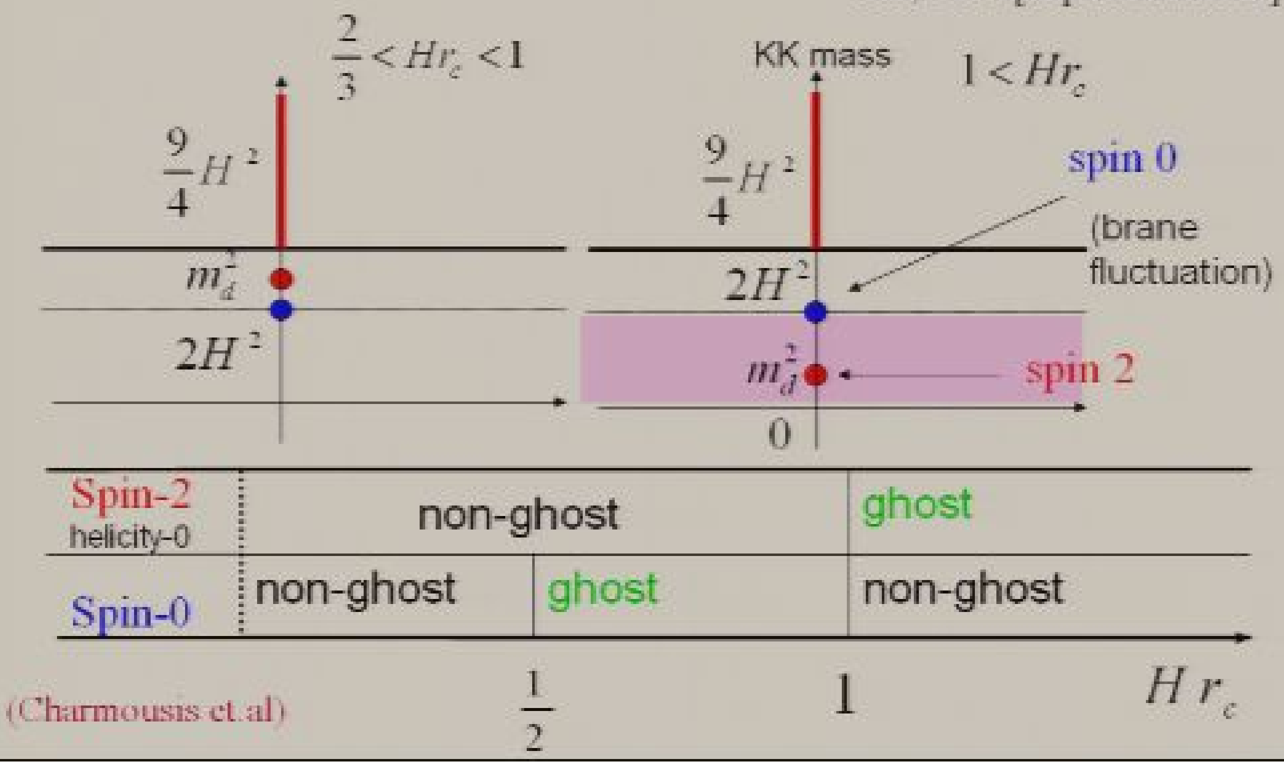

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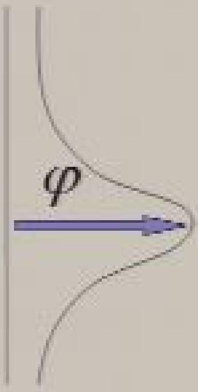

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Connection between Spin-2 / Spin-0

Izumi, KK and Tanaka JHEP [hep-th/0610282]

- Spin-2 and Spin-0 are mixed $m^2 = 2H^2$

$$h_{\mu\nu}(y,x) = (\nabla_\mu \nabla_\nu + H^2 \gamma_{\mu\nu})\chi(x), \quad \left(\Box + 4H^2\right)\chi(x) = 0$$

$$\nabla^\mu h_{\mu\nu} = h = 0$$

- Coupling to matter

  massive spin-2 perturbations cannot be coupled to T

  for $m^2 = 2H^2$

  e.o.m

$$(2H^2 - m^2)(\Box + 4H^2)h = \frac{8H^2\kappa_4^2}{3}T$$

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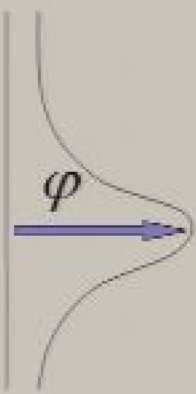

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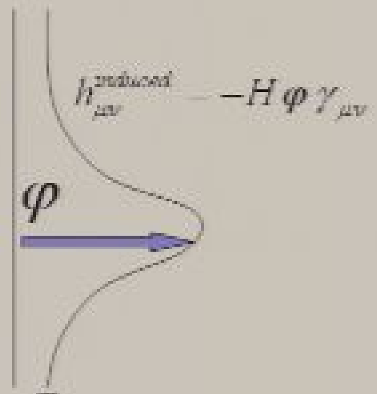
- DGP

brane bending mode is coupled to T

$$(1-2Hr_c)\left(\Box+4H^2\right)\varphi=\frac{\kappa^2}{6}T$$



amplitude

$$A \equiv h_{\mu\nu}^{induced}T^{\mu\nu}=-H\varphi T=-\frac{\kappa^2}{6}\frac{H}{1-2Hr_c}T\frac{1}{\Box+4H^2}T$$

$$=\left[-\frac{\kappa^2}{3}\sum_i\frac{H^2u_i(0)^2}{m_i^2-2H^2}-\frac{\kappa^2 H}{12}\frac{1}{(1-Hr_c)(1-2Hr_c)}\right]T\frac{1}{\Box+4H^2}T$$

Spin-2 Spin-0

$u_i(0)$ wave funciton of spin-2 perturbations

no singularity at $m^2 \to 2H^2$. $(Hr_c \to 1)$

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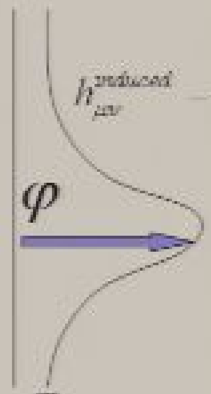
$h_{\mu\nu}^{induced} \sim -H\varphi\gamma_{\mu\nu}$



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Spin-2   Spin-0

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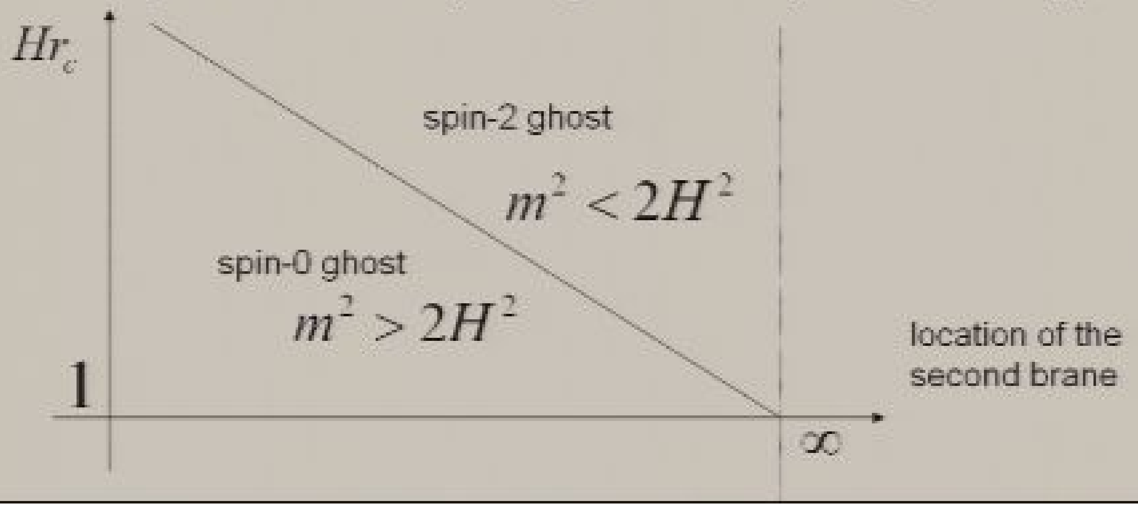
- Spin-0 perturbations MUST exist

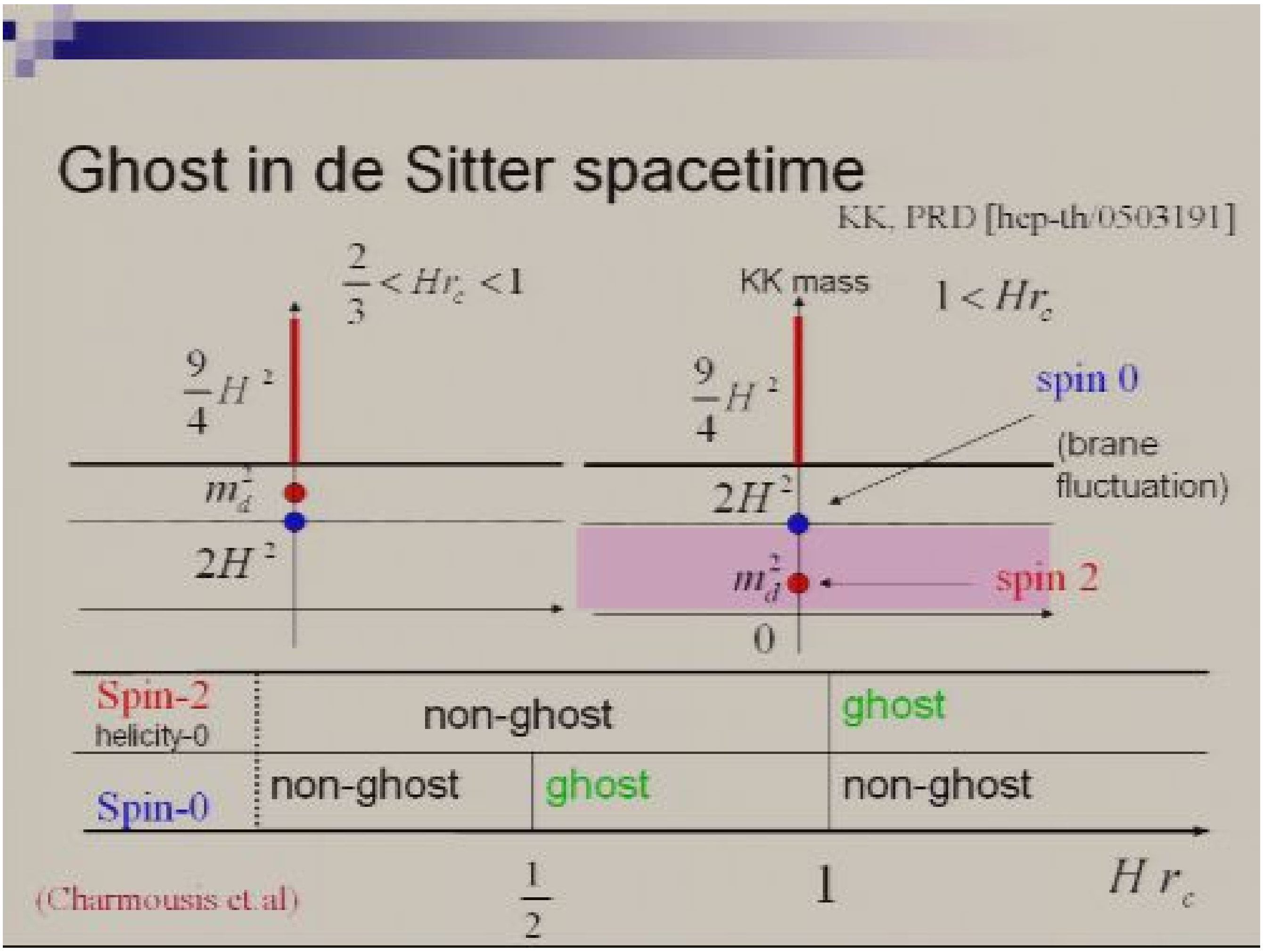
  contribution to amplitude is OPPOSITE to spin-2

  ⇨ spin-2 ghost is transferred to spin-0

- Two branes

  easy to eliminate spin-2 ghost but spin-0 ghost appears




Ghost in de Sitter spacetime
KK, PRD [hep-th/0503191]
$\frac{2}{3} < Hr_c < 1$
$\frac{9}{4}H^2$
m_d^2
$2H^2$
KK mass
$1 < Hr_c$
$\frac{9}{4}H^2$
spin 0
(brane fluctuation)
$2H^2$
m_d^2
spin 2
0
Spin-2
helicity-0
non-ghost
ghost
Spin-0
non-ghost
ghost
non-ghost
(Charmousis et.al)
$\frac{1}{2}$
1
$H r_c$

Connection between Spin-2 / Spin-0

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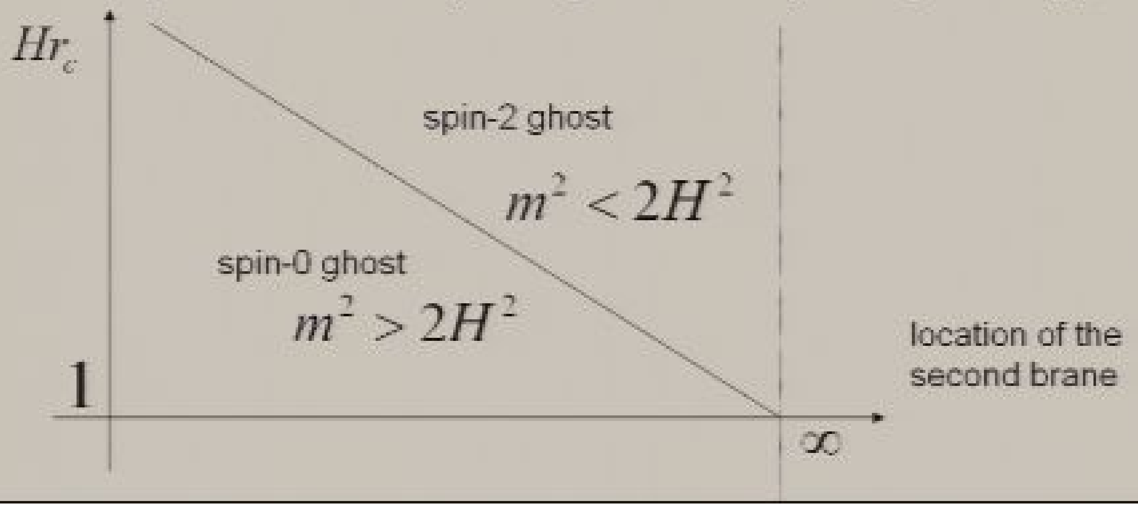
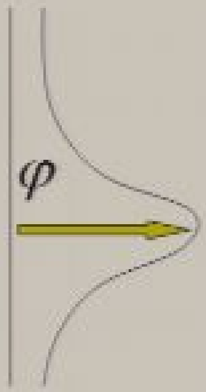


Non-linear evolution

Silva and KK
PRD [hep-th/0702169]

- Non-linearity of brane bending mode

$$ds^2 = -N^2(1+2\Psi)dt^2 + A^2(1+2\Phi)d\bar{x}^2 + (1+2G)dy^2 + 2r_c\varphi_{,i}\,dy\,dx^i$$



Solving bulk perturbations

imposing regularity condition in the bulk

junction conditions on a brane

$$-\nabla^2\Phi = 4\pi G a^2 \rho\delta + \frac{1}{2}\nabla^2\varphi, \qquad \Phi + \Psi = -\varphi \qquad \beta = 1 - 2Hr_c\left(1 + \frac{\dot{H}}{3H^2}\right)$$

$$3\beta(t)\nabla^2\varphi + r_c^2\left\{\partial_j\left(\partial^j\varphi\nabla^2\varphi\right) - \partial_j\left(\partial^i\varphi\,\partial_i\partial^j\varphi\right)\right\} = 8\pi G a^2\rho\delta$$

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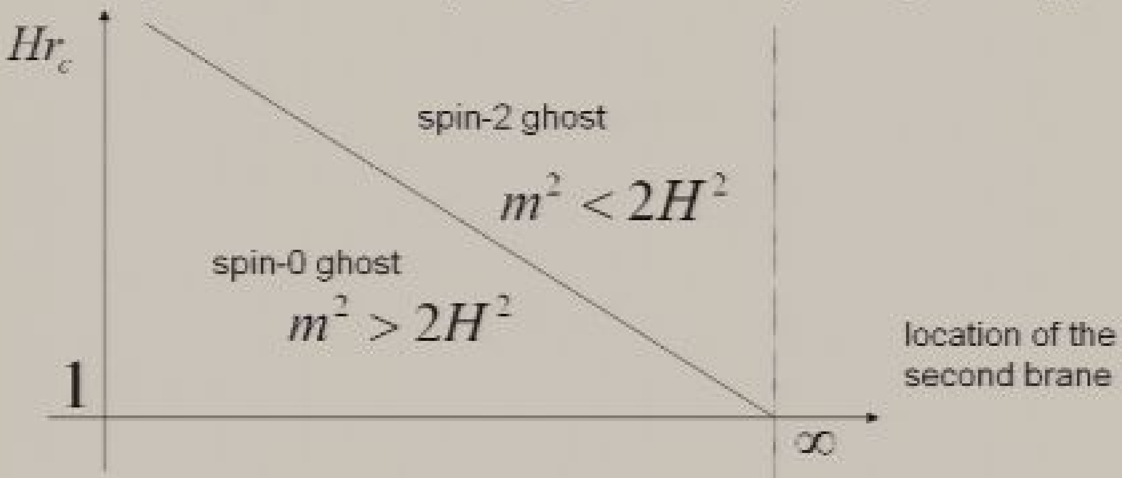

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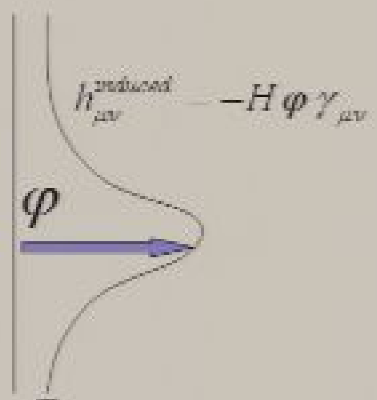

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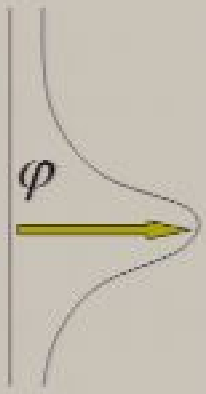

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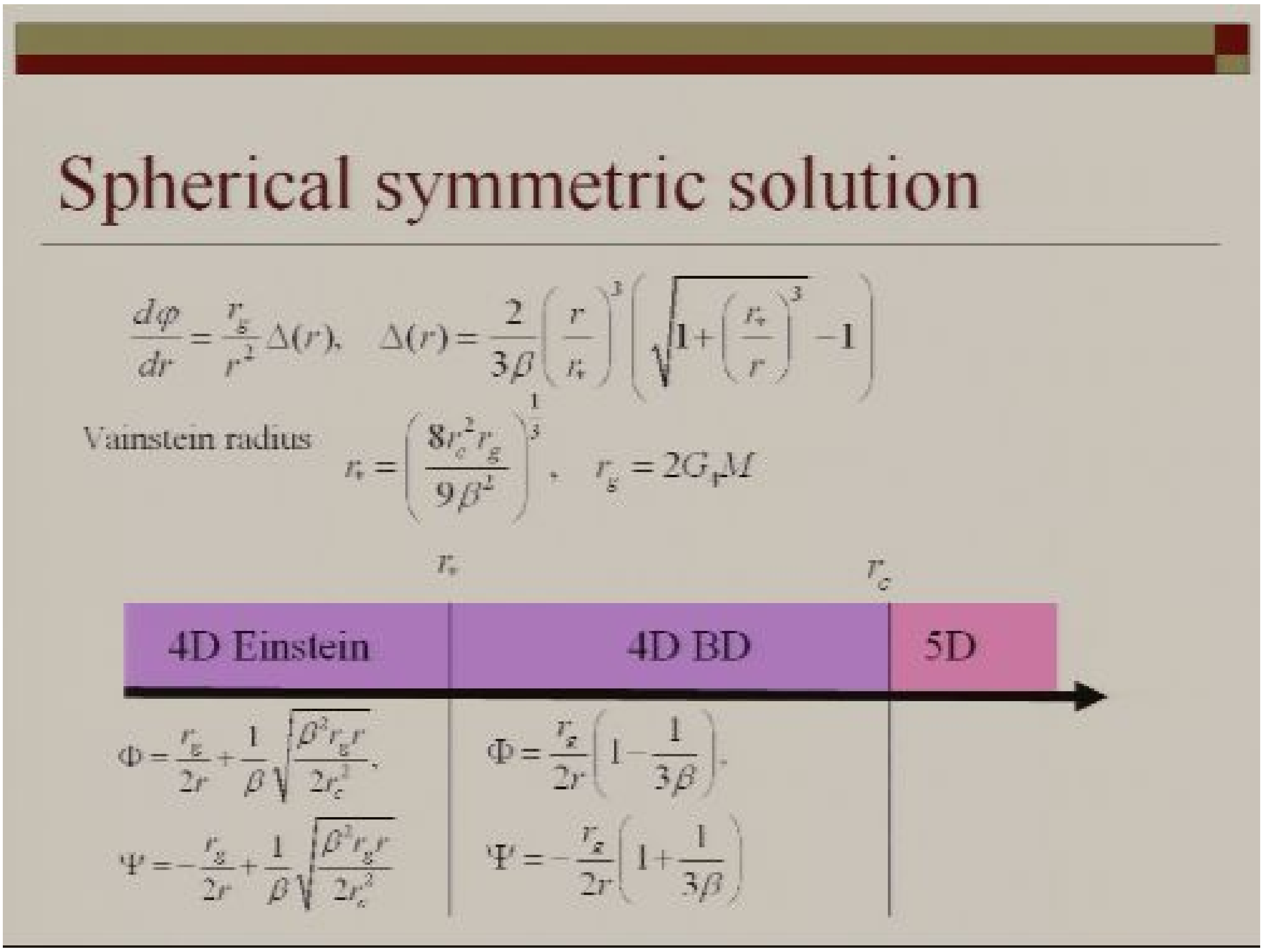
Spherical symmetric solution
$\frac{d\varphi}{dr} = \frac{r_g}{r^2}\Delta(r), \quad \Delta(r) = \frac{2}{3\beta}\left(\frac{r}{r_*}\right)^3\left(\sqrt{1+\left(\frac{r_*}{r}\right)^3}-1\right)$
Vainstein radius
$r_* = \left(\frac{8r_c^2 r_g}{9\beta^2}\right)^{\frac{1}{3}}, \quad r_g = 2G_4M$
r_*
r_c
4D Einstein
4D BD
5D
$\Phi = \frac{r_g}{2r} + \frac{1}{\beta}\sqrt{\frac{\beta^3 r_g r}{2r_c^2}},$
$\Psi = -\frac{r_g}{2r} + \frac{1}{\beta}\sqrt{\frac{\beta^3 r_g r}{2r_c^2}}$
$\Phi = \frac{r_g}{2r}\left(1-\frac{1}{3\beta}\right),$
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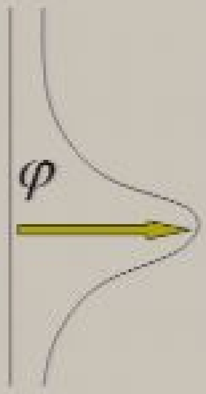

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$$3\beta(t)\nabla^2\varphi + r_c^2\left\{\partial_j\left(\partial^j\varphi\nabla^2\varphi\right) - \partial_j\left(\partial^i\varphi\,\partial_i\partial^j\varphi\right)\right\} = 8\pi G a^2\rho\delta$$

Spherical symmetric solution

$$\frac{d\varphi}{dr} = \frac{r_g}{r^2}\Delta(r), \quad \Delta(r) = \frac{2}{3\beta}\left(\frac{r}{r_*}\right)^3\left(\sqrt{1+\left(\frac{r_*}{r}\right)^3}-1\right)$$

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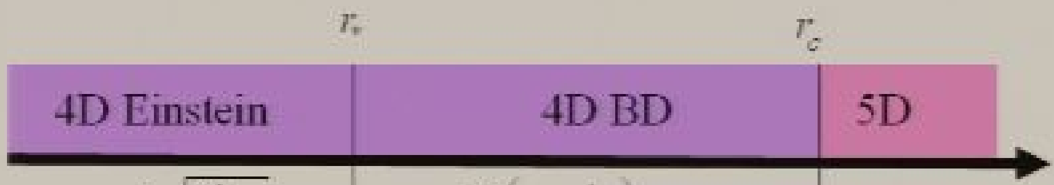

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Non-linear evolution

Silva and KK
PRD [hep-th/0702169]

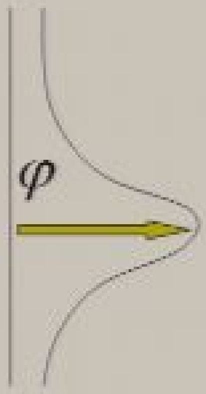
- Non-linearity of brane bending mode

$$ds^2 = -N^2(1+2\Psi)dt^2 + A^2(1+2\Phi)d\bar{x}^2 + (1+2G)dy^2 + 2 r_c \varphi_{,i}\, dy\, dx^i$$

Solving bulk perturbations

imposing regularity condition in the bulk

junction conditions on a brane



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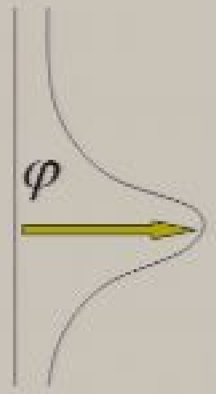

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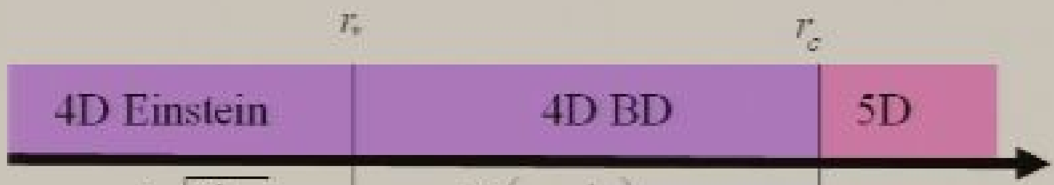

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Validity of linear theory

- Linearized solutions are smoothly matched to GR solution
- Boundary condition in the bulk is crucial

  cf Gabadadze and Iglesias (Deffayet et.al)

  impose a relation between Φ, Ψ ⟺ $ds^2 = -e^{-\lambda} dt^2 + e^{\lambda} dr^2 + r^2 d\Omega^2 + \gamma dr dy + e^{\sigma} dy^2$

  ⇒ linear solution is different

  approaches to 5D even below r_c with a screened mass

  $$M_{eff} = M\left(r_g / r_c\right)^{1/3}$$

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4D BD:

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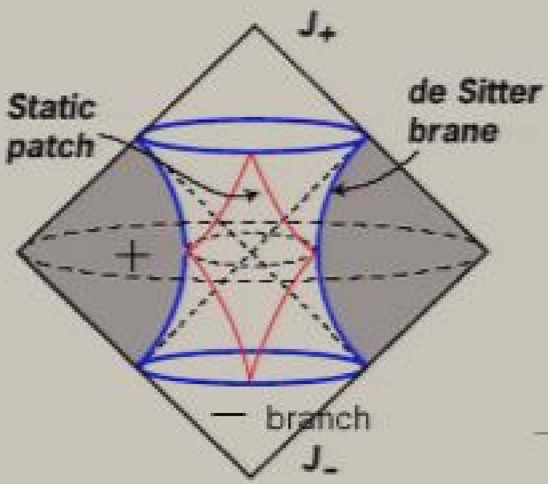
What is an end state of the ghost?

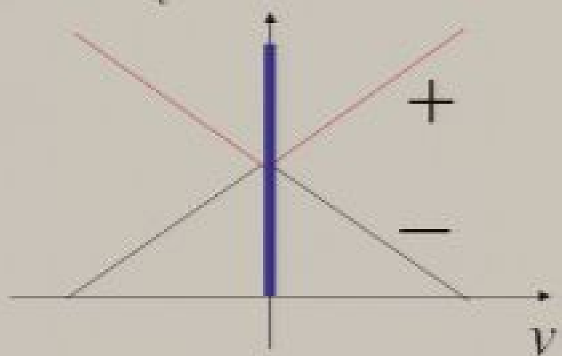
Izumi, KK, Tanaka, Pujolas in preparation

- Two branches

  Embedding of a brane in 5D Minkowski



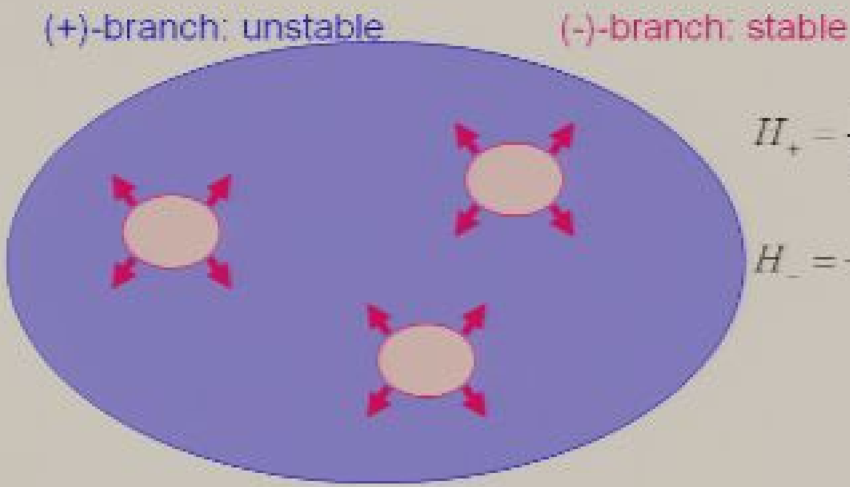
$$\pm \frac{H}{r_c} = H^2 - \frac{8\pi G}{3}\sigma$$



- − (normal) branch

  bound state is a 0-mode ⇨ no ghost $\beta = 1 \mp 2Hr_c\left(1+\frac{\dot{H}}{3H^2}\right)$

- Bubbles of normal branch formation?



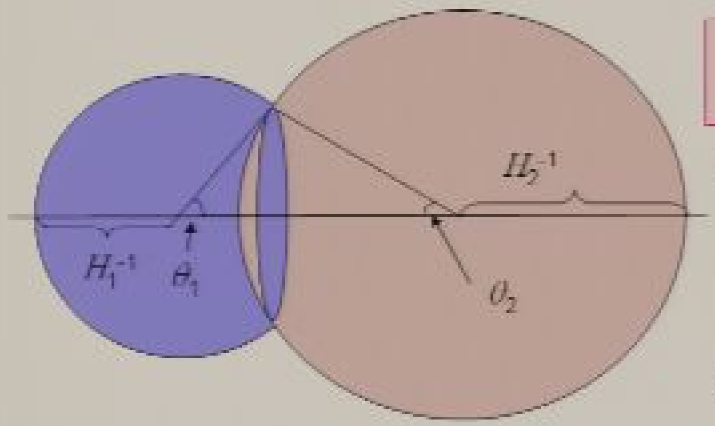
$$H_+ = \frac{1}{2r_c} + \sqrt{\frac{1}{2r_c} + \frac{8\pi G}{3}\sigma},$$

$$H_- = -\frac{1}{2r_c} + \sqrt{\frac{1}{2r_c} + \frac{8\pi G}{3}\sigma}$$

"false" vacuum decay? $H_+ > H_-$

Is there a Coleman-DeLucia type instanton?

- Can we construct such an instanton?



$$G^{(5)}_{\mu\nu} + 2r_c G^{(4)}_{\mu\nu}\delta(y) = 0$$

$$G^{(5)}_{\mu\nu} \approx \Delta\theta\,\gamma_{\mu\nu}\delta^2(x)$$
$$= -2(\theta_1 + \theta_2)\gamma_{\mu\nu}\delta^2(x)$$
(deficit angle)

$$G^{(4)}_{\mu\nu} \approx \left[K_{\mu\nu} - \gamma_{\mu\nu}K\right]\delta(y)$$

$$H_2 - H_1 = r_c^{-1} \quad \Rightarrow \quad = \frac{2}{r_c}\tan\frac{\theta_1+\theta_2}{2}\gamma_{\mu\nu}\delta^2(x)$$

$$4(\tan\theta_+ - \theta_+) = 0 \qquad \theta_+ \equiv \frac{\theta_1+\theta_2}{2}$$ ⇒ **no solution!**

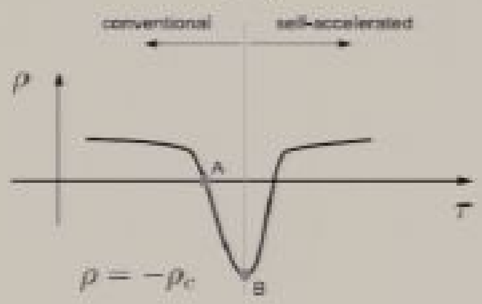
- **Solution with a 3D domain wall tension**

  There is a solution in thin wall limit $G^{(5)}_{\mu\nu} + 2r_c G^{(4)}_{\mu\nu}\delta(y) = \frac{2}{M_5^2}\mu\gamma_{\mu\nu}\delta^2(x)$

  However, it is impossible to construct such a configuration from a scalar field

  $$\dot{\rho}_\pi = 3\frac{\dot{r}}{r}\dot{\phi}^2$$

  $$\frac{1-\dot{r}^2}{r^2} = \frac{1 \pm \sqrt{1+4r_c^2\kappa_4^2\rho_\pi/3}}{2r_c}$$

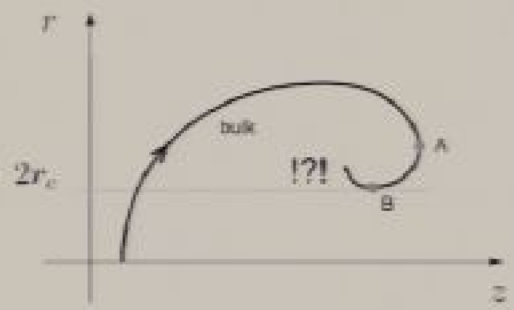


- **Branch changing solution**

  Branch changing solution passes through $\rho = -\frac{3}{4\kappa_4^2 r_c^2}\left(Hr_c - \frac{1}{2}\right)$

  ⇨ the brane bending mode becomes strongly coupled

  $$\beta = 1 - 2Hr_c = 0$$



Conclusion

- Self-accelerating universe

  Opens up a new perspective to dark energy problem

  Great opportunity to exploit future cosmological observations

- Theoretical challenges

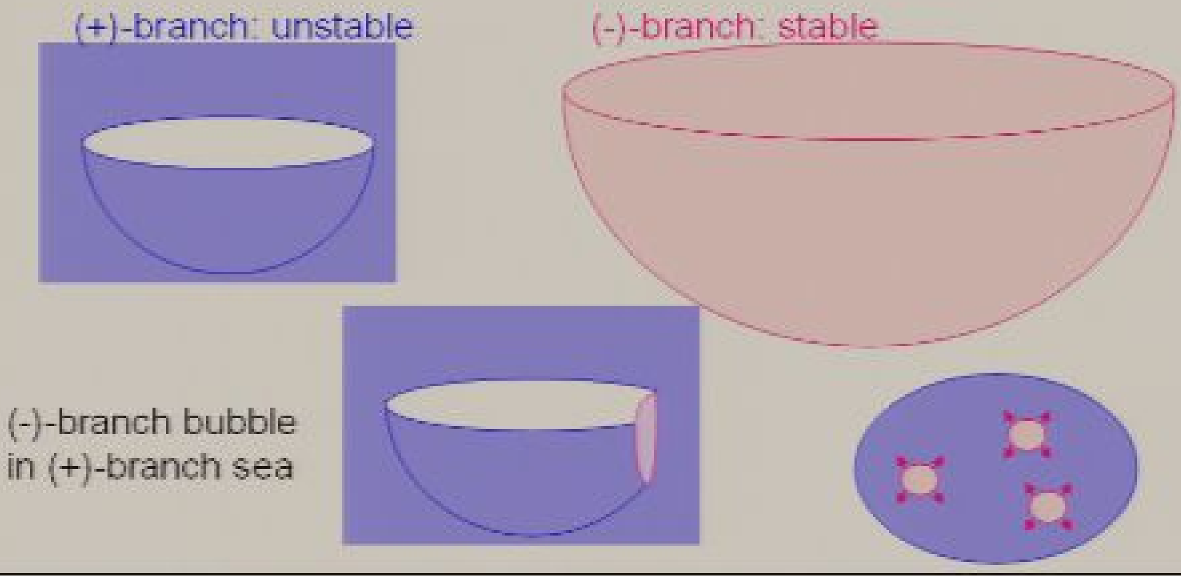
  ghosts, strong coupling problem, superluminal modes…

- instanton connecting (+)-branch with (-)-branch.

  instanton is a Euclidean classical solution

  connecting the initial state and final state

  Euclidian de Sitter S^4



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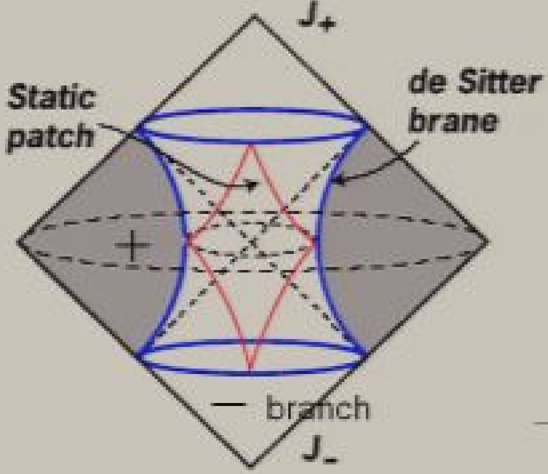

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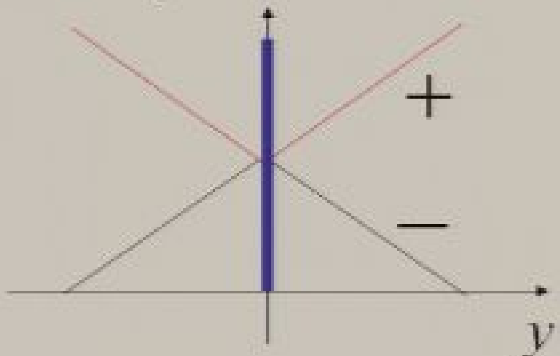

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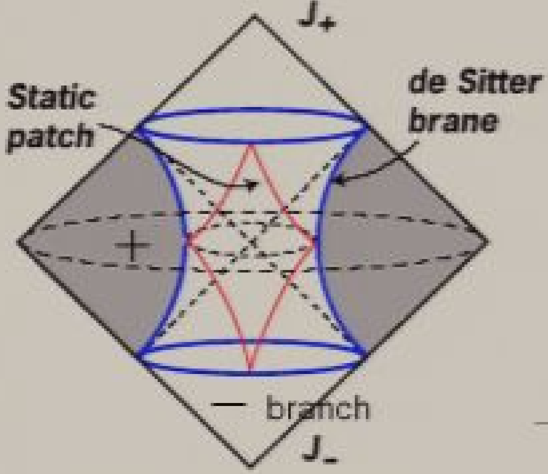

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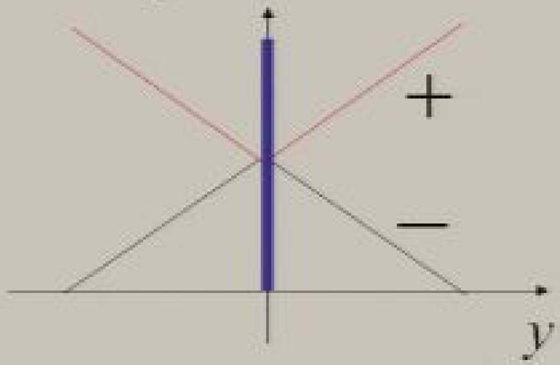

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$$\frac{3}{r_c}K - K^2 + K_{\mu\nu}K^{\mu\nu} = 0 \qquad K = H\gamma_{\mu\nu} = \frac{1}{r_c}\gamma_{\mu\nu}$$

- Self-accelerating universe

  Non-linear terms are comparable to the linear term

  the strong coupling scale is $\frac{1}{r_c} = H$?

Then we cannot trust anything below H^{-1}

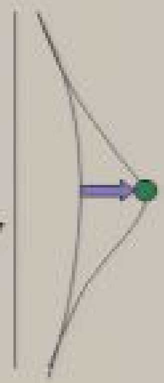
Re-definition of background

(Nicolis and Rattazzi, Deffayet)

$$K_{\mu\nu} = K_{0\,\mu\nu} - r_c \nabla_\mu \nabla_\nu \varphi, \quad K_{\mu\nu} \sim -2\nabla_{(\mu}N_{\nu)}, \quad N_\mu = r_c \nabla_\mu \varphi.$$

$$\frac{3}{r_c}K - K^2 + K_{\mu\nu}K^{\mu\nu} = 0$$

$$3\underbrace{\left(1-2Hr_c\right)}\Box\varphi + r_c^2\left[\left(\nabla^2\varphi\right)^2 - \left(\nabla_\mu\nabla_\nu\varphi\right)^2\right] = -\kappa_4^2 T$$



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