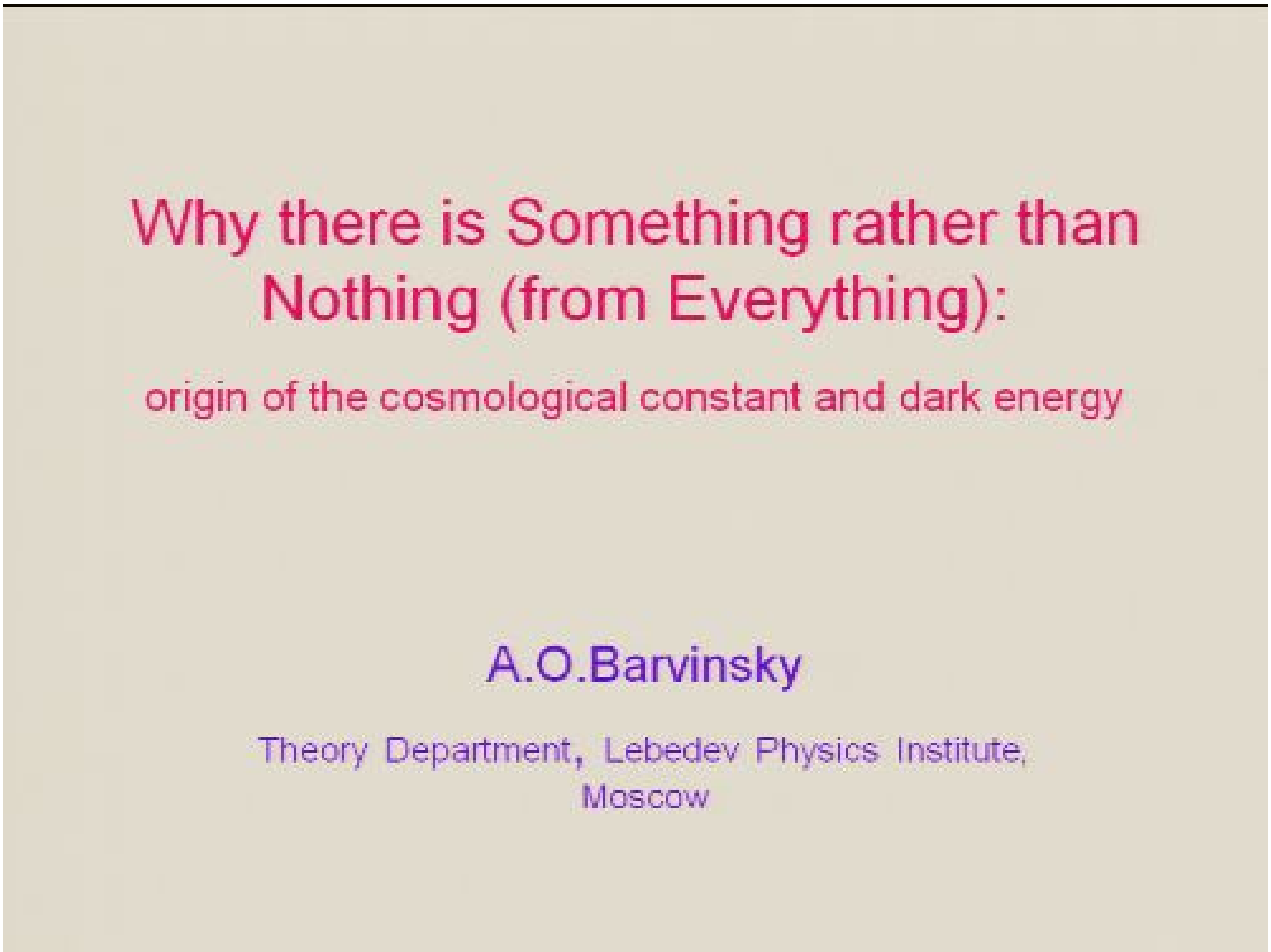
Title: Why there is something rather than nothing

Date: 2007-05-20 16:00:00

Abstract:

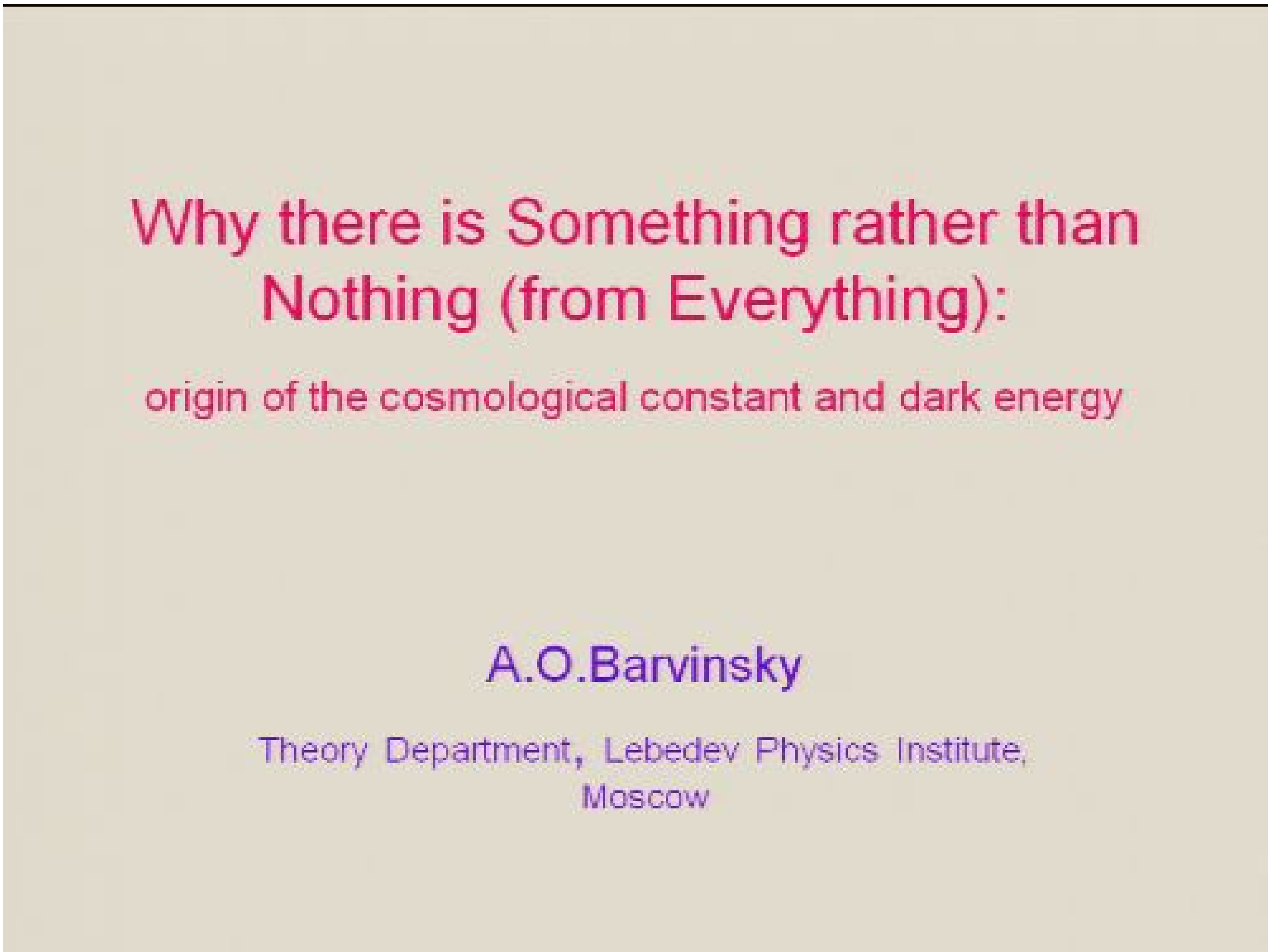


Why there is Something rather than Nothing (from Everything):

origin of the cosmological constant and dark energy

A.O.Barvinsky

Theory Department, Lebedev Physics Institute,
Moscow



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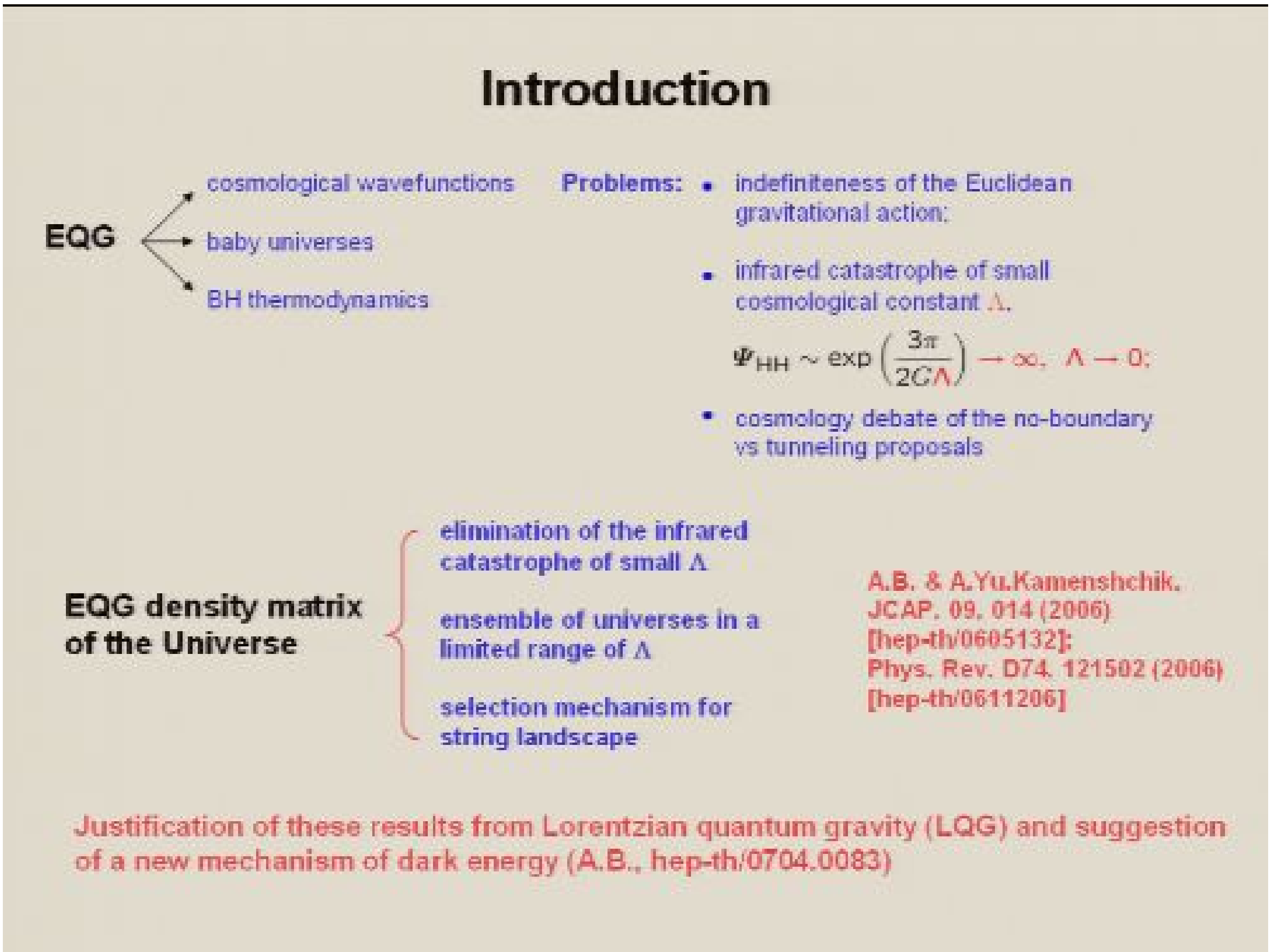


Introduction

EQG

- cosmological wavefunctions
- baby universes
- BH thermodynamics

Problems:

- indefiniteness of the Euclidean gravitational action;
- infrared catastrophe of small cosmological constant Λ,

$$\Psi_{\rm HH} \sim \exp\left(\frac{3\pi}{2G\Lambda}\right) \to \infty, \quad \Lambda \to 0;$$

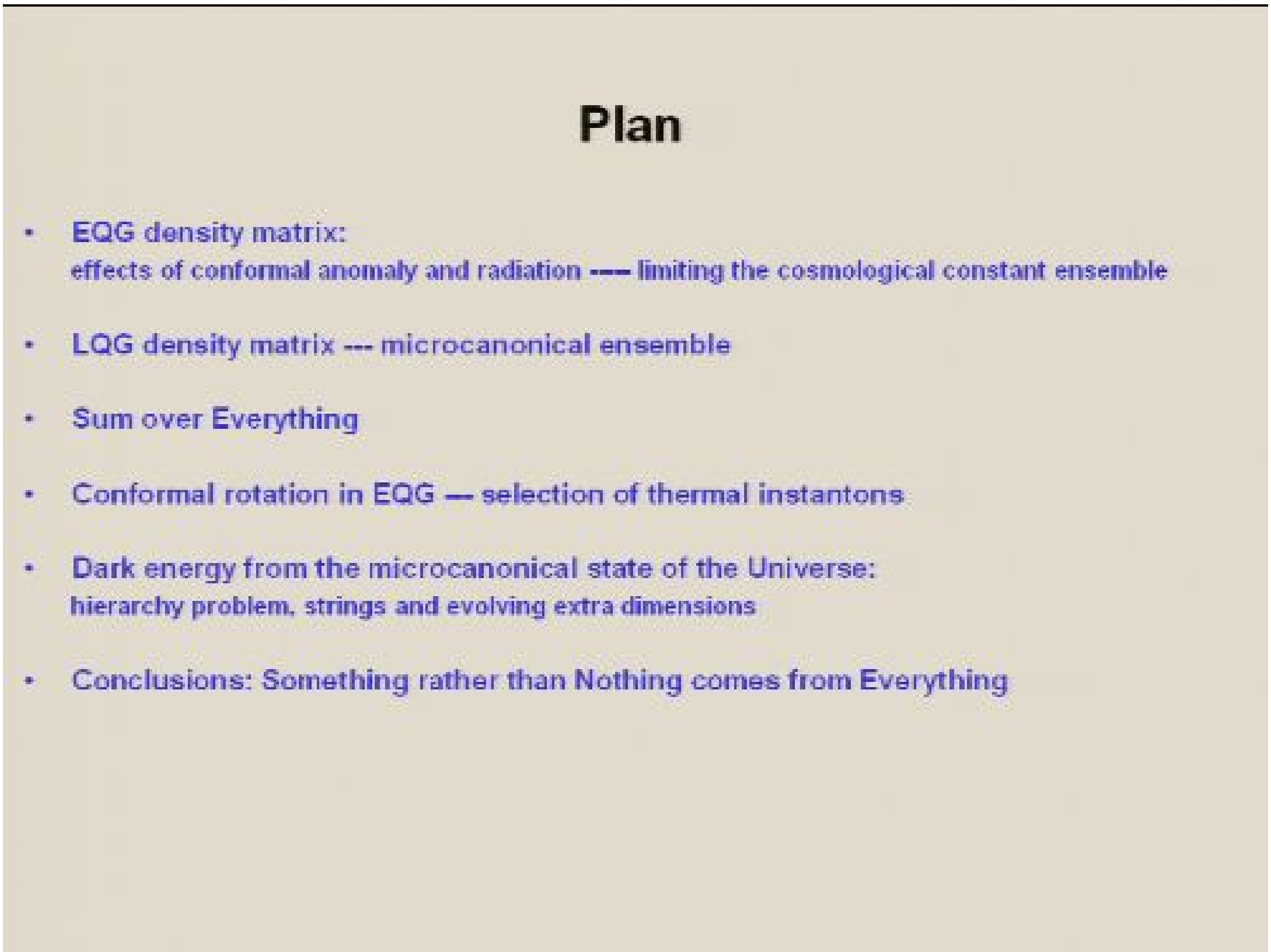
- cosmology debate of the no-boundary vs tunneling proposals

EQG density matrix of the Universe

- elimination of the infrared catastrophe of small Λ
- ensemble of universes in a limited range of Λ
- selection mechanism for string landscape

A.B. & A.Yu.Kamenshchik,
JCAP, 09, 014 (2006)
[hep-th/0605132];
Phys. Rev. D74, 121502 (2006)
[hep-th/0611206]

Justification of these results from Lorentzian quantum gravity (LQG) and suggestion of a new mechanism of dark energy (A.B., hep-th/0704.0083)

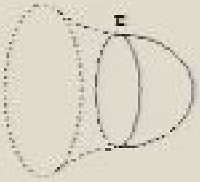


Plan

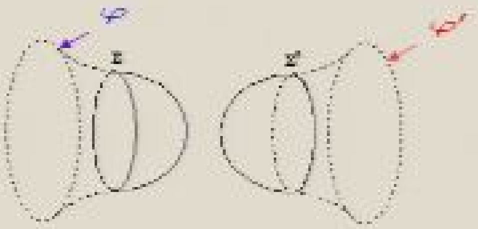
- EQG density matrix:
  effects of conformal anomaly and radiation ---- limiting the cosmological constant ensemble
- LQG density matrix --- microcanonical ensemble
- Sum over Everything
- Conformal rotation in EQG — selection of thermal instantons
- Dark energy from the microcanonical state of the Universe:
  hierarchy problem, strings and evolving extra dimensions
- Conclusions: Something rather than Nothing comes from Everything

From the pure Hartle-Hawking state to a statistical ensemble – the density matrix:

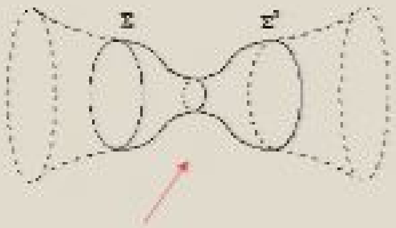
$$|\Psi_{HH}\rangle = \Psi_{HH}[\varphi]$$



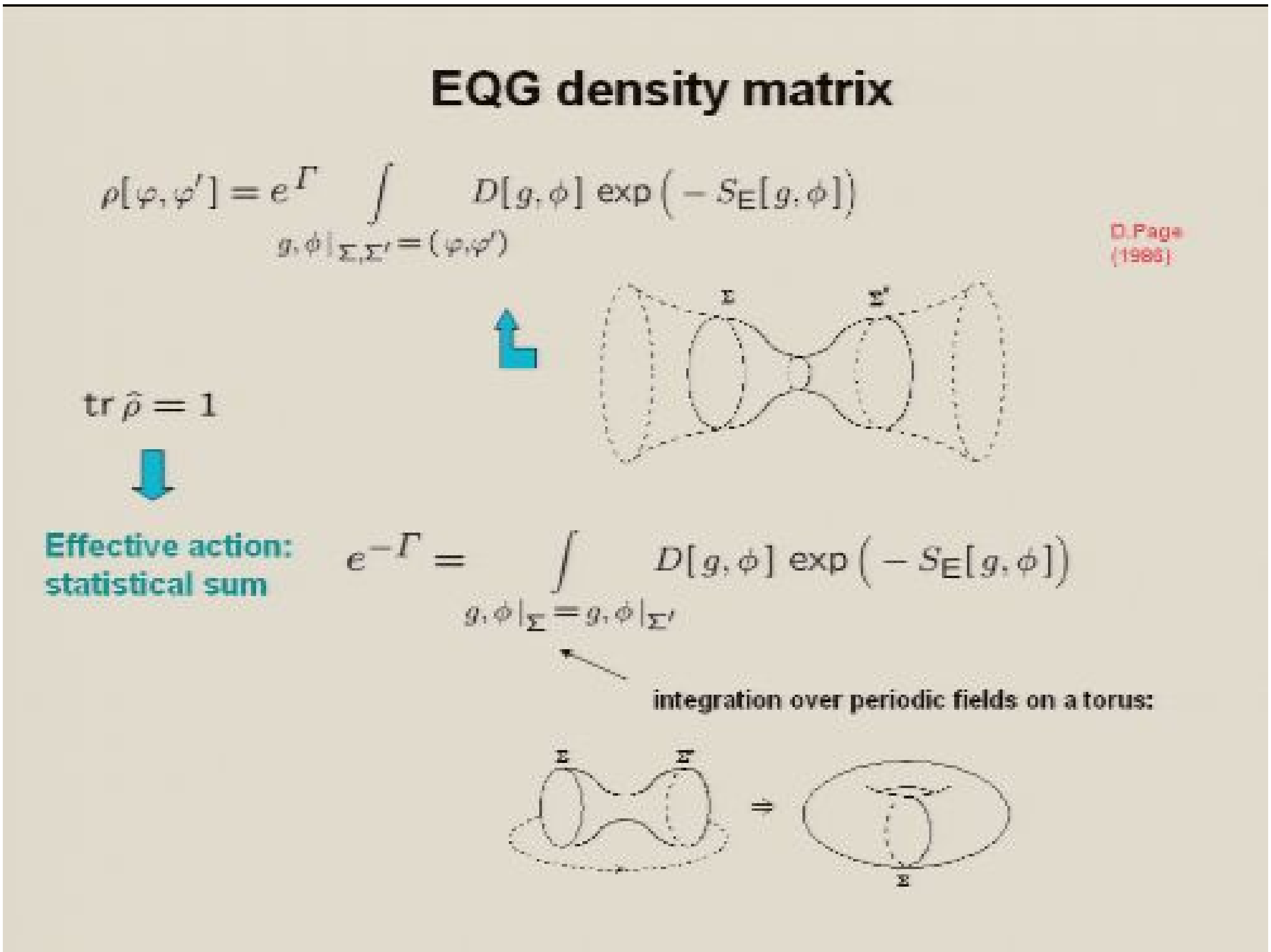
$$|\Psi_{HH}\rangle\langle\Psi_{HH}| \\ = \rho_{HH}[\varphi, \varphi']$$



$$\hat{\rho}_{\text{mixed}} \\ = \rho_{\text{mixed}}[\varphi, \varphi']$$



instanton bridge mediates
density matrix correlations



EQG density matrix

$$\rho[\varphi,\varphi'] = e^{\Gamma} \int\limits_{g,\phi|_{\Sigma,\Sigma'}=(\varphi,\varphi')} D[g,\phi]\,\exp\big(-S_{\mathrm{E}}[g,\phi]\big)$$

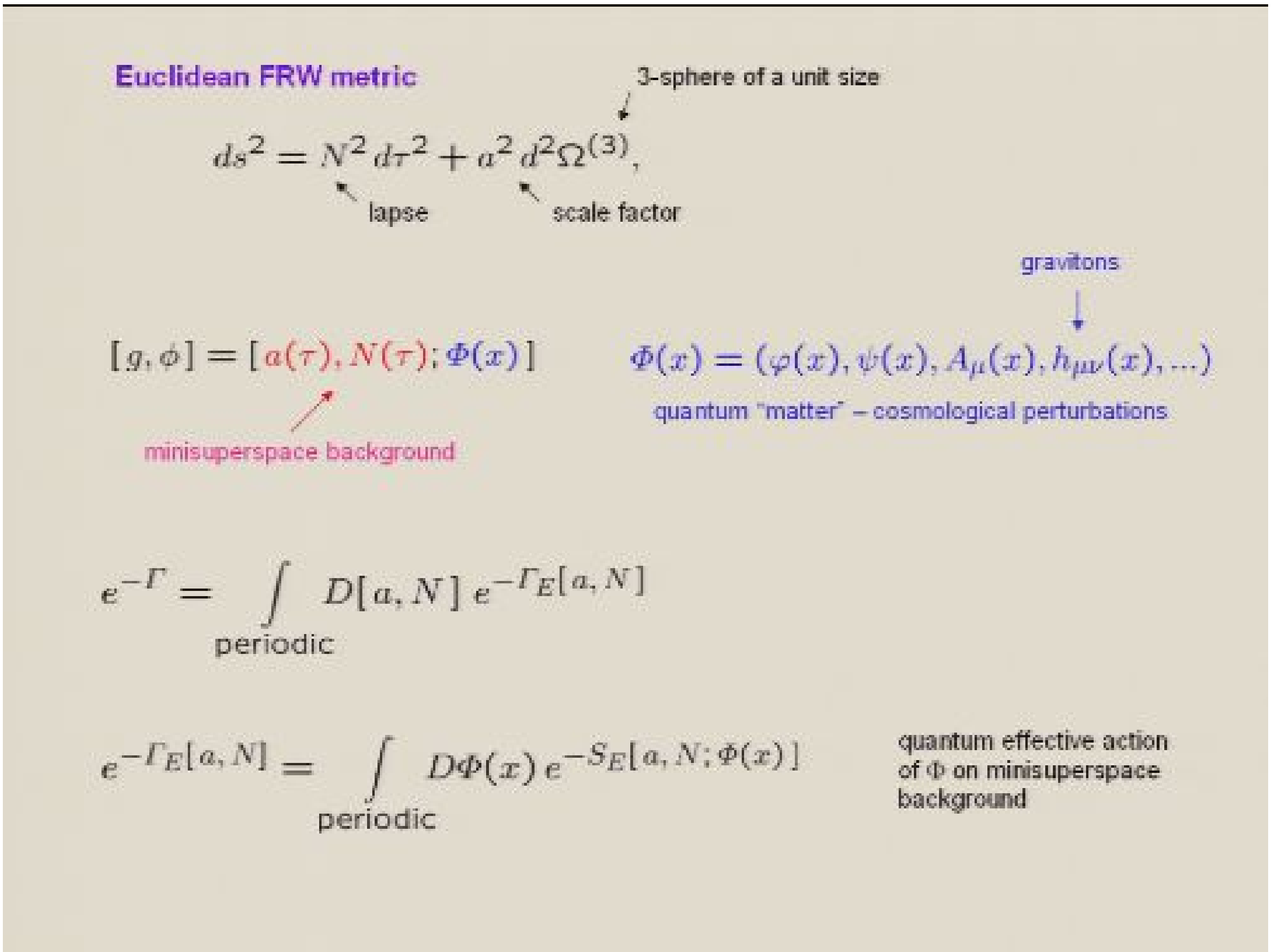
D.Page (1986)

$$\mathrm{tr}\,\hat{\rho} = 1$$

Effective action: statistical sum

$$e^{-\Gamma} = \int\limits_{g,\phi|_{\Sigma}=g,\phi|_{\Sigma'}} D[g,\phi]\,\exp\big(-S_{\mathrm{E}}[g,\phi]\big)$$

integration over periodic fields on a torus:



Euclidean FRW metric

3-sphere of a unit size

$$ds^2 = N^2\, d\tau^2 + a^2\, d^2\Omega^{(3)},$$

lapse

scale factor

gravitons

$$[g,\phi] = [\,a(\tau), N(\tau); \Phi(x)\,]$$

$$\Phi(x) = (\varphi(x), \psi(x), A_\mu(x), h_{\mu\nu}(x), \dots)$$

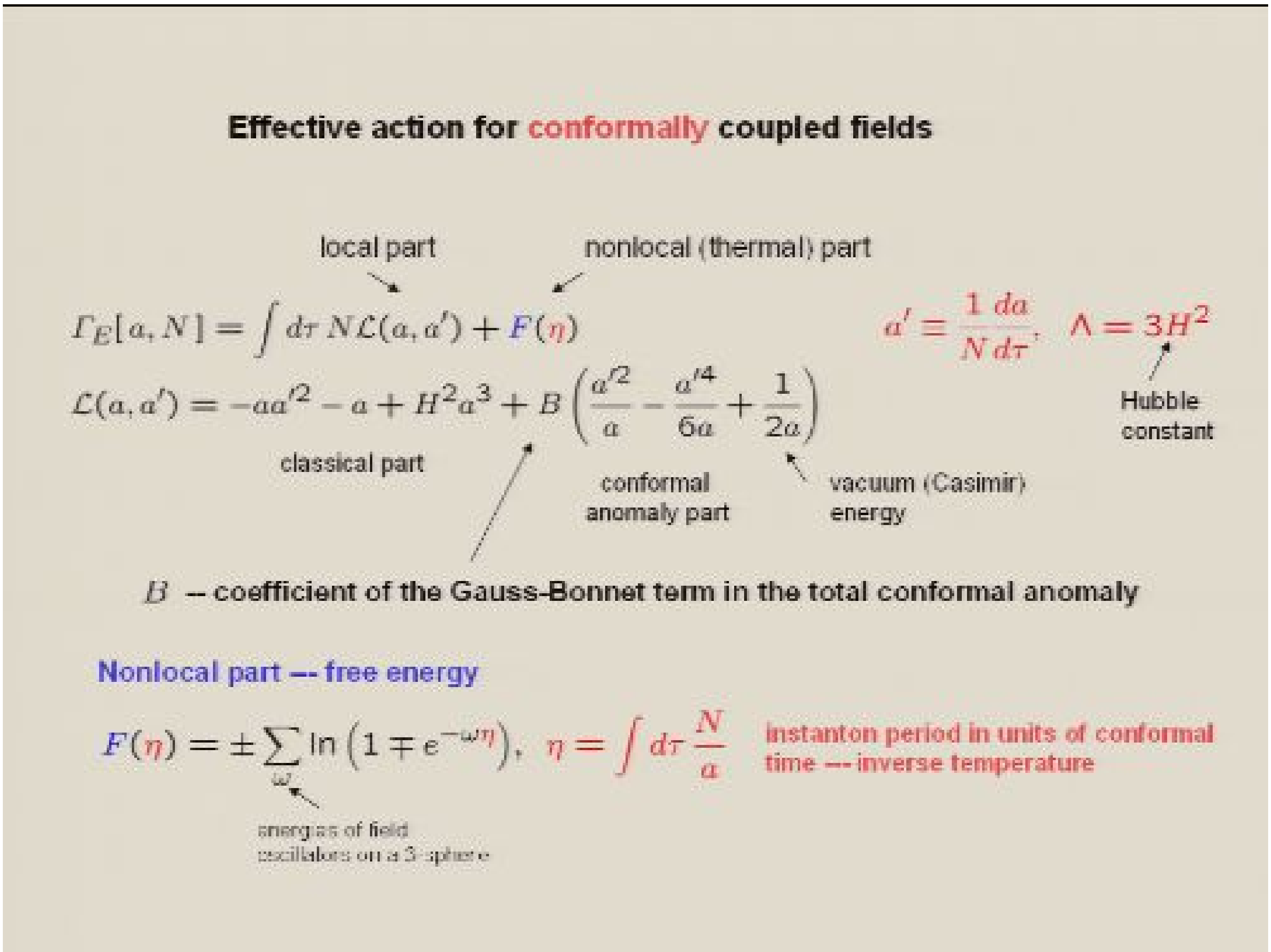
quantum "matter" – cosmological perturbations

minisuperspace background

$$e^{-\Gamma} = \int_{\text{periodic}} D[\,a, N\,]\, e^{-\Gamma_E[\,a,N\,]}$$

$$e^{-\Gamma_E[\,a,N]} = \int_{\text{periodic}} D\Phi(x)\, e^{-S_E[\,a,N;\Phi(x)\,]}$$

quantum effective action of Φ on minisuperspace background



Effective action for conformally coupled fields

local part

nonlocal (thermal) part

$$\Gamma_E[a,N] = \int d\tau\, N\mathcal{L}(a,a') + F(\eta)$$

$$a' = \frac{1}{N}\frac{da}{d\tau}, \quad \Lambda = 3H^2$$

$$\mathcal{L}(a,a') = -aa'^2 - a + H^2a^3 + B\left(\frac{a'^2}{a} - \frac{a'^4}{6a} + \frac{1}{2a}\right)$$

classical part

conformal
anomaly part

vacuum (Casimir)
energy

Hubble
constant

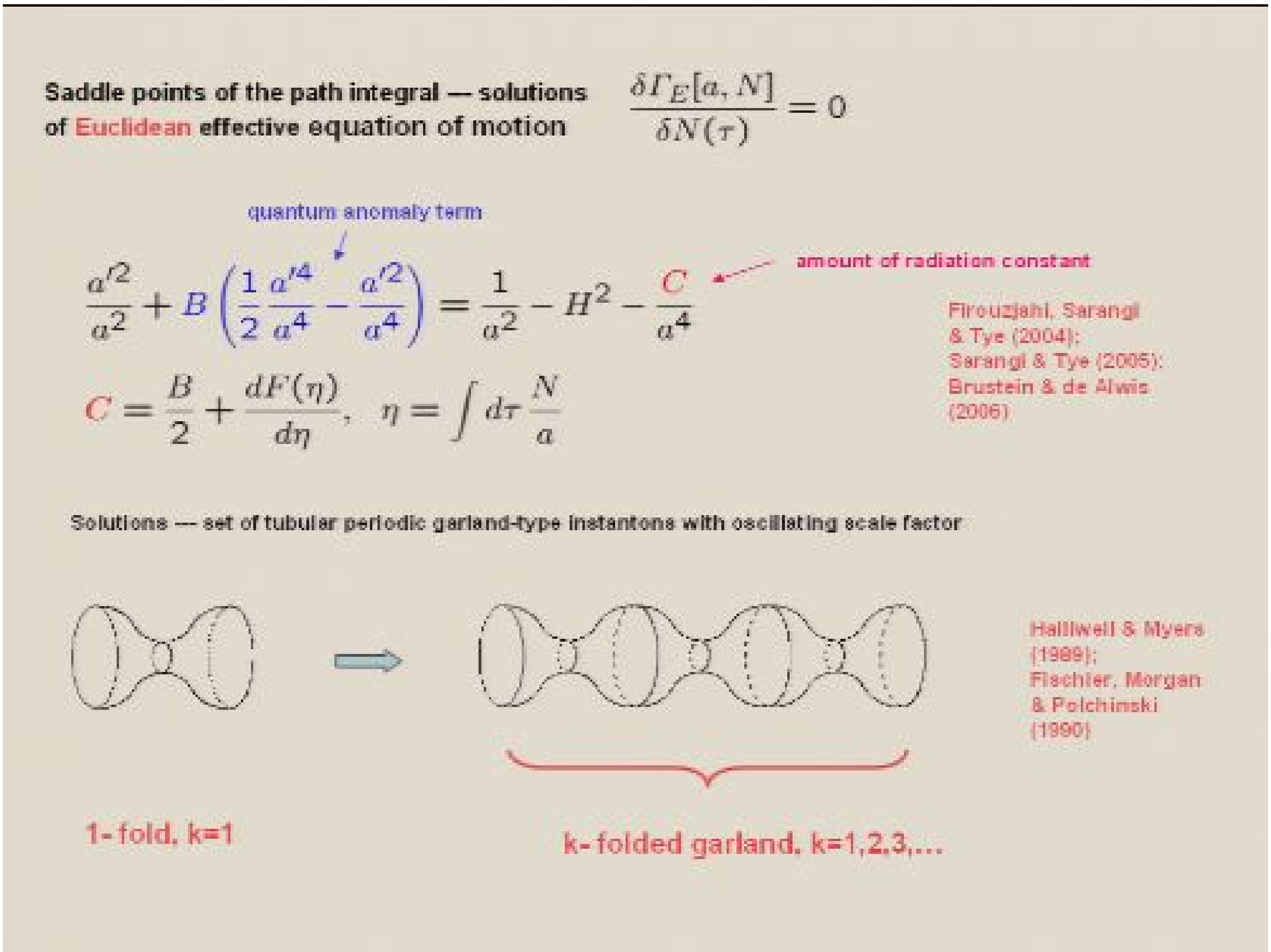
B -- coefficient of the Gauss-Bonnet term in the total conformal anomaly

Nonlocal part --- free energy

$$F(\eta) = \pm\sum_\omega \ln\left(1 \mp e^{-\omega\eta}\right), \quad \eta = \int d\tau \frac{N}{a}$$

instanton period in units of conformal
time --- inverse temperature

energies of field
oscillators on a 3-sphere



Saddle points of the path integral — solutions of Euclidean effective equation of motion $\dfrac{\delta\Gamma_E[a,N]}{\delta N(\tau)}=0$

quantum anomaly term

$$\frac{a'^2}{a^2}+B\left(\frac{1}{2}\frac{a'^4}{a^4}-\frac{a'^2}{a^4}\right)=\frac{1}{a^2}-H^2-\frac{C}{a^4}$$

amount of radiation constant

$$C=\frac{B}{2}+\frac{dF(\eta)}{d\eta},\quad \eta=\int d\tau\,\frac{N}{a}$$

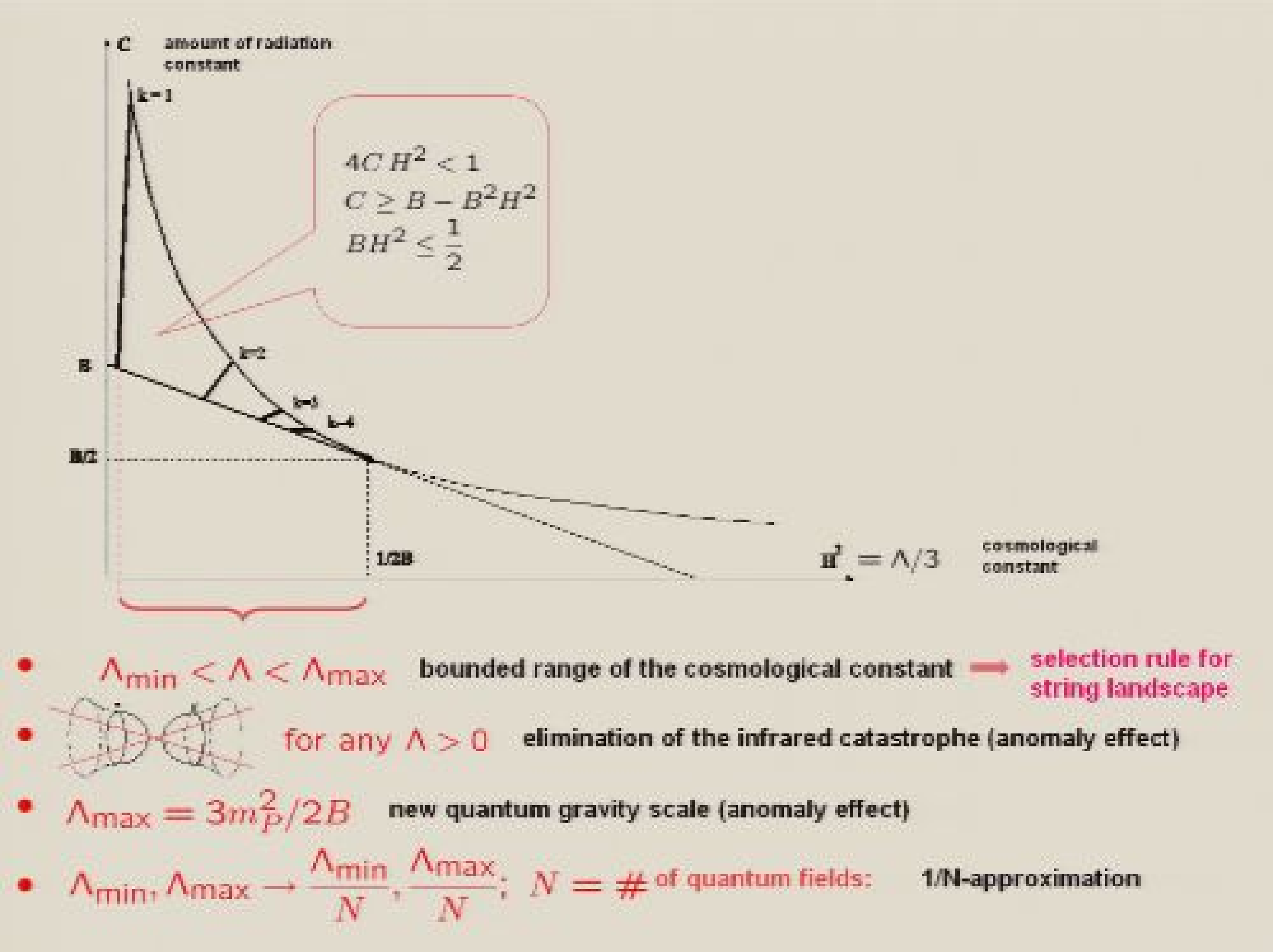
Firouzjahi, Sarangi & Tye (2004);
Sarangi & Tye (2005);
Brustein & de Alwis (2006)

Solutions — set of tubular periodic garland-type instantons with oscillating scale factor

Halliwell & Myers (1989);
Fischler, Morgan & Polchinski (1990)

1-fold, k=1

k-folded garland, k=1,2,3,...

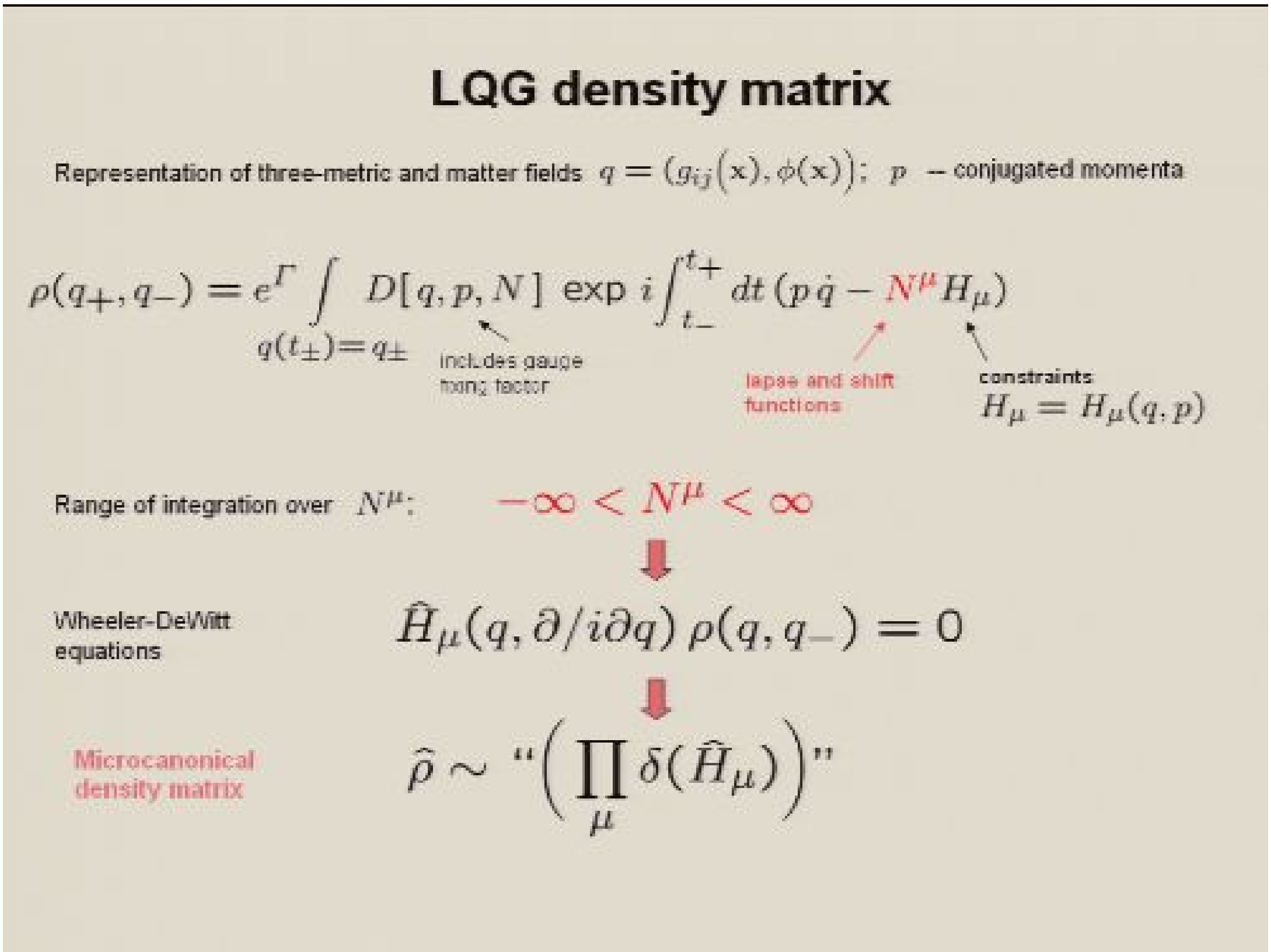


$4CH^2 < 1$

$C \geq B - B^2H^2$

$BH^2 \leq \frac{1}{2}$

- $\Lambda_{\rm min} < \Lambda < \Lambda_{\rm max}$ bounded range of the cosmological constant ➡ selection rule for string landscape
- for any $\Lambda > 0$ elimination of the infrared catastrophe (anomaly effect)
- $\Lambda_{\rm max} = 3m_P^2/2B$ new quantum gravity scale (anomaly effect)
- $\Lambda_{\rm min}, \Lambda_{\rm max} \to \frac{\Lambda_{\rm min}}{N}, \frac{\Lambda_{\rm max}}{N}$; $N = \#$ of quantum fields: 1/N-approximation



LQG density matrix

Representation of three-metric and matter fields $q = \big(g_{ij}(\mathbf{x}), \phi(\mathbf{x})\big)$; p -- conjugated momenta

$$\rho(q_+, q_-) = e^{\Gamma} \int_{q(t_\pm)=q_\pm} D[\,q, p, N\,] \;\exp\, i\int_{t_-}^{t_+} dt\,(p\,\dot{q} - N^\mu H_\mu)$$

includes gauge fixing factor

lapse and shift functions

constraints
$H_\mu = H_\mu(q, p)$

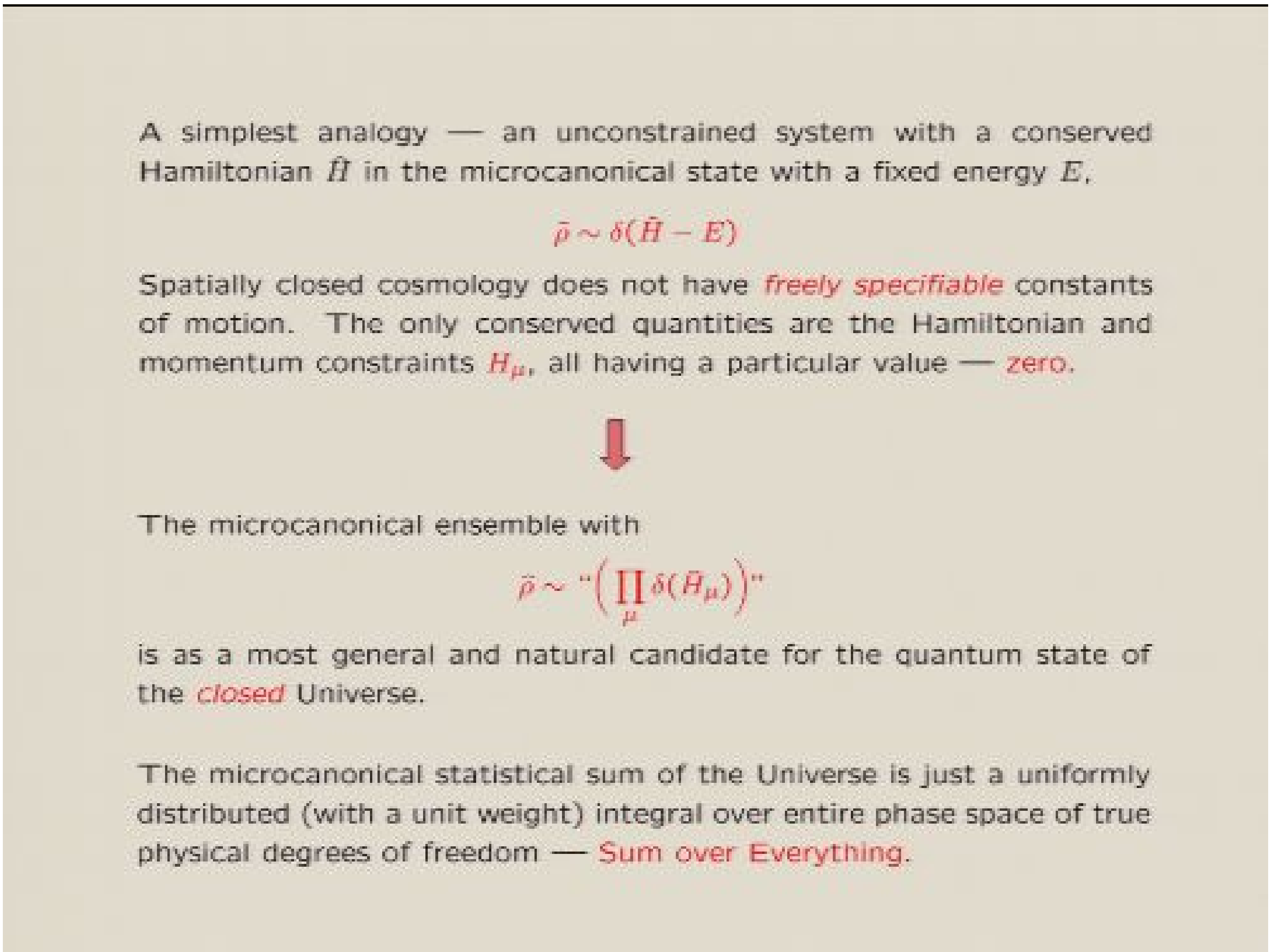
Range of integration over N^μ: $-\infty < N^\mu < \infty$

⇓

Wheeler-DeWitt equations $\hat{H}_\mu(q, \partial/i\partial q)\,\rho(q, q_-) = 0$

⇓

Microcanonical density matrix $\hat{\rho} \sim$ "$\Big(\prod_\mu \delta(\hat{H}_\mu)\Big)$"



A simplest analogy — an unconstrained system with a conserved Hamiltonian $\hat{H}$ in the microcanonical state with a fixed energy E,

$$\hat{\rho} \sim \delta(\hat{H} - E)$$

Spatially closed cosmology does not have *freely specifiable* constants of motion. The only conserved quantities are the Hamiltonian and momentum constraints H_μ, all having a particular value — zero.

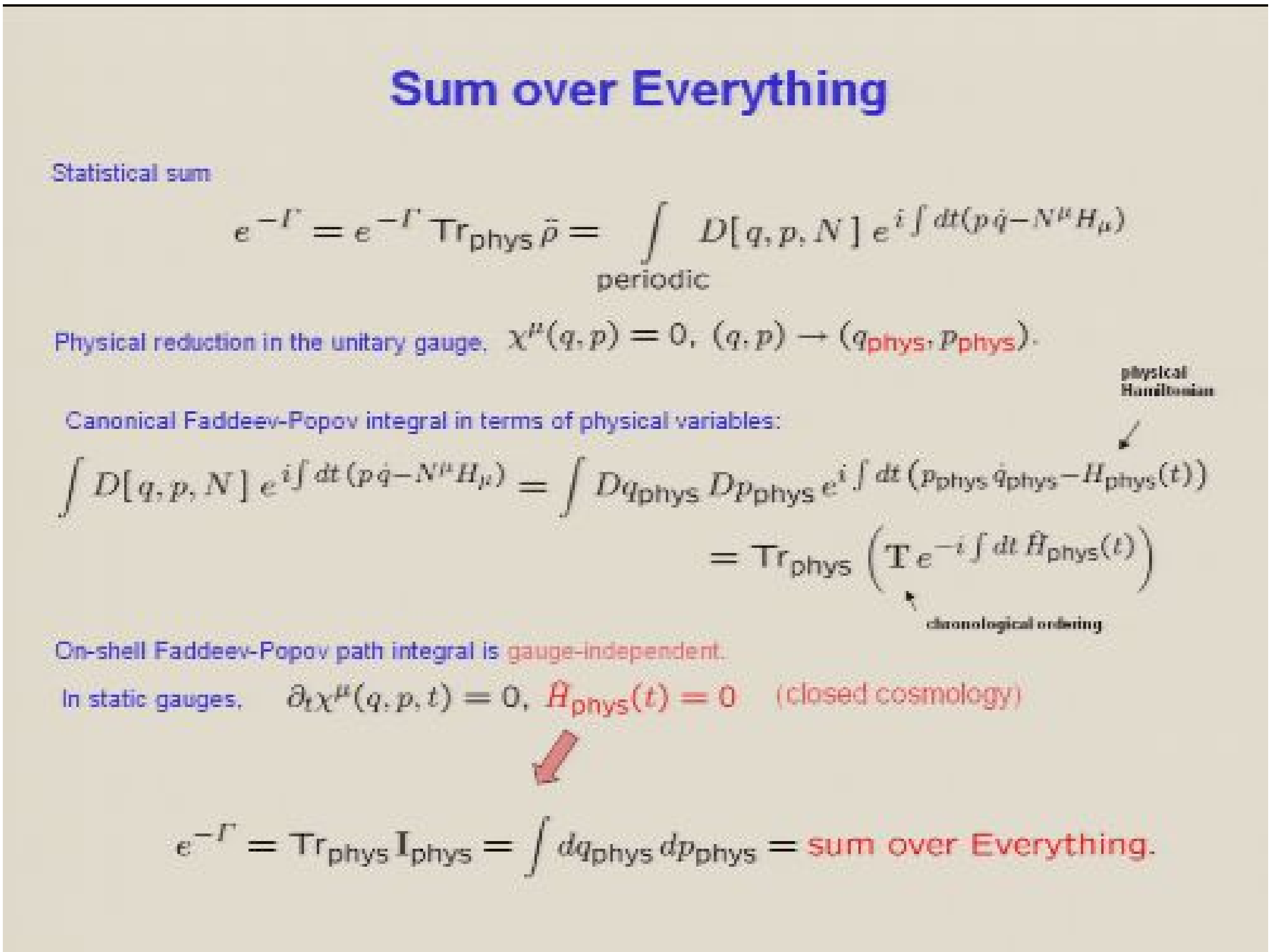
⇓

The microcanonical ensemble with

$$\hat{\rho} \sim \text{``}\Big(\prod_{\mu} \delta(\hat{H}_\mu)\Big)\text{''}$$

is as a most general and natural candidate for the quantum state of the *closed* Universe.

The microcanonical statistical sum of the Universe is just a uniformly distributed (with a unit weight) integral over entire phase space of true physical degrees of freedom — Sum over Everything.



Sum over Everything

Statistical sum

$$e^{-\Gamma} = e^{-\Gamma}\,\mathrm{Tr}_{\mathrm{phys}}\,\hat{\rho} = \int_{\mathrm{periodic}} D[\,q,p,N\,]\; e^{i\int dt(p\dot{q}-N^{\mu}H_{\mu})}$$

Physical reduction in the unitary gauge, $\chi^{\mu}(q,p)=0,\ (q,p)\to(q_{\mathrm{phys}},p_{\mathrm{phys}})$.

Canonical Faddeev-Popov integral in terms of physical variables:

physical Hamiltonian

$$\int D[\,q,p,N\,]\; e^{i\int dt\,(p\dot{q}-N^{\mu}H_{\mu})} = \int Dq_{\mathrm{phys}}\,Dp_{\mathrm{phys}}\,e^{i\int dt\,(p_{\mathrm{phys}}\dot{q}_{\mathrm{phys}}-H_{\mathrm{phys}}(t))}$$

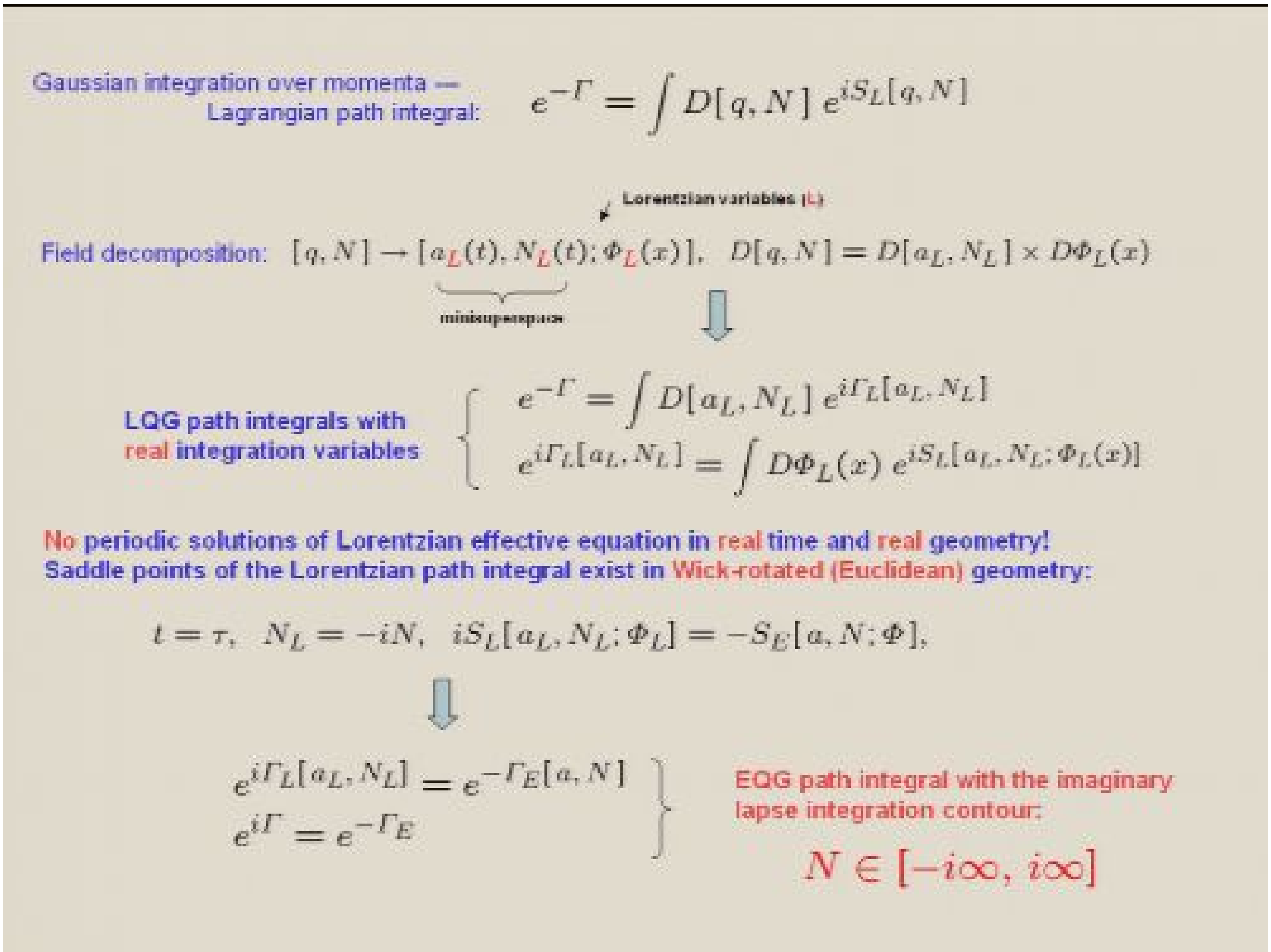
$$= \mathrm{Tr}_{\mathrm{phys}}\left(\mathbf{T}\,e^{-i\int dt\,\hat{H}_{\mathrm{phys}}(t)}\right)$$

chronological ordering

On-shell Faddeev-Popov path integral is gauge-independent.

In static gauges, $\partial_t\chi^{\mu}(q,p,t)=0,\ \hat{H}_{\mathrm{phys}}(t)=0$ (closed cosmology)

$$e^{-\Gamma} = \mathrm{Tr}_{\mathrm{phys}}\,\mathbf{I}_{\mathrm{phys}} = \int dq_{\mathrm{phys}}\,dp_{\mathrm{phys}} = \text{sum over Everything.}$$



Gaussian integration over momenta —
Lagrangian path integral:

$$e^{-\Gamma}=\int D[\,q,N\,]\;e^{iS_L[\,q,N\,]}$$

Lorentzian variables (L)

Field decomposition: $[\,q,N\,]\to[\underbrace{a_L(t),N_L(t)}_{\text{minisuperspace}};\Phi_L(x)]$, $D[\,q,N\,]=D[\,a_L,N_L\,]\times D\Phi_L(x)$

$$\Downarrow$$

LQG path integrals with
real integration variables

$$\begin{cases} e^{-\Gamma}=\int D[\,a_L,N_L\,]\;e^{i\Gamma_L[\,a_L,N_L\,]} \\ e^{i\Gamma_L[\,a_L,N_L\,]}=\int D\Phi_L(x)\;e^{iS_L[\,a_L,N_L;\Phi_L(x)]} \end{cases}$$

No periodic solutions of Lorentzian effective equation in real time and real geometry!
Saddle points of the Lorentzian path integral exist in Wick-rotated (Euclidean) geometry:

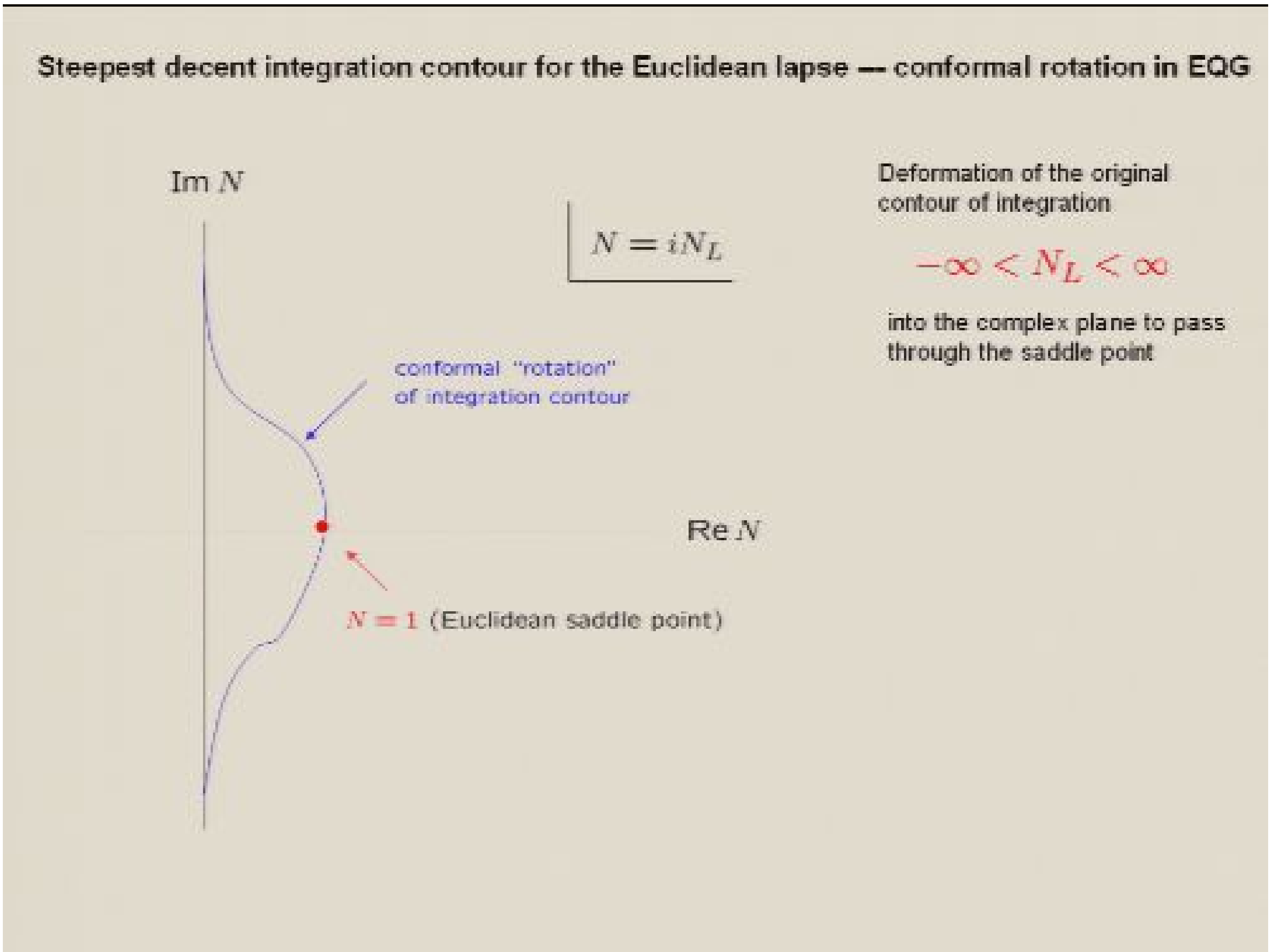
$$t=\tau,\quad N_L=-iN,\quad iS_L[\,a_L,N_L;\Phi_L]=-S_E[\,a,N;\Phi\,],$$

$$\Downarrow$$

$$\left.\begin{array}{l} e^{i\Gamma_L[a_L,N_L]}=e^{-\Gamma_E[\,a,N\,]} \\ e^{i\Gamma}=e^{-\Gamma_E} \end{array}\right\}$$

EQG path integral with the imaginary
lapse integration contour:

$$N\in[-i\infty,\,i\infty]$$



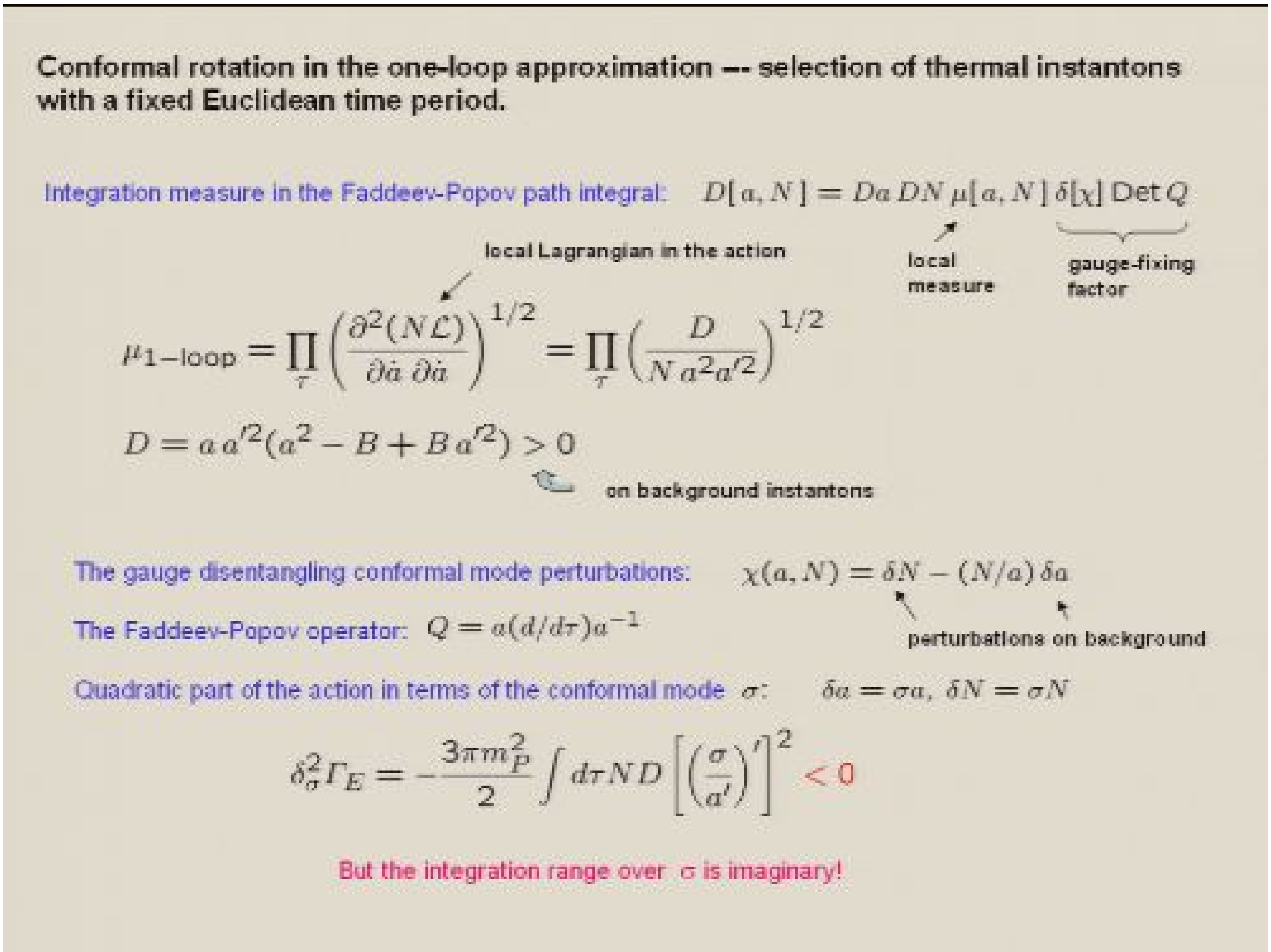
Steepest decent integration contour for the Euclidean lapse — conformal rotation in EQG

$N = iN_L$

Deformation of the original contour of integration

$-\infty < N_L < \infty$

into the complex plane to pass through the saddle point



Conformal rotation in the one-loop approximation -- selection of thermal instantons with a fixed Euclidean time period.

Integration measure in the Faddeev-Popov path integral: $D[\,a,N\,] = Da\,DN\,\mu[\,a,N\,]\,\underbrace{\delta[\chi]\,\mathrm{Det}\,Q}$

local measure

gauge-fixing factor

local Lagrangian in the action

$$\mu_{1-\mathrm{loop}} = \prod_\tau \left(\frac{\partial^2(N\mathcal{L})}{\partial\dot{a}\,\partial\dot{a}}\right)^{1/2} = \prod_\tau \left(\frac{D}{N\,a^2 a'^2}\right)^{1/2}$$

$$D = a\,a'^2(a^2 - B + B\,a'^2) > 0$$

on background instantons

The gauge disentangling conformal mode perturbations: $\chi(a,N) = \delta N - (N/a)\,\delta a$

perturbations on background

The Faddeev-Popov operator: $Q = a(d/d\tau)a^{-1}$

Quadratic part of the action in terms of the conformal mode σ: $\delta a = \sigma a,\ \delta N = \sigma N$

$$\delta^2_\sigma \varGamma_E = -\frac{3\pi m_P^2}{2}\int d\tau N D\left[\left(\frac{\sigma}{a'}\right)'\right]^2 < 0$$

But the integration range over σ is imaginary!

Density Matrix Reloaded:

Minimum set of assumptions — an ultimate equipartition in physical phase space in the form of the microcanonical state of closed quantum cosmology

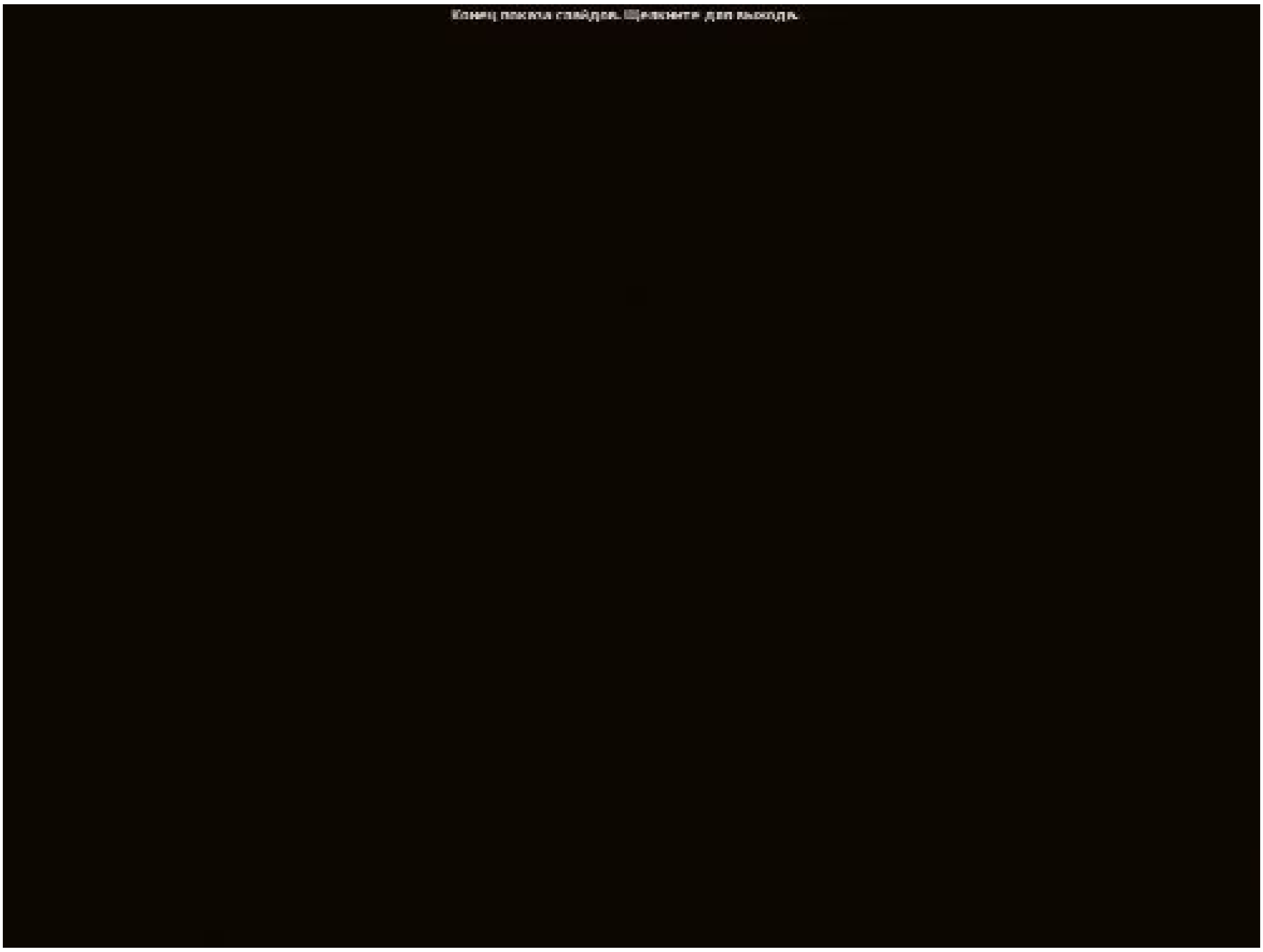
⬇

- Constraining the ensemble of Λ (and possibly landscape of string vacua)
- A new dark energy mechanism transcending the inflationary and matter-domination stages as a quasi-equilibrium decay of the initial microcanonical state

``Nothing comes from Nothing''

Sidney Coleman: ``There is Nothing rather than Something''

Something (rather than Nothing) comes from Everything



Конец показа слайдов. Щелкните для выхода.

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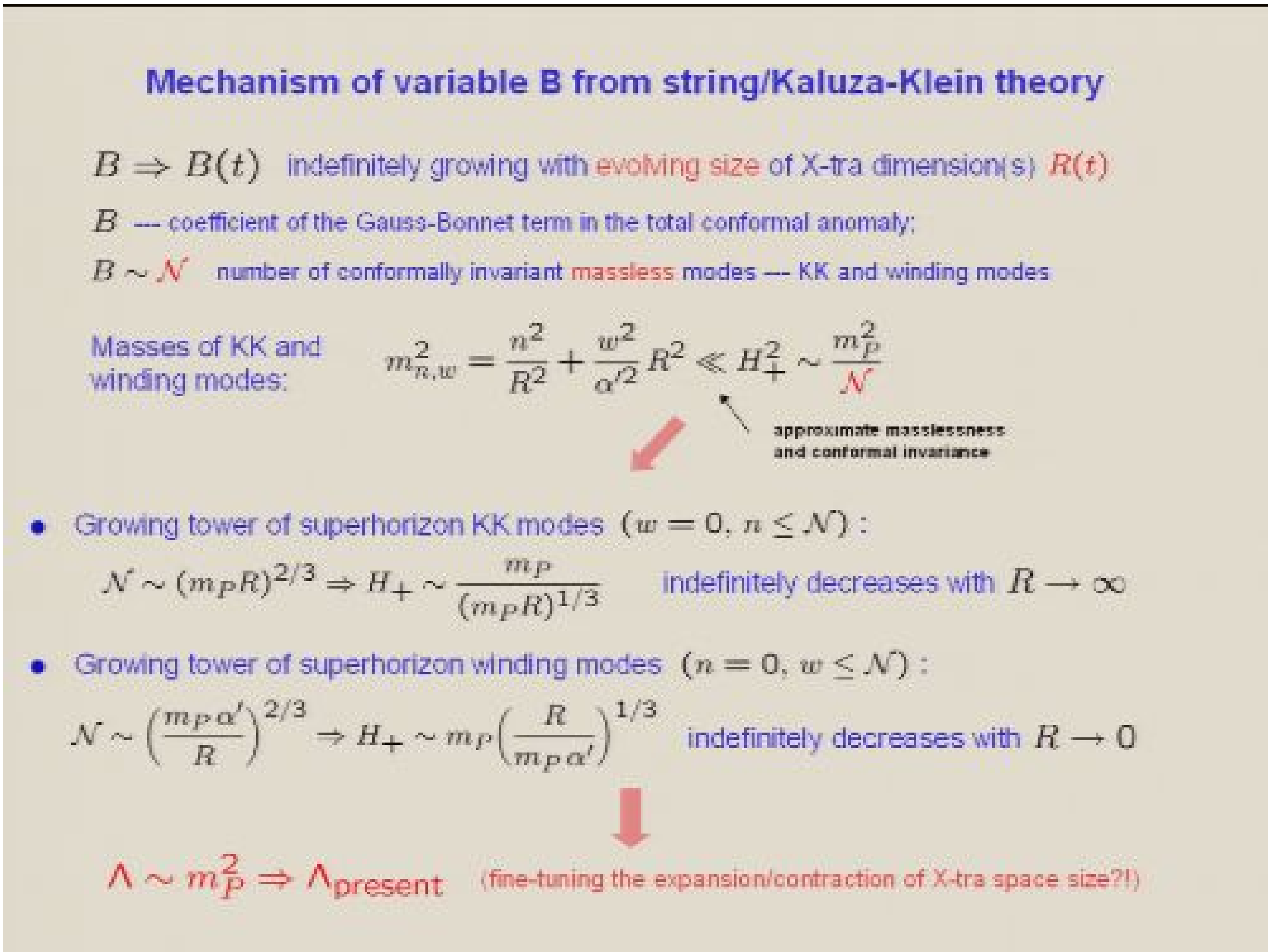

Mechanism of variable B from string/Kaluza-Klein theory

$B \Rightarrow B(t)$ indefinitely growing with evolving size of X-tra dimension(s) $R(t)$

B --- coefficient of the Gauss-Bonnet term in the total conformal anomaly;

$B \sim \mathcal{N}$ number of conformally invariant massless modes --- KK and winding modes

Masses of KK and winding modes: $$m_{n,w}^2 = \frac{n^2}{R^2} + \frac{w^2}{\alpha'^2} R^2 \ll H_+^2 \sim \frac{m_P^2}{\mathcal{N}}$$

approximate masslessness and conformal invariance

- Growing tower of superhorizon KK modes $(w = 0,\ n \leq \mathcal{N})$:

$$\mathcal{N} \sim (m_P R)^{2/3} \Rightarrow H_+ \sim \frac{m_P}{(m_P R)^{1/3}}$$ indefinitely decreases with $R \to \infty$

- Growing tower of superhorizon winding modes $(n = 0,\ w \leq \mathcal{N})$:

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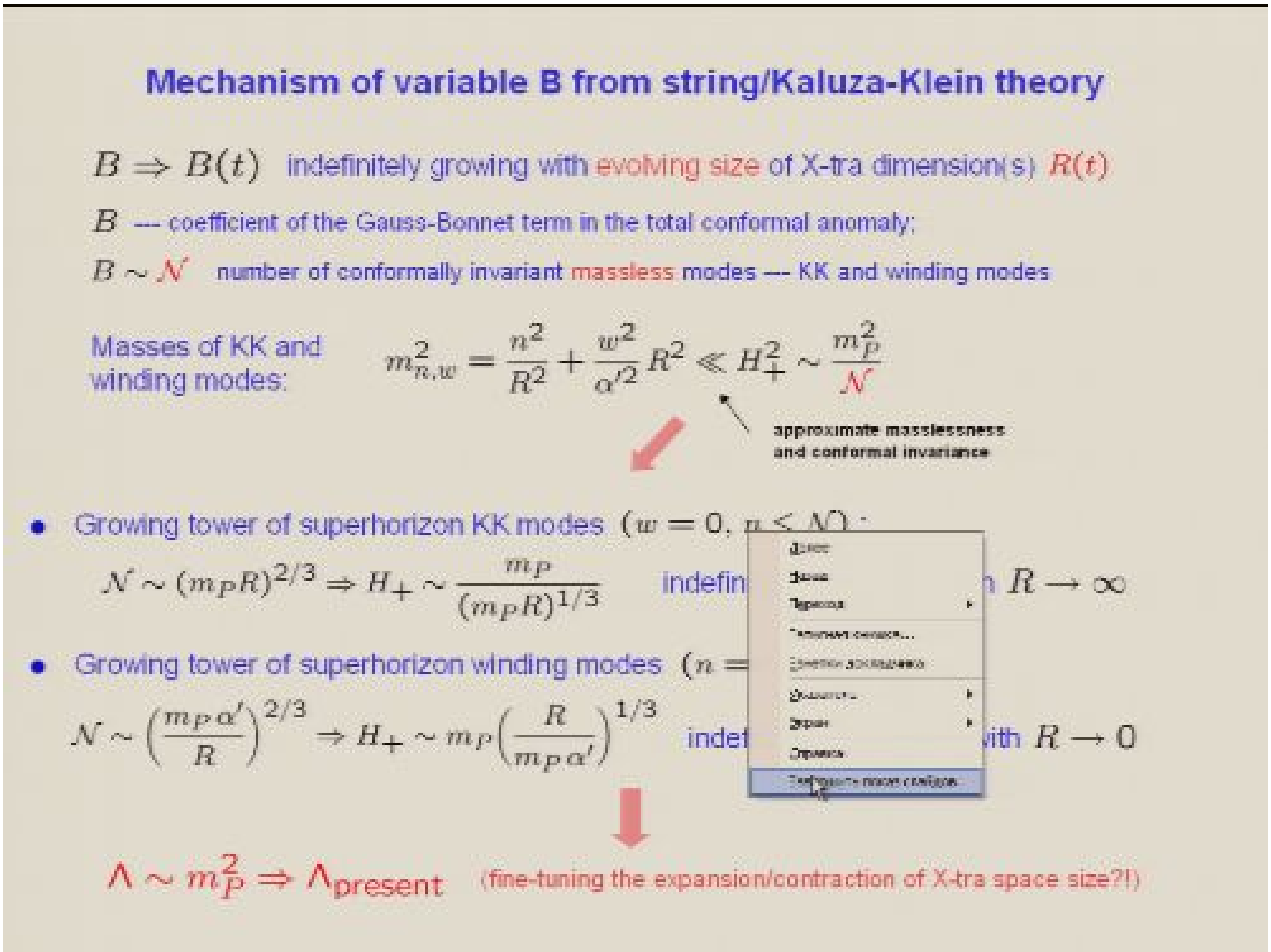
$\Lambda \sim m_P^2 \Rightarrow \Lambda_{\rm present}$ (fine-tuning the expansion/contraction of X-tra space size?!)



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indefin n $R \to \infty$

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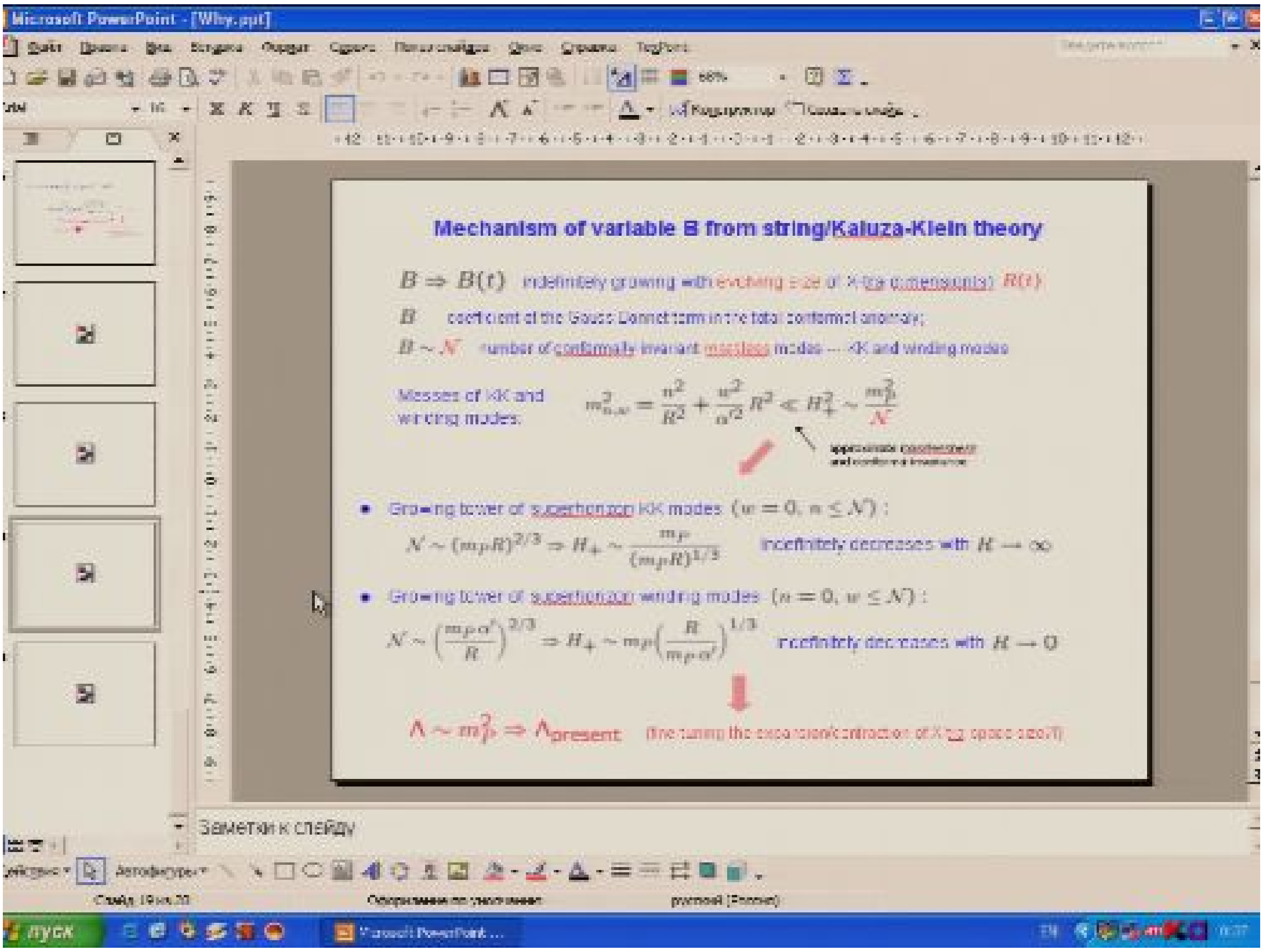

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B coefficient of the Gauss-Bonnet term in the total conformal anomaly;

$B \sim N$ number of conformally invariant massless modes — KK and winding modes

Masses of KK and winding modes: $m^2_{n,w} = \frac{n^2}{R^2} + \frac{w^2}{\alpha'^2} R^2 \ll H_+^2 \sim \frac{m_P^2}{N}$

approximate masslessness and conformal invariance

- Growing tower of superhorizon KK modes $(w = 0,\ n \leq N)$:

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- Growing tower of superhorizon winding modes $(n = 0,\ w \leq N)$:

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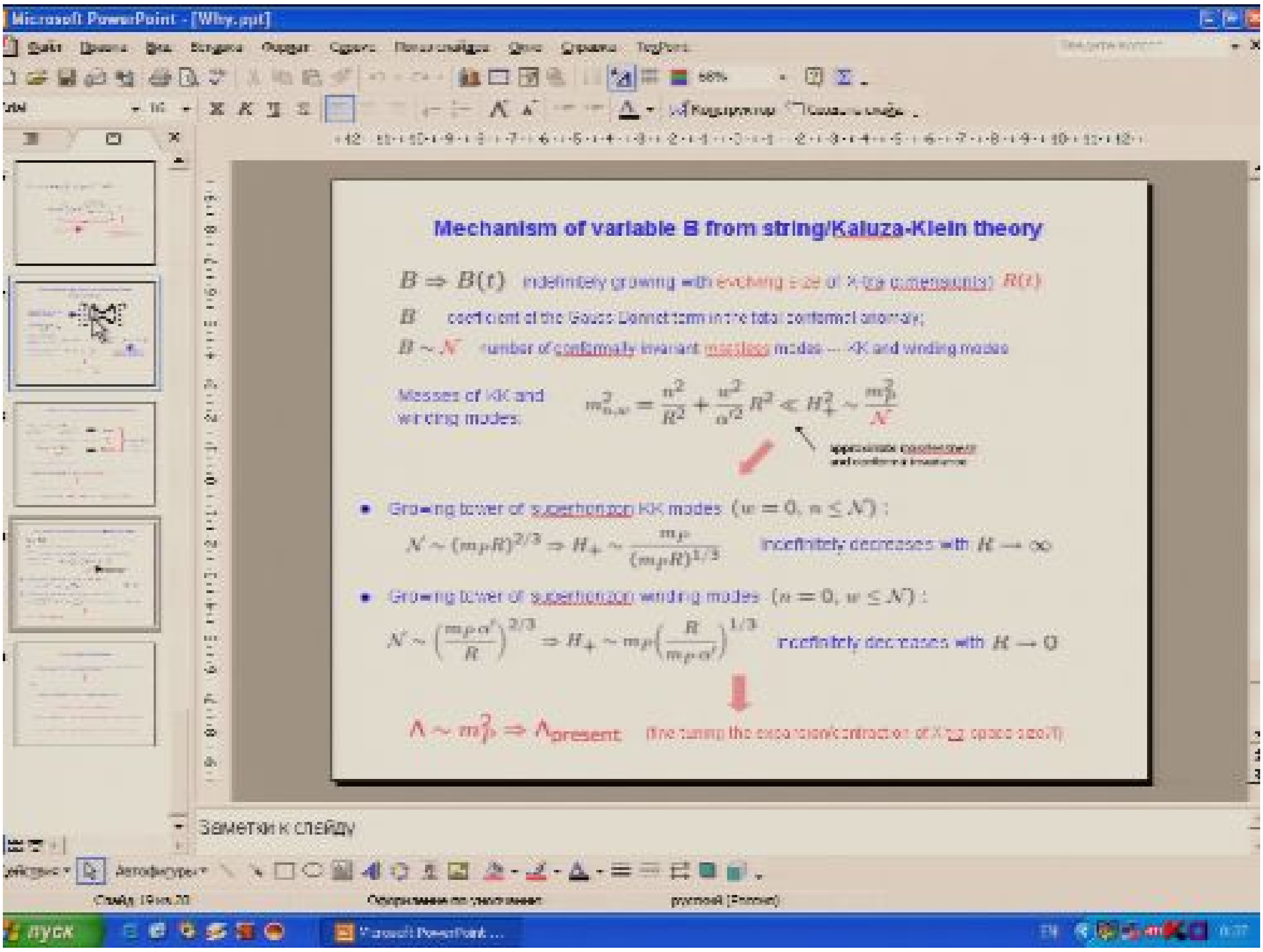
$\Lambda \sim m_P^2 \Rightarrow \Lambda_{\text{present}}$ (fine tuning the expansion/contraction of X (no space size?))



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approximate masslessness and conformal invariance

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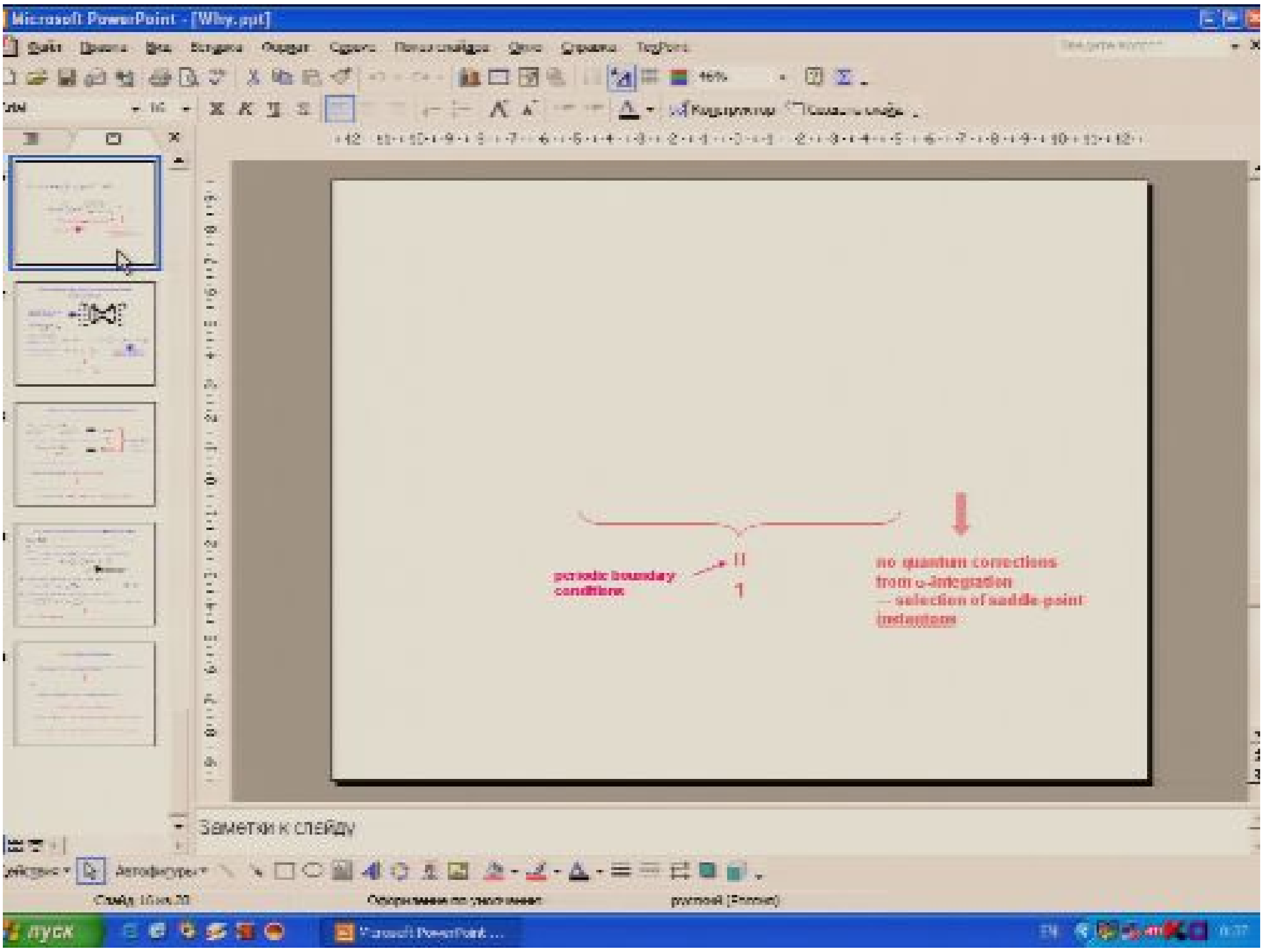
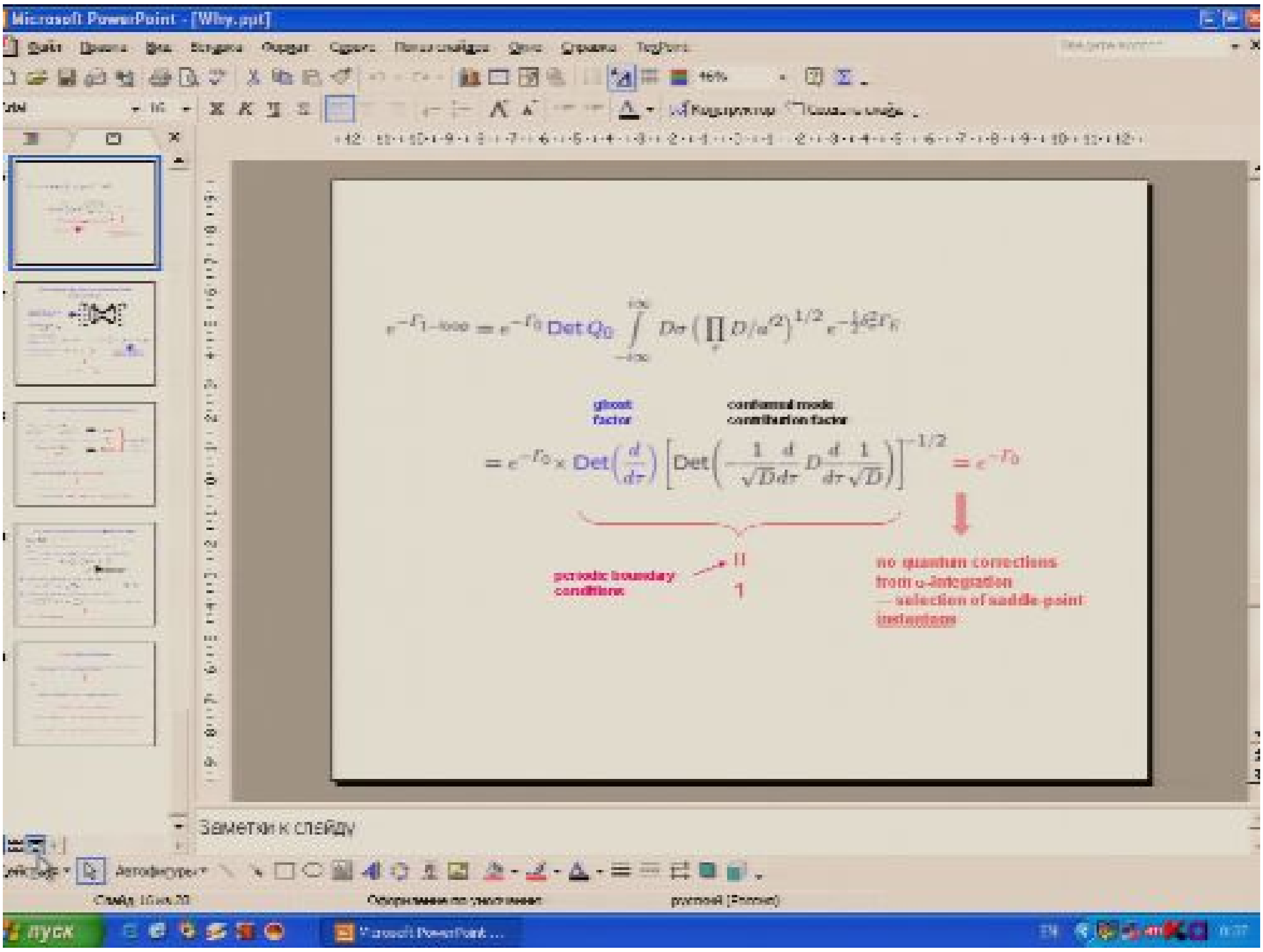
$\Lambda \sim m_P^2 \Rightarrow \Lambda_{\text{present}}$ (fine tuning the expansion/contraction of X to space size?)



periodic boundary conditions

no quantum corrections from ω-integration
— selection of saddle-point instantons



$$e^{-\Gamma_{1-loop}} = e^{-\Gamma_0}\,\mathrm{Det}\,Q_0 \int\limits_{-\infty}^{+\infty} D\sigma\,\Big(\prod_\tau D/a^2\Big)^{1/2} e^{-\frac{1}{2}\delta^2\Gamma_0}$$

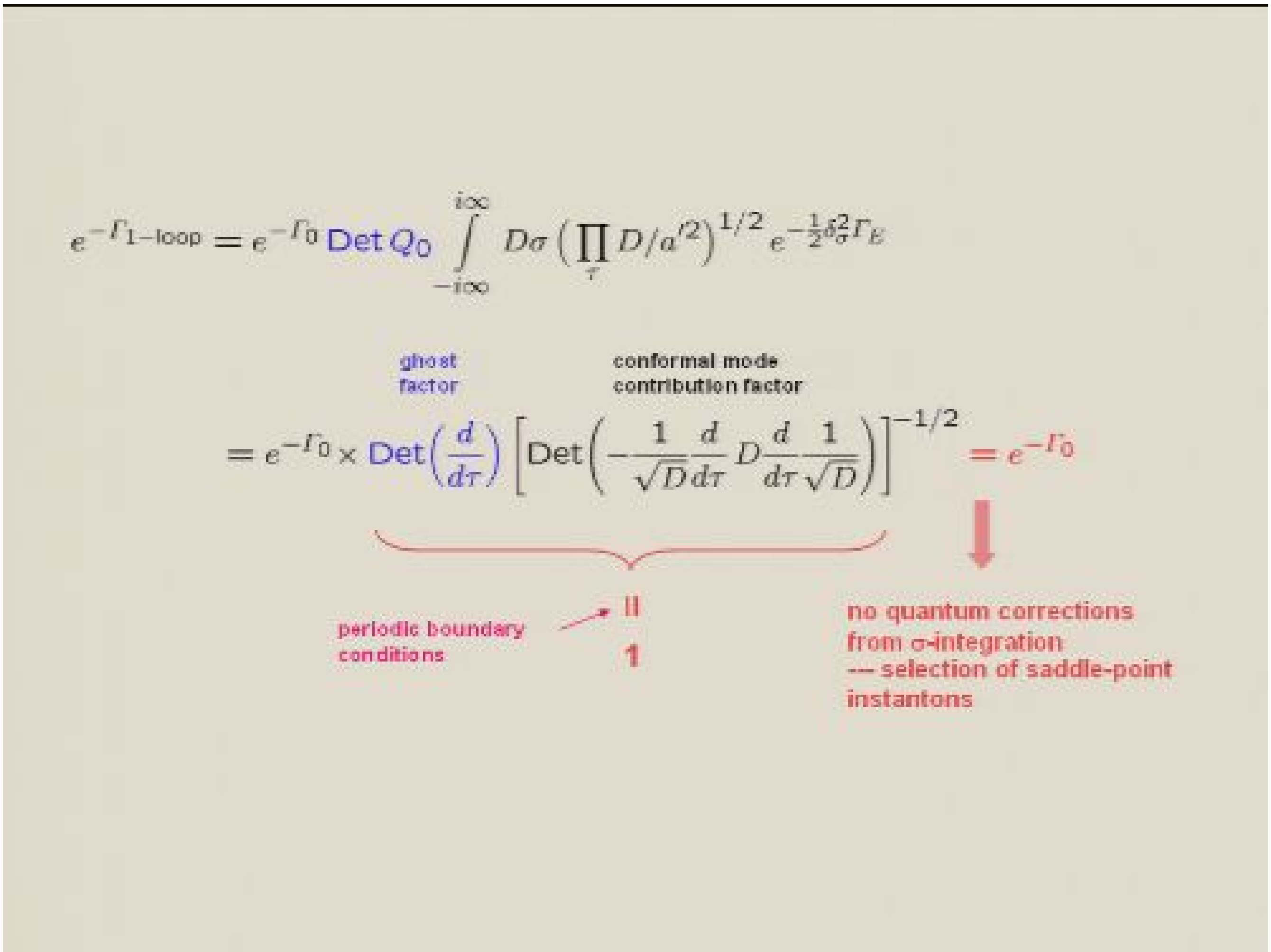
ghost factor

conformal mode contribution factor

$$= e^{-\Gamma_0}\times \underbrace{\mathrm{Det}\Big(\frac{d}{d\tau}\Big)\Big[\mathrm{Det}\Big(-\frac{1}{\sqrt{D}}\frac{d}{d\tau}D\frac{d}{d\tau}\frac{1}{\sqrt{D}}\Big)\Big]^{-1/2}}_{\begin{array}{c}\| \\ 1\end{array}} = e^{-\Gamma_0}$$

periodic boundary conditions

no quantum corrections from σ-integration — selection of saddle-point instantons



$$e^{-\Gamma_{1-\text{loop}}} = e^{-\Gamma_0}\,\mathrm{Det}\,Q_0 \int\limits_{-i\infty}^{i\infty} D\sigma \Big(\prod_{\tau} D/a'^2\Big)^{1/2} e^{-\frac{1}{2}\delta_\sigma^2 \Gamma_E}$$

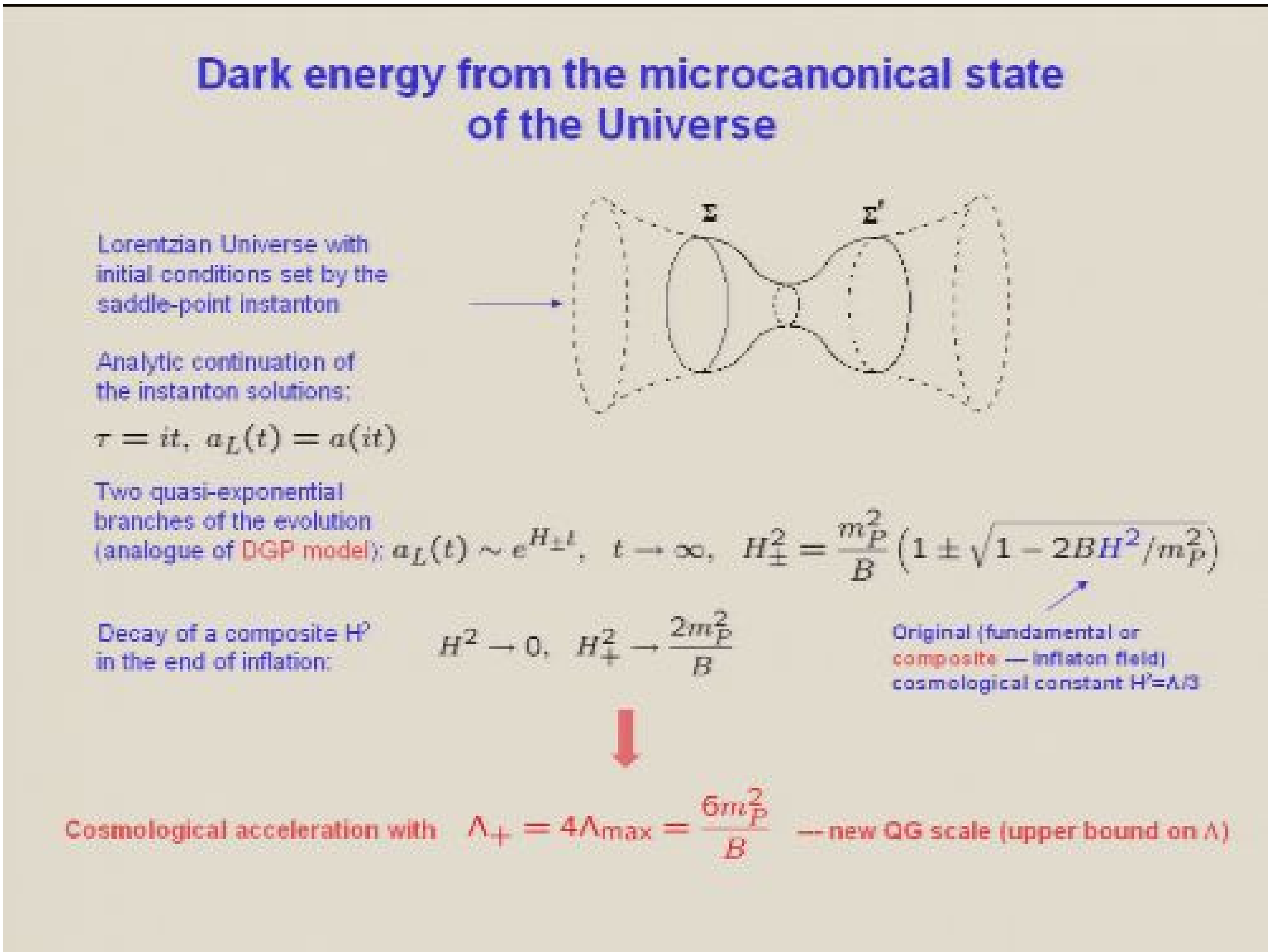
ghost factor

conformal mode contribution factor

$$= e^{-\Gamma_0} \times \underbrace{\mathrm{Det}\Big(\frac{d}{d\tau}\Big)\Big[\mathrm{Det}\Big(-\frac{1}{\sqrt{D}}\frac{d}{d\tau}D\frac{d}{d\tau}\frac{1}{\sqrt{D}}\Big)\Big]^{-1/2}}_{\overset{\|}{1}} = e^{-\Gamma_0}$$

periodic boundary conditions

no quantum corrections from σ-integration
--- selection of saddle-point instantons



Dark energy from the microcanonical state of the Universe

Lorentzian Universe with initial conditions set by the saddle-point instanton

Analytic continuation of the instanton solutions:

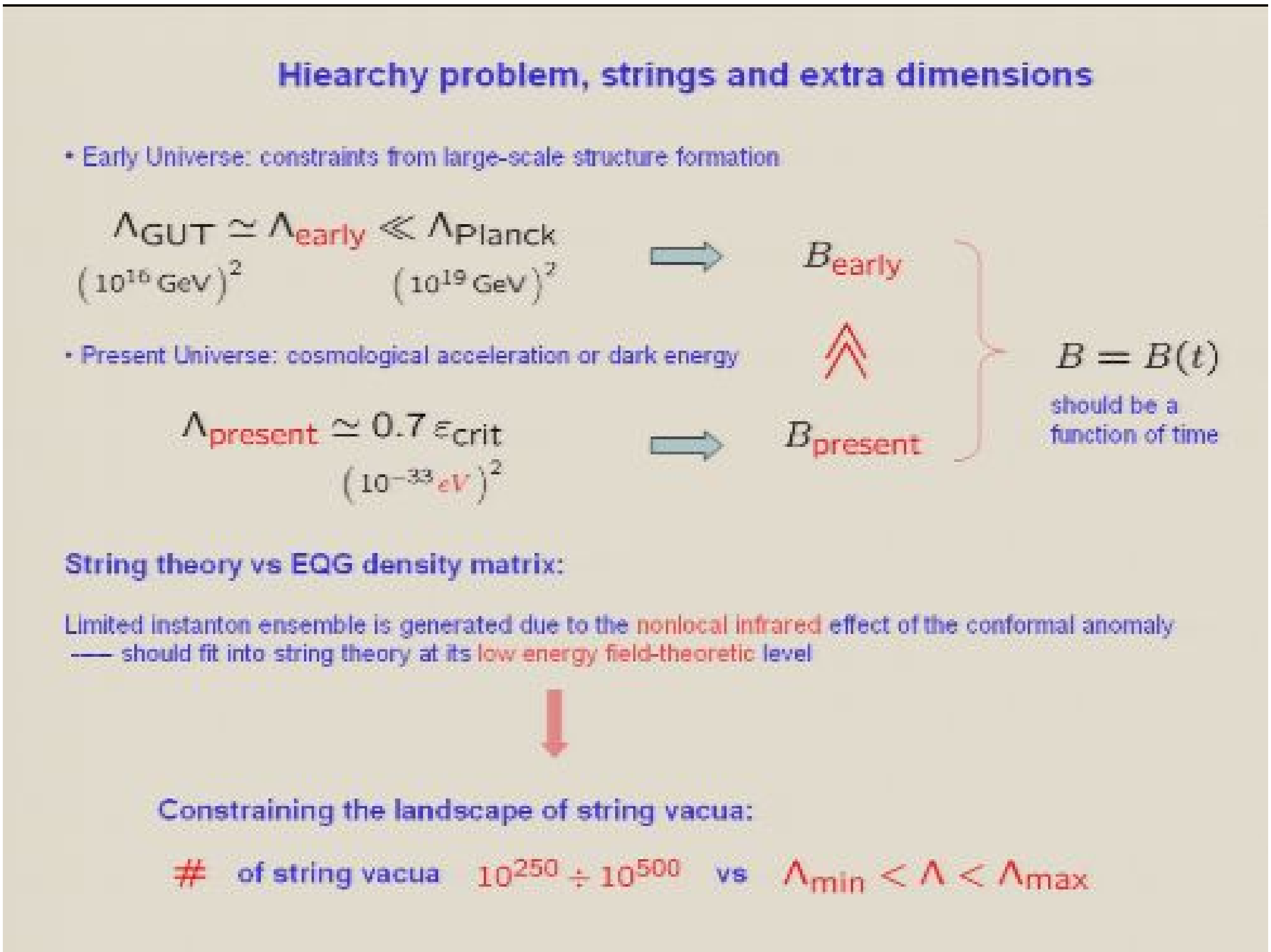
$$\tau = it,\ a_L(t) = a(it)$$

Two quasi-exponential branches of the evolution (analogue of DGP model): $a_L(t) \sim e^{H_\pm t},\ \ t \to \infty,\ \ H_\pm^2 = \frac{m_P^2}{B}\left(1 \pm \sqrt{1 - 2BH^2/m_P^2}\right)$

Decay of a composite H^2 in the end of inflation: $H^2 \to 0,\ \ H_+^2 \to \frac{2m_P^2}{B}$

Original (fundamental or composite — inflaton field) cosmological constant $H^2=\Lambda/3$

Cosmological acceleration with $\Lambda_+ = 4\Lambda_{\rm max} = \frac{6m_P^2}{B}$ -- new QG scale (upper bound on Λ)



Hiearchy problem, strings and extra dimensions

- Early Universe: constraints from large-scale structure formation

$$\underset{(10^{16}\,\text{GeV})^2}{\Lambda_{\text{GUT}}} \simeq \Lambda_{\text{early}} \ll \underset{(10^{19}\,\text{GeV})^2}{\Lambda_{\text{Planck}}} \quad \Longrightarrow \quad B_{\text{early}}$$

- Present Universe: cosmological acceleration or dark energy

$$\Lambda_{\text{present}} \simeq 0.7\,\underset{(10^{-33}\,eV)^2}{\varepsilon_{\text{crit}}} \quad \Longrightarrow \quad B_{\text{present}}$$

$$\left.\begin{array}{c} B_{\text{early}} \\ \rotatebox{-90}{$\ll$} \\ B_{\text{present}} \end{array}\right\} \quad B = B(t)$$

should be a function of time

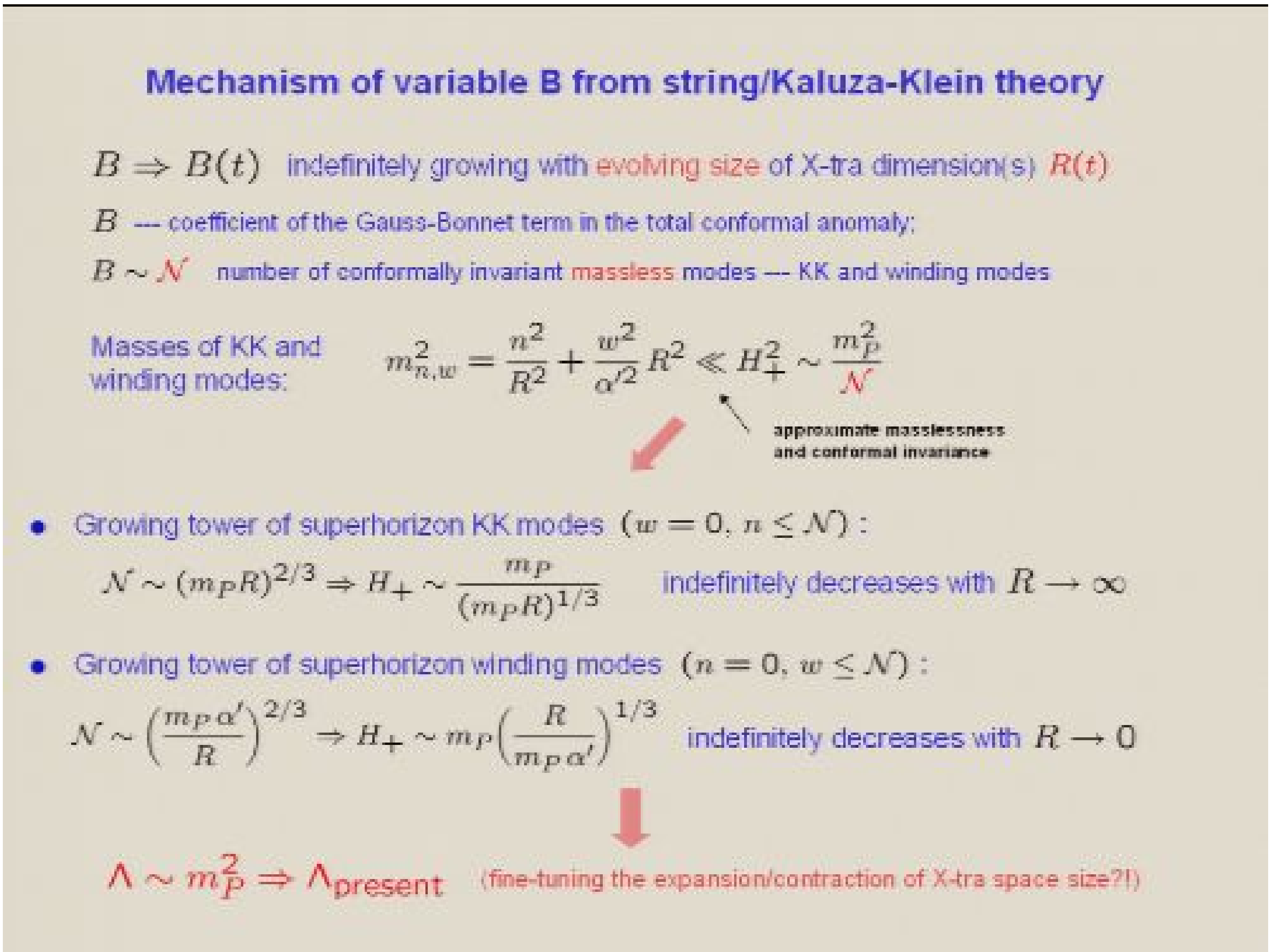
String theory vs EQG density matrix:

Limited instanton ensemble is generated due to the nonlocal infrared effect of the conformal anomaly
—— should fit into string theory at its low energy field-theoretic level

$$\Downarrow$$

Constraining the landscape of string vacua:

\# of string vacua $10^{250} \div 10^{500}$ vs $\Lambda_{\text{min}} < \Lambda < \Lambda_{\text{max}}$



Mechanism of variable B from string/Kaluza-Klein theory

$B \Rightarrow B(t)$ indefinitely growing with evolving size of X-tra dimension(s) $R(t)$

B --- coefficient of the Gauss-Bonnet term in the total conformal anomaly;

$B \sim \mathcal{N}$ number of conformally invariant massless modes --- KK and winding modes

Masses of KK and winding modes:

$$m^2_{n,w} = \frac{n^2}{R^2} + \frac{w^2}{\alpha'^2} R^2 \ll H^2_+ \sim \frac{m^2_P}{\mathcal{N}}$$

approximate masslessness and conformal invariance

- Growing tower of superhorizon KK modes $(w = 0,\ n \le \mathcal{N})$:

$$\mathcal{N} \sim (m_P R)^{2/3} \Rightarrow H_+ \sim \frac{m_P}{(m_P R)^{1/3}}$$

indefinitely decreases with $R \to \infty$

- Growing tower of superhorizon winding modes $(n = 0,\ w \le \mathcal{N})$:

$$\mathcal{N} \sim \left(\frac{m_P \alpha'}{R}\right)^{2/3} \Rightarrow H_+ \sim m_P \left(\frac{R}{m_P \alpha'}\right)^{1/3}$$

indefinitely decreases with $R \to 0$

$\Lambda \sim m^2_P \Rightarrow \Lambda_{\rm present}$ (fine-tuning the expansion/contraction of X-tra space size?!)

Density Matrix Reloaded:

Minimum set of assumptions — an ultimate equipartition in physical phase space in the form of the microcanonical state of closed quantum cosmology

⬇

- Constraining the ensemble of Λ (and possibly landscape of string vacua)
- A new dark energy mechanism transcending the inflationary and matter-domination stages as a quasi-equilibrium decay of the initial microcanonical state

``Nothing comes from Nothing''

Sidney Coleman: ``There is Nothing rather than Something''

Something (rather than Nothing) comes from Everything