The Entropic Appoach to Predicting Λ

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The Causal Entropic Principle

Work in progress, building on:

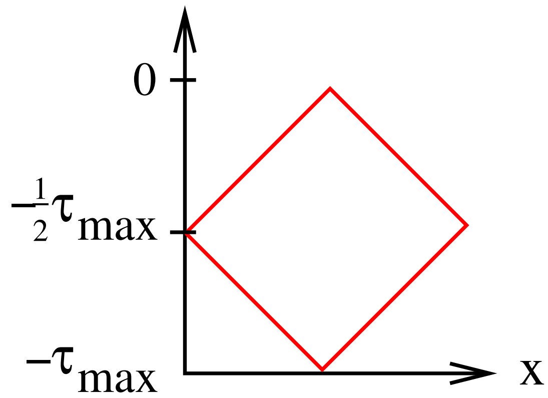
R. Bousso, R. Harnik, G. D. Kribs and G. Perez,
“Predicting the Cosmological Constant from the Causal
Entropic Principle,” arXiv:hep-th/0702115.

A replacement for the anthropic principle:

$$[\text{ Probability of finding observers }] \propto [\text{ entropy produced in causal diamond }]$$

- Simple, quantitative, makes minimal assumptions about nature of observers;
- Solves measure problem of eternal inflation

The Causal Diamond



- Conformal time $ds^2 = a^2(\tau)(d\tau^2 - d\vec{x}^2)$
- Start at some early time, *e.g.*, reheating
- End at $t = \infty$, but τ is finite,

In Planck units,

$$\tau_{\text{max}} = \int \frac{dt}{a(t)} \sim 2.8\, t_\Lambda^{1/3} \equiv 2.8 \left(\frac{3}{\Lambda}\right)^{1/6}$$

Computed for universe which is previously matter-dominated,

$$a(t) = \left[t_\Lambda \sinh\left(\frac{3t}{2\Lambda}\right)\right]^{2/3}$$

Comoving volume of diamond $V_c = \frac{4\pi}{3} r^3(\tau)$ is finite at all times

Entropic Principle

Amount of entropy created inside the diamond determines probability for evolving complexity, structure, observers:

$$\frac{dP}{d\Lambda} = \Delta S(\Lambda)$$

ΔS indicates free energy was available, needed for making measurements. Minimal assumption: observers must obey laws of thermodynamics!

Oppposite situation: universe which remains in thermal equilibrium. No complexity can arise.

Same approach can be used to predict other variables besides Λ.

How to compute ΔS

Product of two factors:

$$\Delta S = \int dt \, V_c(t) \, \frac{dS}{dV_c dt}$$

$V_c =$ comoving volume of causal diamond; determined by cosmology;

$\frac{dS}{dV_c dt}$ = entropy production rate per unit volume; depends on astrophysics

Entropy production in our universe

Starlight is the biggest source of entropy; UV photons scattered by dust to become many 20 meV photons.

$$\frac{dS}{dV_c dt}(t) = \int_0^t dt' \, \frac{d^2S}{dM_* dt}(t - t') \, \dot{\rho}_*(t')$$

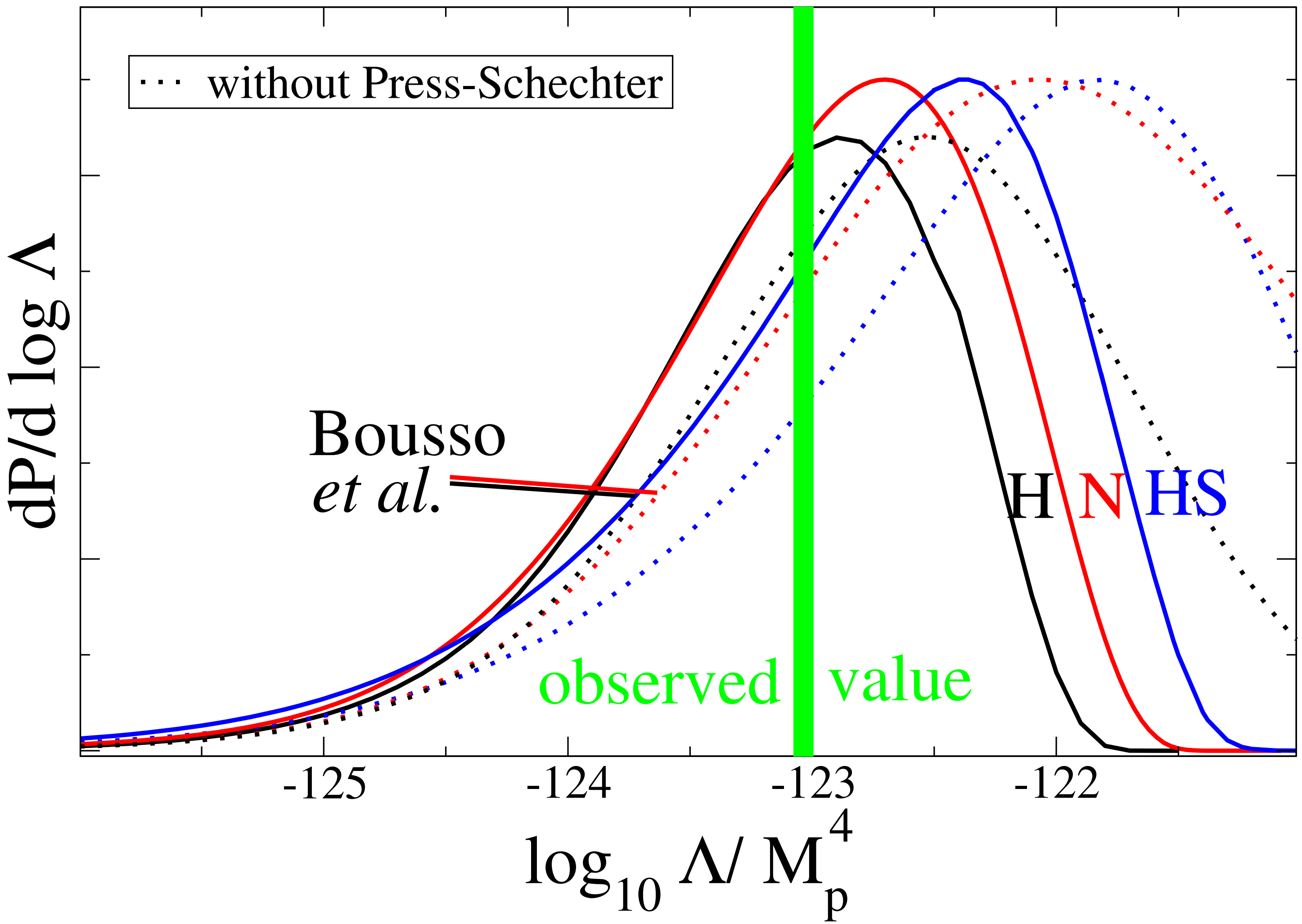
$\frac{d^2S}{dM_* dt}(t - t')$ = rate of entropy production per stellar mass, at time t, from stars born at time t',

$$\frac{d^2S}{dM_* dt}(t - t') = \frac{1}{\langle M \rangle} \int_{0.08 M_\odot}^{M_{\max}(t-t')} dM \, \xi_{\mathrm{IMF}}(M) \, \frac{d^2 s}{dN_* dt}$$

and $\dot{\rho}_* =$ stellar mass formation rate: depends on cosmological parameters (Λ) through Press-Schechter fraction $F(M, t)$ of matter in DM halos of mass $\geq M$ by time t.

Probability Distributions for Λ

For different published star formation rates
(Hopkins-Beacom, Nagamine *et al.*, Hernquist-Springel)



Probability to observe $\Lambda = 10^{-123} M_p^4$ is $O(1)$!

Press-Schechter Formalism

Fraction of matter collapsed into DM halos of mass $\geq M$:

$$F(M, x) = \text{erfc}\left[\frac{\rho_\Lambda^{1/3}}{Q f(x) g(M)}\right]$$

$Q = \delta\rho/\rho =$ density contrast; $x = \rho_\Lambda/\rho_m(t) =$ "time";

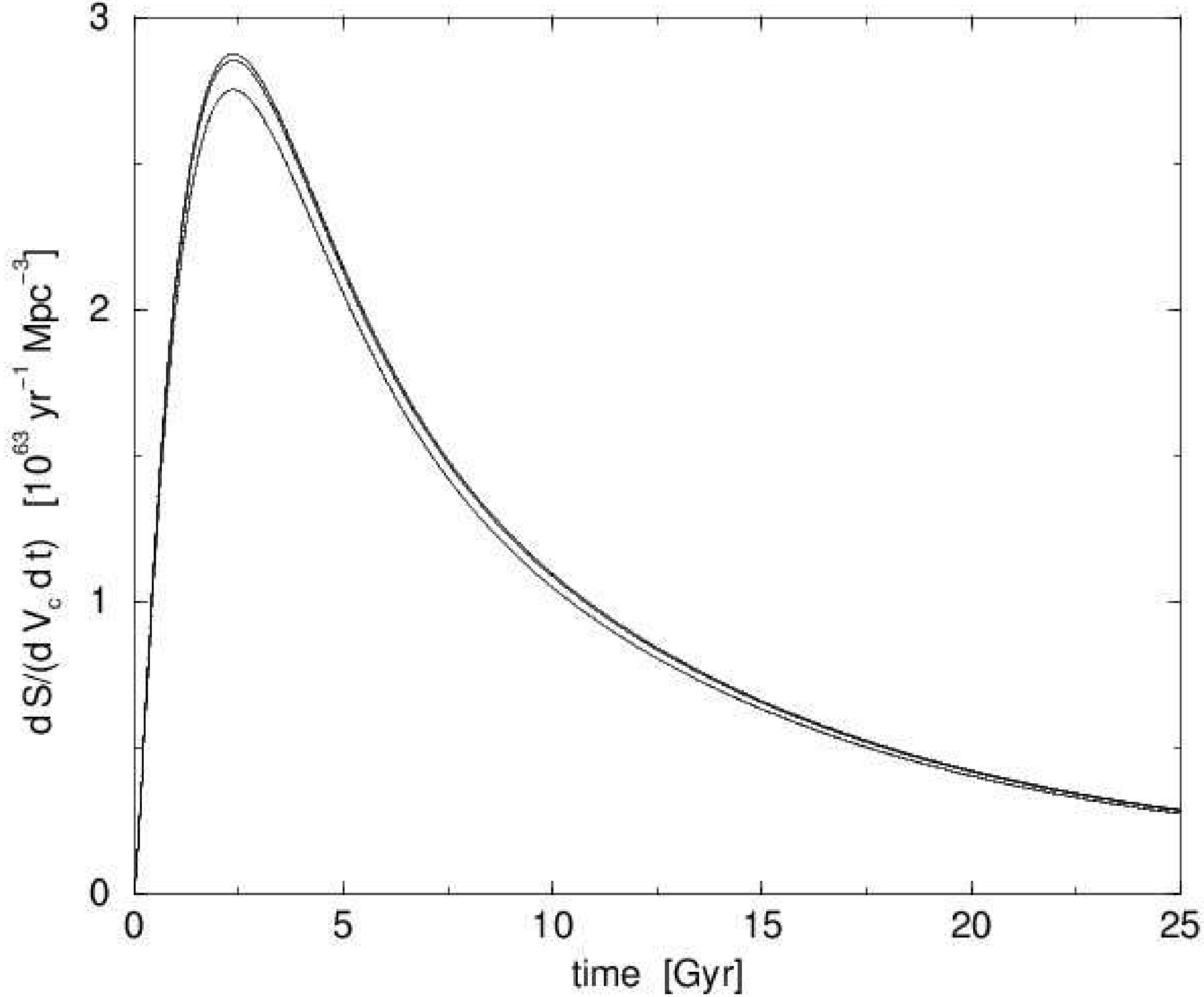
To model Λ-dependence of $\dot{\rho}_*$, Bousso *et al.* rescale

$$\frac{dS}{dV_c dt} \;\rightarrow\; F(10^7 M_\odot,\; x_{\rm max})\,\frac{dS}{dV_c dt}$$

where $x_{\rm max} =$ "time" when $\dot{\rho}_*$ is maximized. They claim the dependence is small:

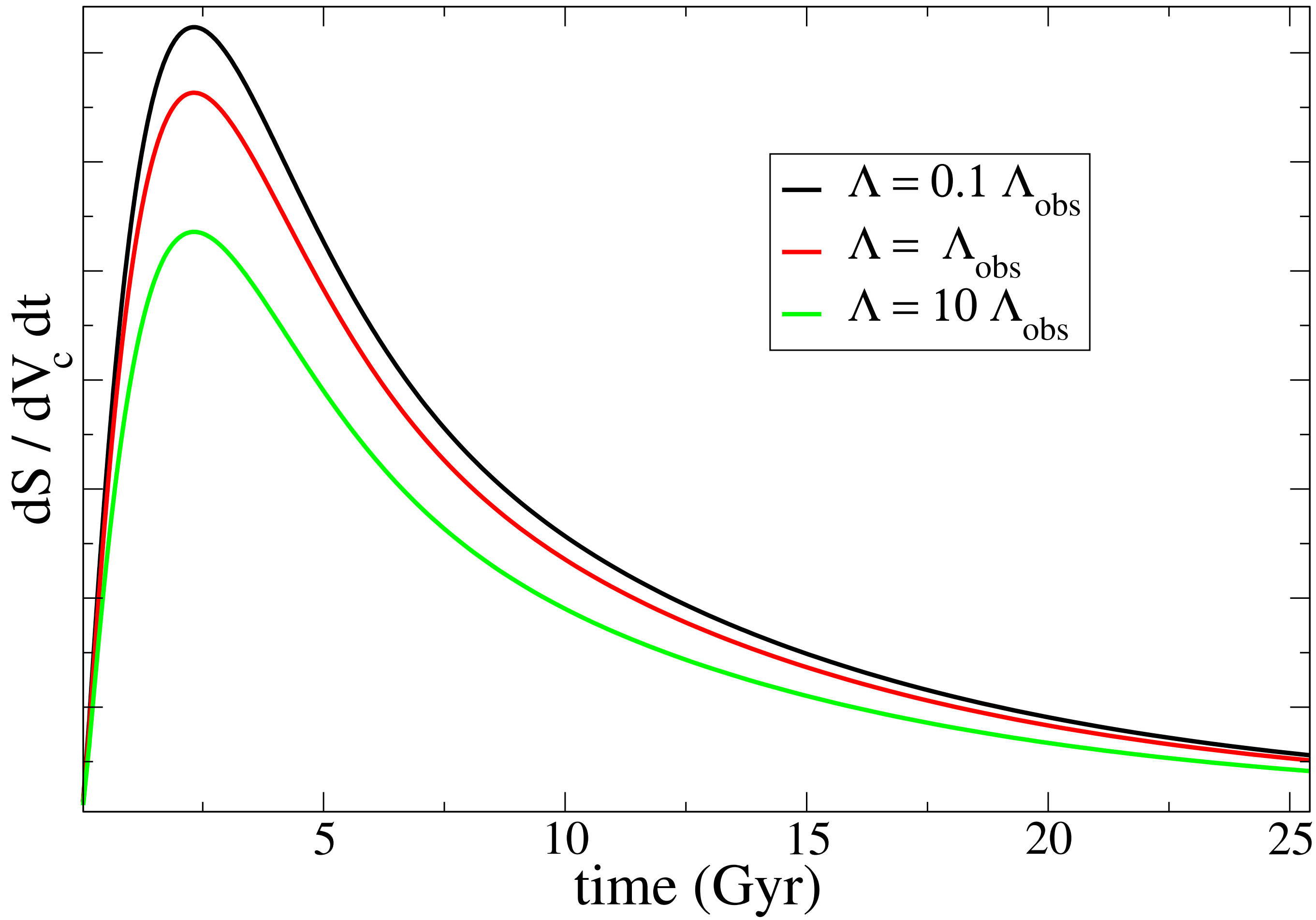
Dependence on PSF

Variation of $\frac{dS}{dV_c dt}$ for $\Lambda = (0.1,\ 1,\ 10) \times \Lambda_{\rm obs}$ found by Bousso *et al.*:



Dependence on PSF

Variation of $\frac{dS}{dV_c dt}$ for $\Lambda = (0.1,\ 1,\ 10) \times \Lambda_{\rm obs}$ found by us:

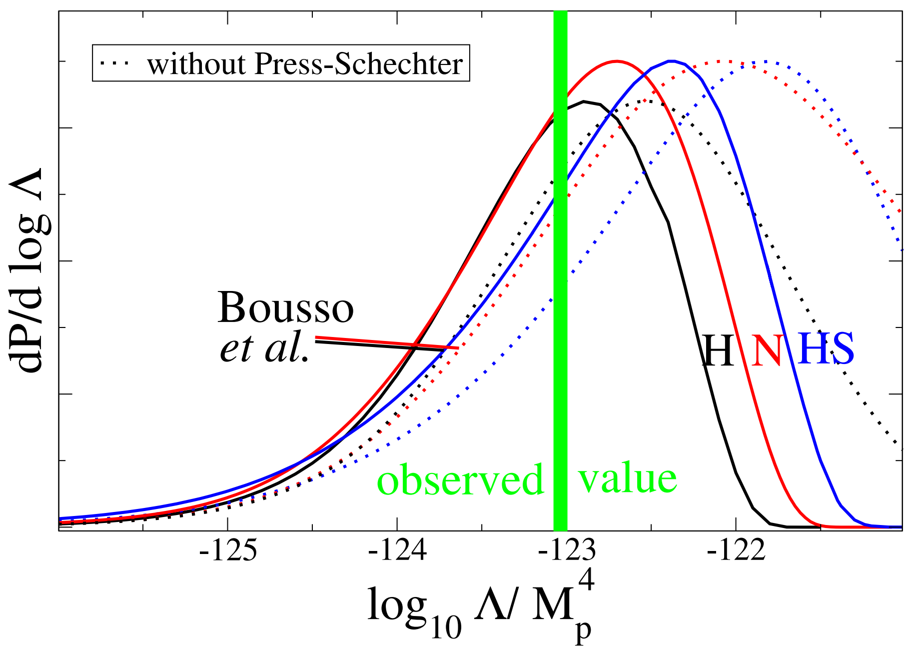


But procedure is crude: they simply multiply $\dot{\rho}_*$ by $F(10^7 M_\odot,\ t_{\rm max})$, where $\dot{\rho}_*(t_{\rm max})$ is maximized.

A better procedure

- Hopkins-Beacom and Nagamine *et al.* star formation rates are empirical fits to data.
- Hernquist-Springel analytically derive form of $\dot{\rho}_*$, incorporating time-dependent Press-Schechter formalism.

$$\dot{\rho}_*(t) = \dot{\rho}_*(0)\frac{\chi^2}{1+\alpha(\chi-1)^3\exp(\beta\chi^{7/4})}, \quad \chi \equiv \left(\frac{H(t)}{H_0}\right)^{2/3}$$

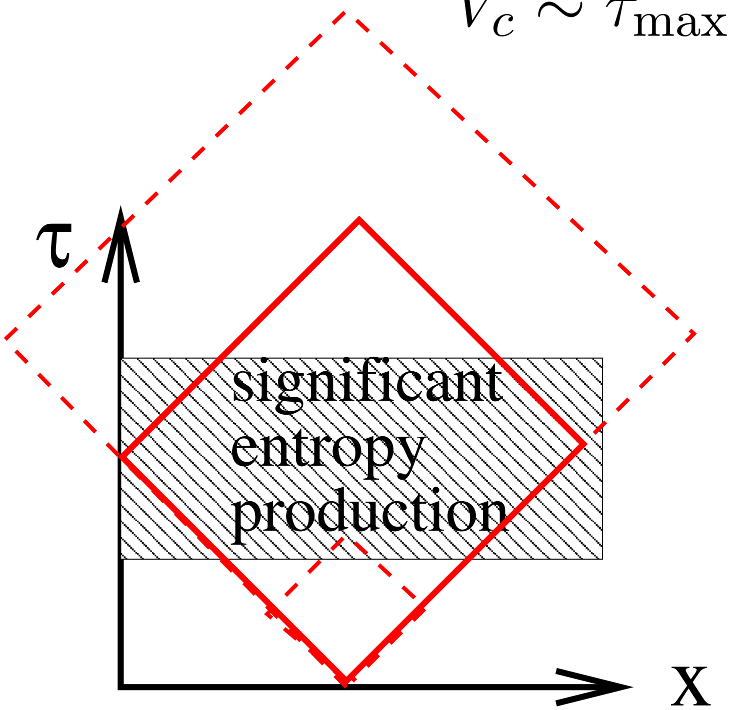


Basic predictions are robust

The basic prediction remains unaffected by these details: $dP/d\log\Lambda$ peaks near $\Lambda_{\rm obs}$.

Easy to understand: Main Λ-dependence is through volume of causal diamond,

$$V_c \sim \tau_{\rm max} \sim \frac{1}{\sqrt{\Lambda}}, \quad \int dt\, V_c \sim \frac{1}{\Lambda}$$



Preferred Λ has time of maximum diamond volume coinciding with time of maximum entropy production by stars.

Contrast to Anthropic Prediction

Suppose
$$\frac{dP}{d\Lambda} = \begin{cases} \text{const.}, & \Lambda < \Lambda_a \\ 0, & \Lambda > \Lambda_a \end{cases}$$

Then
$$\frac{dP}{d\log\Lambda} = \begin{cases} c\Lambda, & \Lambda < \Lambda_a \\ 0, & \Lambda > \Lambda_a \end{cases}$$

sharply peaked at maximum anthropically allowed value Λ_a.

Causal diamond volume's Λ-dependence changes this to

$$\frac{dP}{d\Lambda} = \begin{cases} \frac{c}{\Lambda}, & \Lambda < \Lambda_a \\ 0, & \Lambda > \Lambda_a \end{cases} \Longrightarrow \frac{dP}{d\log\Lambda} = \begin{cases} c, & \Lambda < \Lambda_a \\ 0, & \Lambda > \Lambda_a \end{cases}$$

ignoring time-dependence of $\frac{dS}{dV_c dt}$. Including $\frac{dS}{dV_c dt}$, $c \to c(\Lambda)$, not peaked at Λ_a

Going beyond Λ

Entropic principle should apply to other cosmological parameters, *e.g.*,

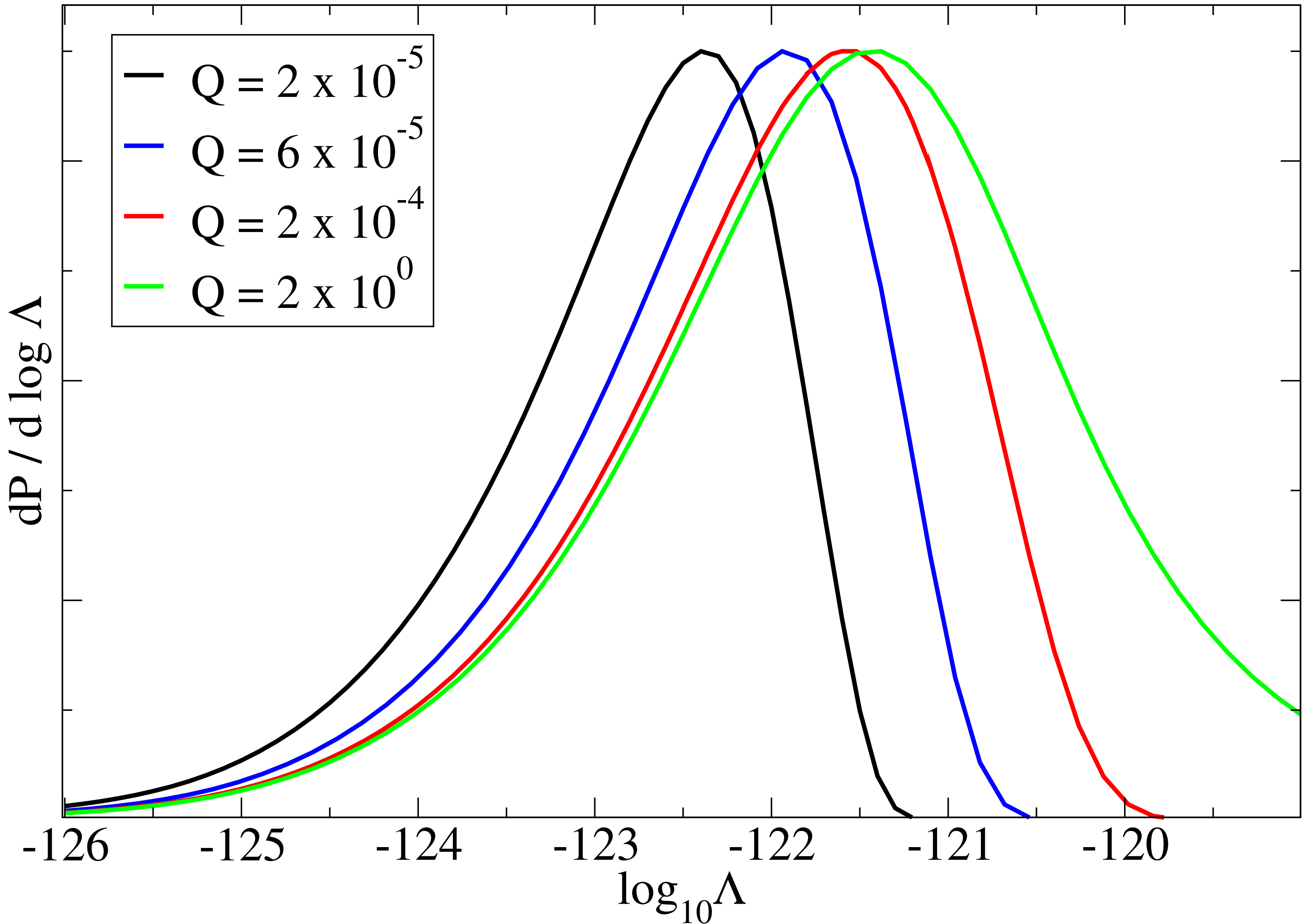
- density contrast $\frac{\delta\rho}{\rho} \equiv Q$
- spatial curvature
- time of matter domination relative to Λ-domination
- ratio of baryonic to dark matter

We should compute joint probability distribution of all parameters scanned in the landscape. Details of $\dot{\rho}_*$ could become important.

Will prediction for Λ be spoiled by letting other parameters vary? Letting $Q \to 1$ spoiled anthropic prediction for Λ, $\Lambda \to 10^5 \Lambda_{\mathrm{obs}}$.

Dependence on Q

In HS $\dot{\rho}_*$, $\beta \sim \frac{\Lambda^{2/3}}{Q^2}$; we find $dP/d\log\Lambda$ has strong overlap with $\Lambda_{\rm obs}$ for large range of Q:



Properties of Dark Matter

- Experimental hint for dark matter decaying or annihilating into e^+e^- in galactic center.
- No other plausible explanation for excess 511 keV emission from galactic center:

instrument	year	flux $[10^{-3}\ ph\ cm^{-2}\ s^{-1}]$	centroid $[keV]$	width (FWHM) $[keV]$	references
HEAO-3[a]	1979 – 1980	1.13 ± 0.13	510.92 ± 0.23	$1.6^{+0.9}_{-1.6}$	Mahoney et al. 1994
GRIS[b]	1988 *and* 1992	0.88 ± 0.07		2.5 ± 0.4	Leventhal et al. 1993
HEXAGONE[b]	1989	1.00 ± 0.24	511.33 ± 0.41	$2.90^{+1.10}_{-1.01}$	Smith et al. 1993
TGRS[c]	1995 – 1997	1.07 ± 0.05	510.98 ± 0.10	1.81 ± 0.54	Harris et al. 1998
SPI	2003	$0.99^{+0.47}_{-0.21}$	$511.06^{+0.17}_{-0.19}$	$2.95^{+0.45}_{-0.51}$	this work

SPI spectrometer on International Gamma-Ray Astrophysics Laboratory (INTEGRAL) gives best current measurement

Observed spectrum:

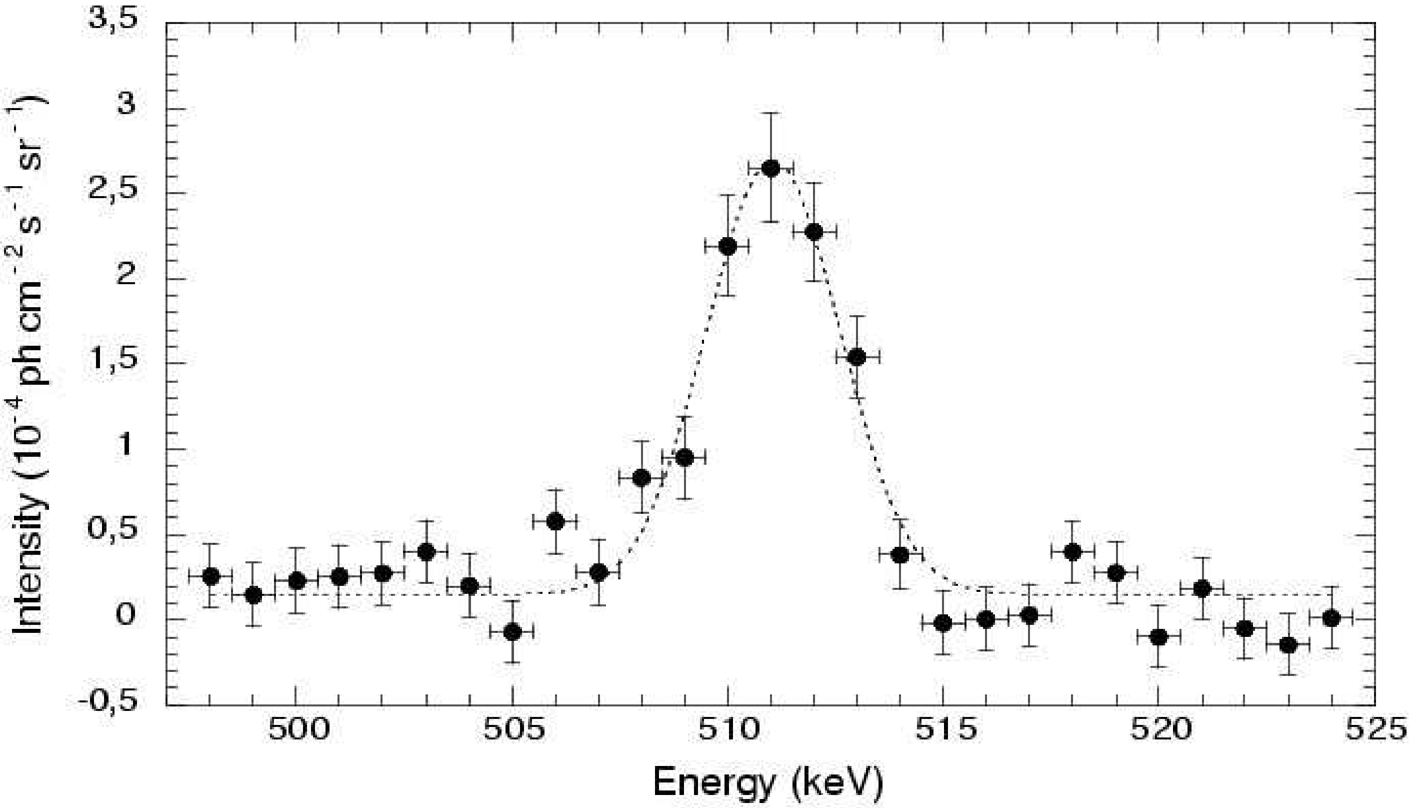


Fig. 3. 511 keV flux spectrum obtained using a gaussian centred on the GC with a FWHM of 10°.

Observed angular distribution:

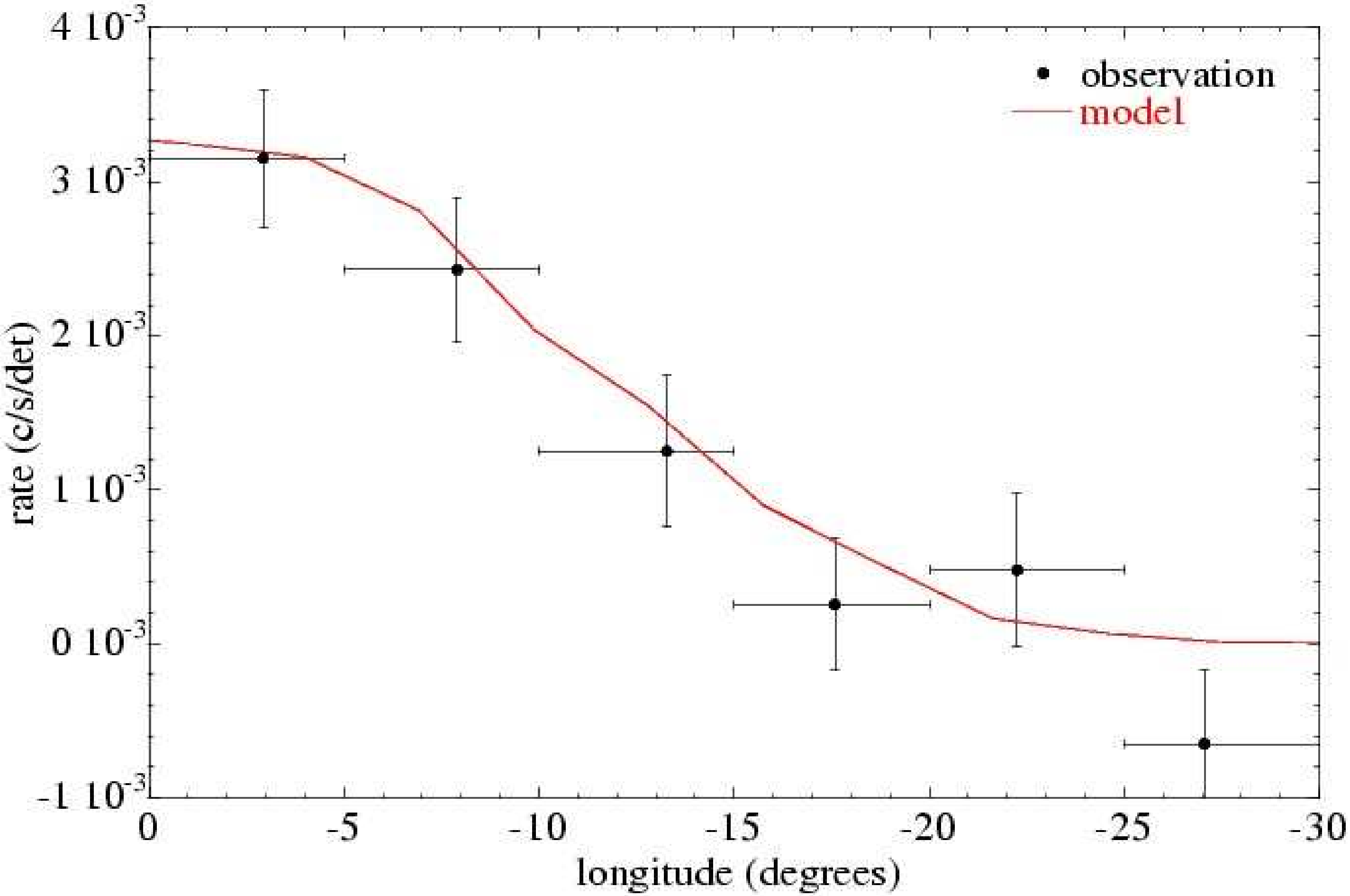


Fig. 2. Rate induced by galactic 511 keV photons as a function of longitude. The response to a gaussian source (FWHM = 10°) is also shown for comparison.

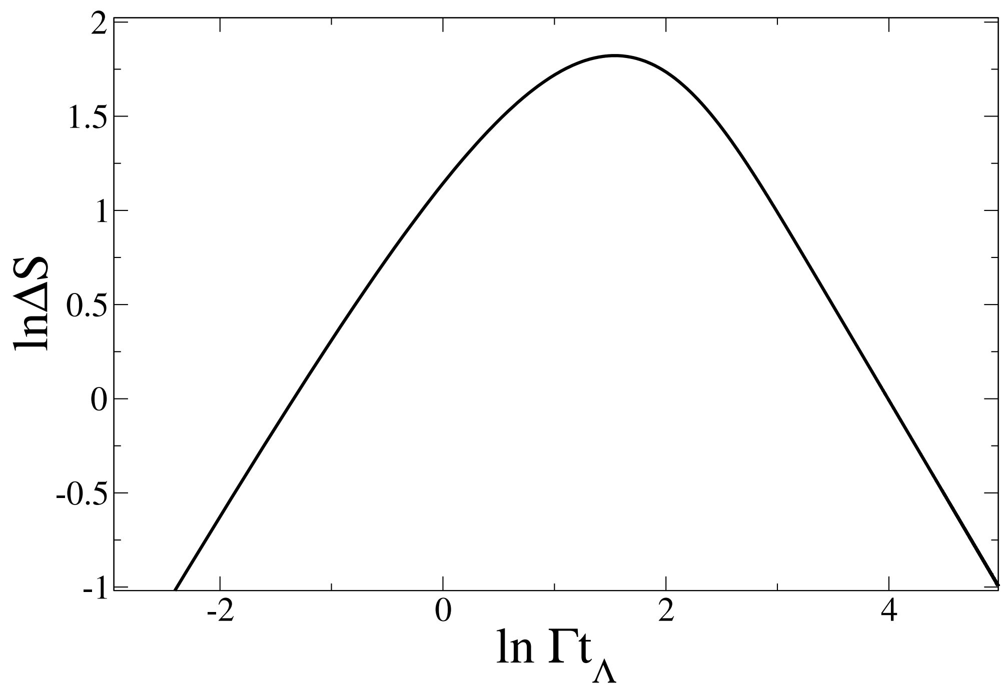
Entropy from Decaying DM

Decaying DM could be major new source of entropy production in causal diamond. Simpler to compute than star contribution:

$$\frac{dS}{dV_c dt} = g_s \, \Gamma \, n \, e^{-\Gamma t}$$

Leads to entropy change $\Delta S_d = t_\Lambda f(\Gamma t_\Lambda)$, where $t_\Lambda \equiv \sqrt{3/\Lambda}$

f is maximized when $1/\Gamma \cong t_\Lambda/4.7$. Since $t_\Lambda = 16.7$ Gyr, could explain why DM is decaying just today



Decaying DM versus stars

How does ΔS_d compare to ΔS from stars? From decays,

$$\frac{dS_d}{dV_c dt} \cong 10^{66} g_s \left(\frac{\Gamma}{\mathrm{Gyr}^{-1}}\right)\left(\frac{\mathrm{eV}}{m}\right)\left(\frac{\rho_{dm}}{(10^{-3}\mathrm{eV})^4}\right) \mathrm{Mpc}^{-3}\,\mathrm{y}^{-1}$$

where $m =$ DM mass, $g_s =$ number of decay products.

From stars, peak value is

$$\frac{dS_*}{dV_c dt} = 10^{63}\,\mathrm{Mpc}^{-3}\,\mathrm{y}^{-1}$$

$$\Longrightarrow \quad \Delta S_d > \Delta S_* \quad \text{if} \quad \frac{m}{g_s} \leq \text{ keV.}$$

Need $m \sim$ MeV, $g_s \sim 10^3$ to explain excess positrons. Such large g_s can come from synchrotron radiation by decay e^-'s.

If decay is GUT-suppressed, then $\Gamma^{-1} \sim \frac{M^2_{\mathrm{GUT}}}{m^3} \sim 10$ Gyr!

Conclusions

- Causal entropic principle overcomes many deficiencies of anthropic principle
- Predicts Λ better than anthropic approach
- No problem defining measure on landscape
- No arbitrary assumptions about nature of observers
- Robust against variation of other parameters
- Can be applied to universes from the landscape which look quite different from ours

“cosmo-ph” archive

the cosmologically-oriented papers in astro-ph:

http://www.physics.mcgill.ca/∼jcline/astroph/