THREE-DIMENSIONAL STATISTICS OF COMETARY APHELIA

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Eight families of comets which obey a modified law of planetary distances for the giant planets have been revealed as a result of the three-dimensional statistical analysis of the distribution of cometary aphelia in the nearest outskirts of the Solar System (at distances from 50 to 4000 a.u.). Three of the families (at distances of 93, 320, and 1100 a.u.) have been studied in detail. An assumption is made concerning the origin of these families of comets.

The paper seeks to reveal statistical regularities in the families of comets. The analysis is based on the data for 438 comets selected from Marsden's catalogue [3]. The group includes 155 short-period comets, 237 long-period comets with a known initial semimajor axis (which they had before entering the planetary system), and 46 long-period comets with an osculating semimajor axis. The following data were taken for calculations: the aphelion distance Q, the perihelion longitude L and latitude B. The direction cosines of perihelia were calculated (this corresponds to the aphelion direction cosines of opposite sign):

$$x = \cos B \cos L\,, \qquad y = \cos B \sin L\,, \qquad z = \sin B\,.$$

A special technique was developed for the statistical analysis of the space distribution of aphelia; this technique gives a possibility to discover toroid-like concentrations of cometary aphelia with their center in the Sun. Just such concentrations are of the greatest interest, as the aphelia of comet families of giant planets display the toroidal concentration in the first approximation, the torus's center lying in the Sun and its principal plane being close to the orbital plane of the planet.

ANALYSIS TECHNIQUE AND RESULTS

The comets were arranged in the increasing order of Q, and a certain number n of comets in succession were chosen. For these comets, the radial (from the Sun) density of aphelia $P = n/\Delta(\log Q)$ was calculated. Then the best central plane (i.e., the plane of the torus) and the root-mean-square deviation S of aphelia from the calculated plane were found by the method of least squares through solving the set of n equations of condition of the form

$$z = ax + by\,. \qquad (1)$$

This technique is nearly identical with that described in [2]. It is easy to show that the quantity S may be interpreted as a certain effective half width of the torus (that has unit radius) in the direction perpendicular to its plane. Then the quantity $1/S$ will be proportional to the cross density of aphelia in the torus (strictly speaking, such a quantity will be $1/(SQ)$ and not $1/S$). An important distinction from paper [2] is that such calculations were carried out for all the groups of n comets in succession. This permitted to plot the radial and cross densities of aphelia as functions of $\log Q$ (Fig. 1). The number n was chosen to be 9, and the only selection criterion was the most efficient isolation of the families of the giant planets. The dotted lines mark the distances that correspond to the semimajor axes of four giant planets. Figure 1,*a* shows the variance minima corresponding to the families B, D, and F (they will be discussed below). Peaks are obvious in Fig. 1,*b*, they correspond to the comet families of Jupiter, Saturn (it practically blends with the Jupiter family), Uranus, and Neptune. Several peaks of the radial density of aphelia in the transneptunian region attract

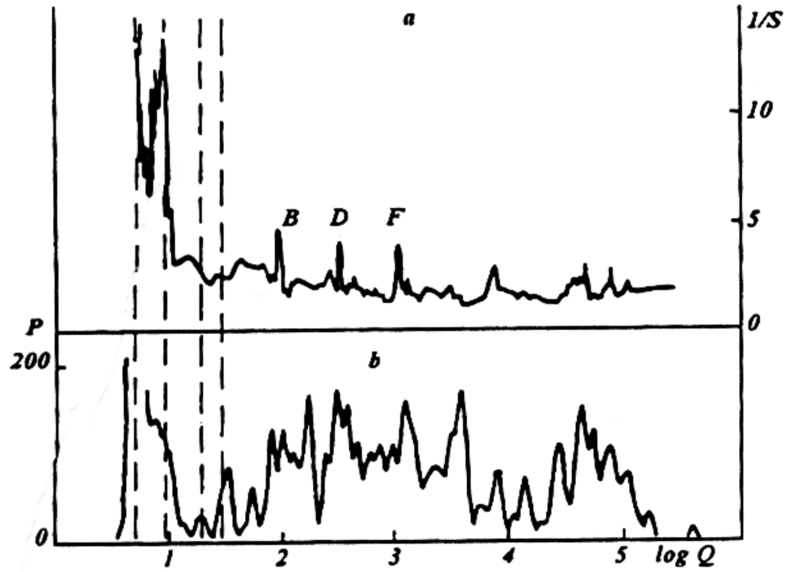

Fig. 1. Distributions of density of cometary aphelia: a) cross density, b) radial density (both are smoothed by the moving-average technique).

attention. It is quite obvious that the maxima of the radial density of comet families of the giant planets should obey the law of planetary distances. At large enough distances, this law may be represented in a linear form:

$$\log a = b + cN ,$$

and, using the method of least squares, we easily obtain for four giant planets:

$$\log a = 0.47 + 0.259N . \tag{2}$$

Here a is the semimajor axis of the orbit, $N = 1$ for Jupiter, $N = 2$ for Saturn, etc. The root-mean-square deviation for one planet is $\sigma = 0.028$.

Figure 2 (an enlarged fragment of Fig. 1,b) shows the radial density of aphelia. Vertical bands correspond to the distances $\log a \pm \sigma$ that were calculated by formula (2) for $N = 1, 2, 3, 4$. This relationship was extrapolated for $N = 5, 6, 7, 8$. The bands coincide clearly enough with the density maxima of the distribution of aphelia both for the families of the giant planets and in the transneptunian region.

In Fig. 3, the density of aphelia in corresponding tori is plotted against log Q. For every point in the plot, the position in space of the plane of the corresponding torus was calculated. The density of aphelia, W, in the general form is calculated approximately with the formula

$$W = \frac{n}{\Delta r \cdot \Delta z \cdot 2\pi Q} ,$$

where Δr is the torus radial thickness, Δz is the lateral thickness. In our specific case $\Delta r = \Delta(\log Q)$, $\Delta z = 2SQ$, and then

$$W = \frac{P}{4\pi Q^2 S} . \tag{3}$$

In Fig. 3, we show the quantity $W' = P/S$ related to log Q in order to present in a single plot both the nearest families and very distant families whose W is very small because of large values of Q. The same figure

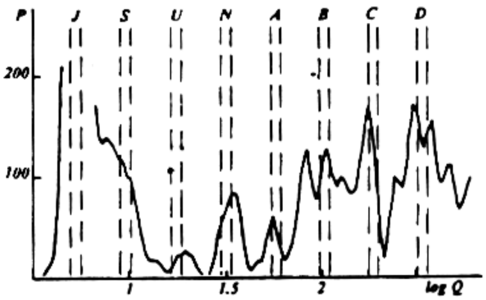

Fig. 2

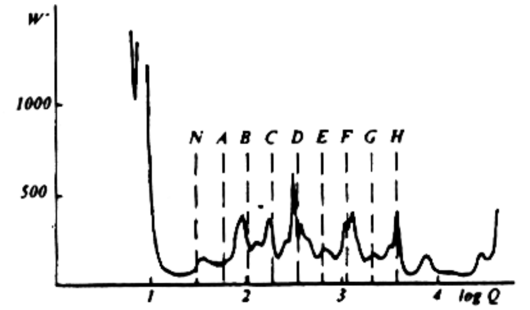

Fig. 3

Fig. 2. Radial density of aphelia.

Fig. 3. Density of toroidal distribution of comet families.

Table 1

Distances of Comet Families

N	Family	log a	log Q	Q, a.u.	N	Family	log a	log Q	Q, a.u.
1	Jovian	0.72	0.73	5.3	7	*C*	2.28	2.25	180
2	Saturnian	0.98	0.95	9.0	8	*D*	2.54	2.50	320
3	Uranian*	1.28	1.29	19.5	9	*E*	2.80	2.81	650
4	Neptunian	1.48	1.56	36	10	*F*	3.06	3.05	1100
5	*A**	1.76	1.75	56	11	*G*	3.32	3.35	2200
6	*B*	2.02	1.97	93	12	*H*	3.57	3.58	3800

Note. The data are taken from Fig. 2. For the families of the giant planets, the column log a gives the true value of the semimajor axis logarithm for the planet, and the value calculated by formula (2) for other families.
*) Density peak is not seen in Fig. 3 because of a small number of comets ($n \ll 9$) in the family.

depicts lines which correspond to the distances for $N = 4, 5, 6, \ldots, 12$. A very good agreement is seen immediately between the W maxima and the distances calculated with formula (2). All these families of comets were lettered by the author: A, B, ..., H. Table 1 gives the data on them. Almost all the deviations lie within limits of $\sigma = 0.03$. The families A and B correspond approximately to two transplutonian families which were marked as the most probable in [2]. It is interesting that, beginning with Uranus, strong and weak density peaks alternate, and the peaks E and G are very weak.

As can be seen from Fig. 1, the families B, D, and F have a distinct variance minimum (the maximum of $1/S$ in the plot). This enables to obtain the information not only on their distances from the Sun, but also on the position of their planes in space, which can be defined by the inclination of the plane to the ecliptic, i, and the longitude of its ascending node, Ω. These quantities can be easily obtained, provided the coefficients a and b from expression (1) are known:

$$i = \arccos \frac{1}{\sqrt{a^2 + b^2 + 1}}, \quad \Omega = \arctan\left(-\frac{a}{b}\right).$$

If $b < 0$, then $\Omega + 180°$ should be taken. Table 2 gives the values obtained. Here n is the number of comets in the family, S_0 is the standard deviation of the family aphelia from the ecliptic plane. The table includes also the data on the families of Jupiter and Saturn. In spite of small inclinations i of their orbits to the ecliptic (which gives a large error in the determination of Ω), a rather good agreement with the orbital elements of these planets is observed, especially for Saturn. The family of Uranus in too small in number ($n = 4$-5), and this does not permit to analyse it even with a smallest degree of certainty. As for the Neptunian

Table 2

Data on the Variance Minima

Family	Interval of log Q variations	i, deg	Ω, deg	S	S_0	n
Jovian	0.60—0.88	1.7	26.0	0.09	—	114
Saturnian	0.88—1.15	0.5	95.8	0.13	—	21
Saturnian^1	—	0.8	69.6	0.08	—	19
B	1.93—2.03	35.3	306.6	0.21	0.58	10
B^2	—	33.4	310.0	0.14	0.54	9
B^3	—	35.9	305.7	0.09	0.57	8
D	2.49—2.54	35.6	308.5	0.25	0.49	9
D^4	—	33.5	298.4	0.16	0.49	8
F	2.99—3.09	45.1	293.6	0.26	0.58	11

Note. 1) excluding two comets (Tuttle and Du Toit) which deviate from the common plane more than by $2S$; 2) excluding comet 1932 I (deviation of approximately $2S$); 3) excluding comets 1932 I and 1874 IV (deviation of approximately $2S$ relative to B^1); 4) excluding comet 1947 IV (deviation of approximately $2S$).

family, it has considerable peculiarities, and paper [1] deals with some attempts to explain them. For the families which are being considered here, we have $S_0 \geq 2S$, which means that specific planes other than the ecliptic plane exist with a high degree of certainty. Attention should be drawn to the closeness of orbital elements of the families B, D, and F (especially B and D). We give the membership of these families (in ascending order of Q):

B - 1840 IV, 1932 I, 1979 X, 1932 V, 1874 IV, 1931 III, 1955 III, 1941 II, 1861 II, 1861 I;

D - 1854 V, 1987 XXIX, 1858 VI, 1911 II, 1911 V, 1909 I, 1769, 1947 IV, 1926 I;

F - 1941 IV, 1890 IV, 1957 V, 1975 XII, 1913 IV, 1849 III, 1988 III, 1987 XXX, 1930 I, 1927 IX, 1982 VI.

CONCLUSIONS

As a results of the statistical three-dimensional analysis of the distribution of cometary aphelia, regular structures which obey a modified law of planetary distances for the giant planets have been discovered in the near outskirts of the Solar System. We can only speculate on the nature of these formations. Their obeying the law of planetary distances indicates their genetic connection with the Solar System, but their large inclination to ecliptic ($i \approx 30$-$45°$) makes one doubt the reality of this connection, especially as the planes of these families lie close enough to the galactic plane ($i' = 62.6°$, $\Omega' = 282.2°$), particularly the plane of the farthest of three families studied (F). This speaks for the galactis origin of these formations. It is possible, nevertheless, that the whole observed family structure has formed as a result of interaction between objects in the outskirts of the Solar System and galactic objects, huge molecular complexes, for example.

It is also not clear what real objects are at the distances corresponding to the families. The existence of major planets there is too doubtful, as the deviations of aphelia from the optimum planes of the families are too great (e.g., the deviations of aphelia reach 1000 a.u. in the family F). In addition, giant planets (of the Neptune type) at the distances of the families A and B would have been certainly discovered — their brightness would be 10.5^m and 12.7^m, respectively. However, there is a possibility that even a planet of a small mass at such great distances from the Sun would be able to have an appreciable family of comets. The author believes the following hypothesis to be more likely: at the distances of these families there are belts of small icy bodies composed of the protoplanetary nebula matter that remained from the times of formation of the Solar System. Their large inclinations may be explained by an effect of the Galaxy.

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